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Debt maturity without commitment

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ABSTRACT

How does sovereign risk shape the maturity structure of public debt? We consider a government that balances benefits of default, due to tax savings, and costs, due to output losses. Debt issuance affects subsequent default and rollover decisions and thus, current debt prices. This induces welfare costs beyond the consumption smoothing benefits from the marginal unit of debt. The equilibrium maturity structure minimises these welfare costs. It is interior with positive gross positions and shortens during times of crisis and low output, consistent with empirical evidence.

1. Introduction

Sovereign borrowers carefully choose the maturity structure of public debt. This is difficult to reconcile with a benchmark model in which the equilibrium allocation concurs with net financial positions while gross positions and the maturity structure are indeterminate (Modigliani and Miller, 1958; Barro, 1974, 1979). Existing theories (discussed below) point to a role for debt maturity in avoiding “bad” equilibria with rollover crises or in improving insurance possibilities for the government. However, the predictions of these theories are not robust or not in line with the empirical evidence, leading Faraglia et al. (2010, p. 835) to conclude that “[w]e remain in search of a plausible theory of debt management.”

In this paper, we pursue an alternative explanation for sovereign borrowers' choice of maturity, arguing that lack of commitment paired with social costs in the wake of a default undermines the neutrality of the maturity structure. Focusing on these two factors appears natural given the pervasiveness of limited contract enforceability in sovereign lending and the significant social costs in the aftermath of defaults.[1] The implications of this alternative explanation turn out to be broadly consistent with the evidence.

We consider a benevolent government issuing real non-contingent debt of different maturities to international investors. Successive governments decide whether, and to what extent, to honour maturing debt. They also choose the level of

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[1] See, for example, Eaton and Fernandez (1995) for an overview over the literature and Reinhart and Rogoff (2004), Sturzenegger and Zettelmeyer (2006, pp. 49–52) or Panizza et al. (2009) for a discussion of the costs of sovereign defaults.

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taxation and debt issuance to finance debt repayment. The desire to redistribute from foreign bondholders to domestic taxpayers provides an ex-post incentive for the government to default.[2] The wish to avoid the costs of a default which take the form of output losses provides a counteracting incentive to repay. Both bondholders and the government form rational expectations. The price of a debt maturity therefore reflects its expected repayment rate, and government policy is subgame perfect.

In equilibrium, the risk-adjusted returns on short- and long-term funding are identical and the maturity structure is determined on the demand side. It is critically shaped by revenue effects on infra-marginal units of debt. A direct consequence of lack of commitment, these revenue effects arise because debt issuance affects default and rollover choices of subsequent governments and thus, prices of the maturities currently issued.[3] When considering whether to sell additional debt, the government weighs the consumption smoothing benefits from the marginal unit of debt against the costs due to the revenue effects on infra-marginal units.

To understand the implications for the maturity structure, we first consider a version of the model that can be solved in closed form. Under a regularity condition, the revenue effects on infra-marginal units of debt relative to the consumption smoothing benefits from the marginal unit are convex. As a consequence, the equilibrium maturity structure is interior and smoothes cost–benefit ratios across maturities, in parallel to tax rates in optimal fiscal policy problems that smooth tax distortions (Barro, 1979; Lucas and Stokey, 1983). While the "tax smoothing" prescription yields a Ramsey tax sequence and associated net government debt sequence that minimises the detrimental effects of tax distortions, the "maturity smoothing" prescription yields gross positions of each maturity (and thus, net debt positions and taxes) that maximise welfare when lack of commitment is binding. These gross positions are positive and the maturity structure generally is tilted towards the long end.

Lack of commitment manifests itself twofold: In the ex-post optimal choice of repayment rate which causes the revenue effects on infra-marginal units in previous periods; and in the ex-post optimal choice of new debt issuance which affects the size of these revenue effects. Due to the convexity of the cost-benefit ratio mentioned earlier, a higher amount of inherited, outstanding debt generally leads the government to reduce its issuance of short-term debt (the second manifestation). Intuitively, if the stock of outstanding debt and thus, future default risk is already high then the revenue effects from fresh debt issuance are large relative to the consumption smoothing benefits. As a consequence of this restraining effect on future short-term debt issuance, long-term debt issuance increases the amount of debt maturing in the long term by less than one-to-one, in contrast with one-period debt issuance which results in a one-to-one increase of the amount of debt coming due in the subsequent period. Ceteris paribus, long-term debt issuance then has a smaller impact than short-term debt issuance on debt coming due and thus, default risk and prices (the first manifestation). This smaller price impact is reflected in smaller revenue effects on infra-marginal units and thus, an advantageous cost-benefit ratio of long-term debt. As a result, the equilibrium maturity structure is tilted towards the long end.

Higher levels of debt worsen the cost–benefit ratio of long-term debt because they lessen the extent to which a successor government's debt issuance responds to the amount of outstanding debt. High debt-to-GDP ratios therefore go hand in hand with a more balanced maturity structure. This has implications for the government's portfolio over the cycle: In periods of low output and high marginal utility, total debt issuance increases and the maturity structure shortens; during booms, total debt issuance falls and the maturity structure lengthens. Output volatility tends to lengthen the equilibrium maturity structure as well. When output is low and marginal utility high, governments find it optimal to issue more debt. Since this increases the risk of default in the future, output is positively correlated with the price of outstanding debt. Long-term debt therefore provides a useful hedge for the government and more long-term debt is issued if the environment is riskier.

The picture that emerges from the model's closed-form solutions is one of an interior maturity structure with positive gross positions, in line with the empirical evidence, but in contrast with predictions from models that stress the role of the maturity structure in completing markets or avoiding bad equilibria with rollover crises. The model predicts a shortening of the maturity structure when debt issuance is high, in line with evidence summarised by Rodrik and Velasco (1999); around times of low output "crises", consistent with the evidence reported by Broner et al. (2013); and in periods with low output volatility.

Simulations of the general model that needs to be solved numerically corroborate the theoretical predictions and show that they are robust. The simulation results also replicate Arellano and Ramanarayanan's (2012) and Broner et al.'s (2013) empirical finding that bond spreads rise when the maturity structure shortens.

Being unable to commit, the government in the model cannot force its successors to pay a certain rate of return, including zero. As a consequence, there is no mechanical link between haircuts on maturing and outstanding debt. In the data, some default episodes feature simultaneous default on maturing and outstanding debt while others do not.[4] To allow the model to capture these diverse episodes under the maintained assumption of no commitment we introduce a "cross-default shock." In the event of a default on maturing debt, this shock makes it costless for subsequent governments to

[2] The incentive to default might alternatively derive from the government's desire to transfer funds from the private to the public sector, in order to avoid tax distortions. Focusing on the redistributive motive is attractive for two reasons. On the one hand, it is empirically relevant. On the other hand, abstracting from tax distortions allows to disregard a second source of time inconsistency, related to the optimal timing of taxes (Lucas and Stokey, 1983).

[3] Revenue effects on currently issued debt are fully internalised, in contrast to the dilution of outstanding debt whose price is a bygone.

[4] For example, Zettelmeyer et al. (2013) report haircuts for the 2012 debt restructuring of Greece that differ widely between short-term (maturing) and outstanding bonds.

default on currently outstanding debt and as a consequence, it generates debt acceleration and cross-default as an equilibrium outcome rather than a choice by the government in place. The introduction of cross-default shocks changes debt prices and revenue effects on infra-marginal units but except for a knife-edge case, the central implications of the model turn out to be robust.

The remainder of the paper is structured as follows. Section 2 reviews related literature and Section 3 presents the model. The government's principal trade-offs—related to the debt repayment and debt issuance choices—are discussed in Section 4. Sections 5 and 6 contain analytical and numerical characterisations of equilibrium, respectively, and Section 7 concludes. The online appendix contains derivations and discussions of extensions.

2. Related literature

To motivate an optimal maturity structure, several authors argue that short-term debt renders a country vulnerable to rollover crises while long-term debt reduces such vulnerability (Calvo, 1988; Alesina et al., 1990; Giavazzi and Pagano, 1990; Rodrik and Velasco, 1999; Cole and Kehoe, 2000). Jeanne (2009) argues that vulnerability may be in the borrower's interest if it helps to check commitment problems. But Chamon (2007) shows that a simple mechanism eliminates the coordination failure associated with rollover crises and Phelan (2004) draws a distinction between the maturity of debt and the sequencing of debt rollovers which matters for such crises. Broner et al. (2013) stress supply side determinants of the maturity structure. In their model, lenders are risk averse and exposed to the price risk of long-term debt; higher quantities of long-term debt drive up term premia and thus, the cost of long-term funding.

Bohn (1990) emphasises the insurance benefits of non-state contingent long-term debt and Barro (1995) discusses maturity choice for hedging purposes. Angeletos (2002) and Buera and Nicolini (2004) demonstrate that a sufficiently rich maturity structure of non-contingent bonds can serve as substitute for state-contingent debt (see also Gale, 1990).[5] But the quantitative implications of this "complete market approach" are completely at odds with the data. Nosbusch (2008) shows that the hedging benefits of all but a few maturities are very minor.

Brunnermeier and Oehmke (2013) study the maturity choice of a corporate borrower whose ability (but not willingness) to repay is in doubt and updated over time (see also Diamond, 1991). Short-term debt dilutes long-term debt and accordingly, the equilibrium maturity structure is short term. In our model, incentives to dilute are absent because debt issuance reduces the market value of outstanding debt without benefiting new lenders.

Starting with Eaton and Gersovitz (1981), credibility problems and the associated difficulty to sustain borrowing take centre stage in the sovereign debt literature. Calvo and Guidotti (1990) and Missale and Blanchard (1994) consider the role of the maturity structure of nominal debt for the government's incentive to engineer surprise inflation. Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) study quantitative sovereign debt models with real long-duration debt. They find that the hedging characteristics of long-duration debt and the effect of dilution on bond prices help improve the model's fit relative to the case with short-term debt.

In independent work, Arellano and Ramanarayanan (2012) analyse a model similar to ours, with both short- and long-term debt (in their case, a perpetuity). They assume more complicated default costs than we do and impose that default necessarily triggers cross-default such that the price of short-term debt fluctuates more strongly than the price of long-term debt. In contrast to these authors, we characterise equilibrium in closed form and prove that short-term debt issuance decreases in the quantity of outstanding debt, which is crucial for the equilibrium maturity structure. In simulations, we establish the robustness of this result while Arellano and Ramanarayanan (2012, p. 202) argue that the policy function is increasing. We also prove that the implications of their assumption that default necessarily triggers cross-default may not be robust.

Aguiar and Amador (2013) also study a model with short-term debt and a perpetuity. De-leveraging lowers the risk premium (reflecting social costs of default) on short-term debt, unlike the yield on the perpetuity which is sunk. To implement the optimal de-leveraging policy subject to lack of commitment the sovereign repays short-term debt and does not engage in the long-term debt market as this could distort future de-leveraging incentives, moving long-term bond prices against the sovereign. In our model, long-term debt has finite maturity and fresh long-term debt is issued along the equilibrium path.

3. Model

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. The small open economy is inhabited by a representative taxpayer and a benevolent government that interacts with foreign investors. The government levies taxes, chooses the repayment rate on maturing debt, $r_t \in [0, 1]$, and issues a finite set of zero-coupon debt maturities. Debt *issued* at time t and maturing in period $j > t$ is denoted by $\iota_{t,j}$ while the *stock* of debt at the beginning of period t that will mature in period $j \geq t$ is denoted by $b_{t,j}$.

Without loss of generality, government spending other than debt repayment is normalised to zero. Throughout the analysis we let the maximal maturity be two periods, but this restriction can be relaxed at the cost of increased complexity. The government's debt issuance in period t therefore is given by $\iota_t \equiv (\iota_{t,t+1}, \iota_{t,t+2})$ and at the beginning of the period, the

[5] Lucas and Stokey (1983) show that state-contingent debt *and* a sufficiently rich maturity structure may render a Ramsey fiscal policy time consistent.

stocks of maturing and of outstanding debt are given by $(b_{t,t}, b_{t,t+1})$. Debt issuance is bounded, for instance by natural borrowing limits, $\iota_t \in \mathcal{I}$.

3.1. Private sector

Taxpayers do not save nor borrow. They have time- and state-additive preferences over consumption, c_t, with strictly increasing and concave felicity function $u(\cdot)$ and discount factor $\delta \in (0,1)$. Welfare of taxpayers in period t is given by

$$\mathbb{E}\left[\sum_{j \geq t} \delta^{j-t} u(c_j) | s_t, r_t, \iota_t\right], \tag{1}$$

where s_t denotes the state (to be specified below). We usually adopt the short-hand notation $\mathbb{E}_t[\cdot]$ for the conditional expectation $\mathbb{E}[\cdot | s_t, r_t, \iota_t]$.

Foreign investors are competitive, risk neutral and require a risk free gross interest rate $\beta^{-1} > 1$. Since taxpayers do not save, all government debt is held by foreign investors. To guarantee positive debt positions, we assume $\delta < \beta$.

The assumption that the sets of taxpayers and investors do not "overlap" is unimportant but simplifies the analysis; modelling a mixed rather than concentrated ownership structure of debt would require a theory of how this ownership structure is determined in equilibrium.[6]

3.2. Government

The government maximises the welfare of taxpayers.[7] Crucially, it cannot commit its successors (or future selves). In each period, the government therefore chooses debt issuance as well as the uniform (pari passu) repayment rate on all maturing debt, $b_{t,t}$. Taxes follow residually from the government's dynamic budget constraint.

3.3. Default costs

A government default—a situation where the repayment rate falls short of unity—triggers temporary income losses for taxpayers. More specifically, a default in period t triggers an income loss $L_t \geq 0$ where L_t is the realisation of an i.i.d. random variable with cumulative distribution function $F(\cdot)$ and associated density function $f(\cdot), f(L) > 0$ for all $L \geq 0$. The government learns about the realisation of L_t at the beginning of the period, before choosing its policy instruments. Letting y_t denote the realisation of the (potentially) stochastic output process in period t and $\mathbf{1}_{[x]}$ the indicator function for event x, pre-tax income of taxpayers is given by $y_t - \mathbf{1}_{[r_t < 1]} L_t$.

The assumption of temporary rather than persistent income losses is motivated by two considerations. First, temporary default costs constitute a natural benchmark.[8] Second, and more importantly, the assumption of temporary losses is more plausible. While permanent exclusion from trade or credit markets and other forms of long-term punishment may serve as threat points they are unlikely to materialise in equilibrium if the parties can renegotiate.[9] Empirical evidence supports the notion of temporary rather than permanent default costs as well as the notion that these costs arise in the form of output losses (see, for example, Panizza et al., 2009). Gelos et al. (2011) report that defaults, if resolved quickly, do not preclude sovereigns from re-accessing financial markets.

3.4. Cross-default and debt accumulation

Being unable to commit, the government cannot force its successors to pay a certain rate of return, including zero. This implies that a government may not *directly* default on outstanding debt that matures in the future. Indirectly, however, such a cross-default may occur. Let the random variable ξ_t take the value 1 with probability $\pi \in [0,1]$ and the value 0 with probability $1-\pi$. If $\xi_t = 1$, then a default on debt maturing in period t (carrying income losses L_t) reduces to zero the costs for subsequent governments of defaulting on debt outstanding in period t and maturing in period $t+1$. If $\xi_t = 1$, therefore, a default on maturing debt also triggers an equilibrium devaluation of outstanding debt since investors know that future governments will find it in their interest to default on the latter.[10] Accordingly, the law of motion for the debt maturities is

[6] The government's default decision depends on the ownership structure of debt relative to the distribution of tax burdens across the population, see below. Changes in the ownership structure therefore affect the default decision ex post and thus, investment decisions ex ante.

[7] If the government maximised a weighted average of taxpayers' and investors' welfare and attached a sufficiently large weight to the welfare of investors, interior repayment rates might result, in contrast to what follows. If the government attached a strictly positive weight to the welfare of investors and if investors were risk averse, investor wealth would constitute a state variable, in contrast to what follows.

[8] The results of the paper remain valid under the assumption of permanent default costs if these costs do not interact with future debt issuance and repayment decisions. This is the case, for example, if the utility function is linear.

[9] Suppose, for example, that upon defaulting the sovereign enters into negotiations with creditors. These negotiations last one period, generating income losses L_t, and result in a settlement where lenders secure a repayment rate $\overline{r}_t$. The analysis in this paper is consistent with this interpretation; for simplicity, it sets $\overline{r}_t = 0$.

[10] Alternatively, cross-default can be interpreted as a debt buyback at very low prices that reflect equilibrium expectations of subsequent governments' default decisions.

given by

$$b_{t+1,j} = b_{t,j}(1-\mathbf{1}_{[r_t<1\ \&\ \xi_t=1]})+\iota_{t,j}, \quad j=t+1,t+2. \tag{2}$$

Eq. (2) states that the stock of debt outstanding at the beginning of period $t+1$ and maturing in period j reflects the $(j-t)$–period debt issued at time t as well as the debt outstanding in period t and maturing in period j. However, in case of a cross-default ($r_t<1$ and $\xi_t=1$) the latter component vanishes. The random variables y_t, ξ_t and L_t are pairwise independent.

While the "cross-default shock" ξ_t allows the model to reconcile the assumption of no commitment with the equilibrium occurrence of cross-default, its stochastic specification captures the different extent of cross-default and restructuring across default episodes, presumably due to varying incentives of the sovereign and certain creditors to delay or prevent acceleration. The latter include lenders that seek to avoid an immediate deterioration of their balance sheet as otherwise implied by mark-to-market regulation, or the government itself if it repurchased debt on secondary markets. In addition to such institutional factors, the structure of government debt securities also might affect the extent of cross-default. For example, zero coupon bonds might be less exposed to cross-default risk than the coupon payments of a single console.

3.5. Equilibrium

Apart from time (in the finite-horizon case), the state in this economy is given by the realisations (y_t, ξ_t, L_t) as well as the quantities of maturing and outstanding debt:

$$s_t = (y_t, \xi_t, L_t, b_{t,t}, b_{t,t+1}). \tag{3}$$

If output does not follow a Markov chain then the state variable y_t may have to be augmented by past or future values of output. The former is the case, for example, if output follows an AR(2) process, the latter if output is deterministic and time varying. Throughout the paper, we exclude non-fundamental state variables of the type sustaining trigger strategies.

Denote by $q_{t,j}(s_t, r_t, \iota_t)$ the price of debt issued in period t state s_t and maturing in period j if the government implements the policy (r_t, ι_t). All governments in period t and earlier take the price functions $q_{t,j}(\cdot)$ as given when choosing their policies. The deficit in a period is defined as the market value of newly-issued debt:

$$d_t(s_t, r_t, \iota_t) \equiv \sum_{j=t+1}^{t+2} \iota_{t,j}\ q_{t,j}(s_t, r_t, \iota_t). \tag{4}$$

Summarising the model's elements, the state s_t as well as the government's policy choices r_t and ι_t determine equilibrium prices $q_{t,j}(s_t, r_t, \iota_t)$, the equilibrium deficit $d_t(s_t, r_t, \iota_t)$ and, from the government's dynamic budget constraint, equilibrium taxes $b_{t,t}r_t - d_t(s_t, r_t, \iota_t)$. From the taxpayers' static budget constraint, equilibrium consumption equals pre-tax income (output minus default costs) net of taxes that is,

$$c_t(s_t, r_t, \iota_t) = y_t - \mathbf{1}_{[r_t<1]}L_t - b_{t,t}r_t + d_t(s_t, r_t, \iota_t). \tag{5}$$

Intuitively, both default costs and debt repayment decrease consumption, by reducing the amount of resources in the economy or transferring more of them to foreigners. At the same time, issuing fresh debt increases consumption because this allows to cut taxes.

Let $G_t(s_t)$ denote the value of the government's program conditional on state s_t. An *equilibrium* is given by price functions $\{q_{t,j}(\cdot)\}_t$, value functions $\{G_t(\cdot)\}_t$, and policy functions $\{r_t(\cdot), \iota_t(\cdot)\}_t$ (of s_t) such that

(i) conditional on the price functions, the value and policy functions solve

$$G_t(s_t) = \max_{r_t\in[0,1],\iota_t\in\mathcal{I}} u(y_t - \mathbf{1}_{[r_t<1]}L_t - b_{t,t}r_t + d_t(s_t, r_t, \iota_t)) + \delta\mathbb{E}_t[G_{t+1}(s_{t+1})]$$
$$\text{s.t.} \quad (2) \quad \text{for all} \quad s_t, t; \tag{6}$$

(ii) the price functions reflect rational expectations by investors

$$q_{t,t+1}(s_t, r_t, \iota_t) = \beta\mathbb{E}_t[r_{t+1}(s_{t+1})], \tag{7}$$

$$q_{t,t+2}(s_t, r_t, \iota_t) = \beta^2\mathbb{E}_t\left[(1-\mathbf{1}_{[r_{t+1}(s_{t+1})<1\ \&\ \xi_{t+1}=1]})r_{t+2}(s_{t+2})\right]$$
$$\text{s.t.} \quad (2) \quad \text{for all} \quad s_t, r_t\in[0,1], \iota_t\in\mathcal{I}, t. \tag{8}$$

The second condition states that investors earn the required rate of return, independently of the particular choice of government policy. This requires that the price of short-term debt in period t be proportional to the expected repayment rate at time $t+1$; and the price of long-term debt be proportional to the expected repayment rate in period $t+2$ conditional on no cross-default in the intermediate period.

The insulation of investors' average return from the effects of government policy contrasts with the exposure of domestic taxpayers whose disposable income and consumption depend on taxes and income losses in the wake of defaults. According to the first condition, the benevolent government chooses the repayment rate and debt issuance in order to minimise the detrimental effects of taxes and income losses, taking into account how subsequent governments respond ex-post optimally to these choices.

Along the equilibrium path, the government's policy choices are (as yet unknown) functions of the state, $r_t(s_t)$ and $\iota_t(s_t)$. Since policy affects prices and other variables, equilibrium prices, the equilibrium deficit and marginal utility along the equilibrium path all are (as yet unknown) functions of the state as well, $q_{t,j}(s_t, r_t(s_t), \iota_t(s_t))$, $d_t(s_t, r_t(s_t), \iota_t(s_t))$ and $u'(y_t - \mathbf{1}_{[r_t(s_t)<1]}L_t - b_{t,t}r_t(s_t) + d_t(s_t, r_t(s_t), \iota_t(s_t)))$. To simplify the notation, we let $S_t \equiv (s_t, r_t(s_t), \iota_t(s_t))$ and adopt the shorthand notation $q_{t,t+j}(S_t)$, $d_t(S_t)$ and $u'(S_t)$ for these objects along the equilibrium path.

4. Trade-offs

We now turn to a characterisation of the government's principal trade-offs, related to the choice of debt repayment and debt issuance.

4.1. Debt repayment

Consider first the government's choice of repayment rate, r_t. Since the marginal cost of reducing r_t equals zero if $r_t < 1$, the optimal repayment rate equals either zero or unity. The threshold value of L_t at which the repayment rate changes depends on the cross-default shock. If $\xi_t = 0$, the government's repayment choice maximises $y_t - \mathbf{1}_{[r_t<1]}L_t - b_{t,t}r_t + d_t(s_t, r_t, \iota_t)$ and therefore derives from a comparison of L_t versus $b_{t,t}$. If $\xi_t = 1$, in contrast, then the choice of repayment rate affects the evolution of outstanding debt (see Eq. (2)). As a result, the default threshold is lower and the government defaults more aggressively than in the case without cross-default shock. Formally,

$$r_t(s_t) = \begin{cases} 1 & \text{if } L_t - b_{t,t} \geq \alpha_t(y_t, b_{t,t}, b_{t,t+1}) \cdot \xi_t \\ 0 & \text{if } L_t - b_{t,t} < \alpha_t(y_t, b_{t,t}, b_{t,t+1}) \cdot \xi_t \end{cases}. \tag{9}$$

The function $\alpha_t(\cdot)$ is defined by the condition that the government's value when repaying be equal to the value when defaulting conditional on $\xi_t = 1$. Absent any outstanding debt, the value of $\alpha_t(y_t, b_{t,t}, 0)$ equals zero. If the stock of outstanding debt is strictly positive, in contrast, then $\alpha_t(y_t, b_{t,t}, b_{t,t+1}) > 0$. (The online appendix contains derivations.)

Condition (9) states that a government defaults when the income losses L_t are relatively small. This is consistent with the notion that governments tend to default when the political costs—specifically income losses of pivotal pressure groups—are low. Governments also tend to default when economic activity is depressed (Borensztein et al., 2006; Tomz and Wright, 2007). The model is consistent with this fact as well if it is slightly extended to include direct default costs for the government in addition to the income losses for taxpayers.[11] As discussed in the online appendix, corner solutions for the optimal repayment rate follow under more general assumptions about default costs than those invoked here.

Eq. (9) pins down expected repayment rates. Conditional on all period-t state variables except L_t, the probability of repayment equals $1 - F(b_{t,t})$ if $\xi_t = 0$, and it equals $1 - F(b_{t,t} + \alpha_t(y_t, b_{t,t}, b_{t,t+1}))$ if $\xi_t = 1$. For brevity, we write $1 - F_t^{\xi=0}$ and $1 - F_t^{\xi=1}$ for these two probabilities and similarly, we write $f_t^{\xi=0}$ and $f_t^{\xi=1}$ for the corresponding density functions. Conditional on all period-t state variables except L_t and ξ_t, the expected repayment rate then equals $\pi(1 - F_t^{\xi=1}) + (1-\pi)(1 - F_t^{\xi=0})$, or $1 - F_t$ for short. That is, the expected repayment rate equals the sum of the probability weighted conditional expected repayment rates with and without a cross-default shock.

From equilibrium condition (8), equilibrium debt prices reflect expected repayment rates. Accordingly, short-term debt costs

$$q_{t,t+1}(s_t, r_t, \iota_t) = \beta \mathbb{E}_t[r_{t+1}(s_{t+1})] = \beta\left(\pi \mathbb{E}_t[1 - F_{t+1}^{\xi=1}] + (1-\pi)(1 - F_{t+1}^{\xi=0} | s_t, \iota_t)\right) \tag{10}$$

and the price of long-term debt is given by

$$\begin{aligned} q_{t,t+2}(s_t, r_t, \iota_t) &= \beta^2 \mathbb{E}_t[(1 - \mathbf{1}_{[r_{t+1}(s_{t+1})<1 \ \& \ \xi_{t+1}=1]})r_{t+2}(s_{t+2})] \\ &= \beta \mathbb{E}_t\left[(1 - F_{t+1})q_{t+1,t+2}(S_{t+1}) | L_{t+1} = \infty\right] + \beta(1-\pi)\mathbb{E}_t\left[\int_0^{b_{t+1,t+1}} q_{t+1,t+2}(S_{t+1}) \, \mathrm{d}F(L_{t+1}) | \xi_{t+1} = 0\right]. \end{aligned} \tag{11}$$

The price of short-term debt in (10) reflects the probability (conditional on s_t, r_t and ι_t) that income losses L_{t+1} are sufficiently high to discourage default. This probability depends on the risk of a cross-default shock, π, because governments default more aggressively when $\xi_{t+1} = 1$.

[11] If default triggers costs K to the government in addition to the income losses for taxpayers, the default decision (in the case with $\xi_t = 0$) reduces to $r_t = 1$ iff $u(y_t - b_{t,t} + d_t) \geq u(y_t - L_t + \tilde{d}_t) - K$. Strict concavity of $u(\cdot)$ implies that low income levels render a default more likely. Alternatively, the result can be generated by assuming appropriate correlation between L_t and y_t.

The price of long-term debt depends on the joint probability of no cross-default in period $t+1$ and sufficiently high income losses L_{t+2}. Equivalently, it reflects the expected equilibrium price of short-term debt conditional on no cross-default; for conditional on no cross-default, the price of outstanding long-term debt and newly issued short-term debt coincide, due to pari passu. The price of long-term debt thus equals β times the sum of four components: First, the price of short-term debt in the subsequent period when $\xi_{t+1}=1$ and $L_{t+1} \geq b_{t+1,t+1}+\alpha_{t+1}(y_{t+1}, b_{t+1,t+1}, b_{t+1,t+2})$, weighted by the corresponding probability $\pi(1-F_{t+1}^{\xi=1})$. Second, the price of short-term debt when $\xi_{t+1}=0$ and $L_{t+1} \geq b_{t+1,t+1}$, weighted by the probability $(1-\pi)(1-F_{t+1}^{\xi=0})$. Third, the average price of short-term debt when $\xi_{t+1}=0$ and $L_{t+1} < b_{t+1,t+1}$. And fourth, the probability weighted price of short-term debt when $\xi_{t+1}=1$ and $L_{t+1} < b_{t+1,t+1}+\alpha_{t+1}(y_{t+1}, b_{t+1,t+1}, b_{t+1,t+2})$ which equals zero. The term that involves $1-F_{t+1}$ in Eq. (11) combines the first two components.[12]

In the special case where $\pi=0$ such that no cross-default risk is present, Eqs. (10) and (11) simplify to

$$q_{t,t+1}(s_t, r_t, \iota_t) = \beta(1-F(b_{t,t+1}+\iota_{t,t+1})), \tag{12}$$

$$q_{t,t+2}(s_t, r_t, \iota_t) = \beta^2 \mathbb{E}_t[1-F(\iota_{t,t+2}+\iota_{t+1,t+2}(s_{t+1}))]. \tag{13}$$

Intuitively, the price of each maturity is decreasing in its quantity because higher debt issuance reduces the probability of repayment. Similarly, higher outstanding debt reduces the price of short-term debt and higher expected future short-term debt issuance reduces the price of long-term debt.

If cross-default risk is maximal, $\pi=1$, then the prices of the two maturities satisfy

$$q_{t,t+2}(s_t, r_t, \iota_t) = q_{t,t+1}(s_t, r_t, \iota_t) \cdot \mathbb{E}_t\left[q_{t+1,t+2}(S_{t+1}) | L_{t+1}=\infty\right] + \beta\Phi_t \tag{14}$$

where Φ_t denotes the covariance between $1-F_{t+1}^{\xi=1}$ and $q_{t+1,t+2}(S_{t+1})|L_{t+1}=\infty$. Intuitively, the prices of short- and long-term debt differ for two reasons in this case. First, because in repayment states in the subsequent period, the return on short-term debt equals unity while on long-term debt it averages the mean price of newly-issued short-term debt. Second, because the insurance characteristics of the two maturities differ, depending on the covariance between $q_{t+1,t+2}(S_{t+1})$ and the repayment decision.

We proceed under the assumption that the price functions $q_{t,j}(s_t, r_t, \iota_t)$ be differentiable in $(b_{t,t+1}, \iota_t)$ and the government's program well behaved such that the policy functions are smooth. Below, when considering special cases of the model, we verify that this is indeed the case.[13]

4.2. Debt issuance

Issuing debt of a particular maturity has two effects on the deficit. On the one hand, it raises revenue from the marginal unit of debt, in proportion to its price. On the other hand, it affects the revenue raised from infra-marginal units of debt, by changing the repayment probability and thus, price of these units. This second effect is a consequence of the government's lack of commitment and reflects the fact that subsequent rollover and repayment decisions are endogenous. Formally,

$$\frac{\mathrm{d}d_t(s_t, r_t, \iota_t)}{\mathrm{d}\iota_{t,t+1}} = q_{t,t+1}(s_t, r_t, \iota_t) + \underbrace{\iota_{t,t+1}\frac{\mathrm{d}q_{t,t+1}(s_t, r_t, \iota_t)}{\mathrm{d}\iota_{t,t+1}}}_{\mathcal{R}_{t,ss}} + \underbrace{\iota_{t,t+2}\frac{\mathrm{d}q_{t,t+2}(s_t, r_t, \iota_t)}{\mathrm{d}\iota_{t,t+1}}}_{\mathcal{R}_{t,sl}}, \tag{15}$$

$$\frac{\mathrm{d}d_t(s_t, r_t, \iota_t)}{\mathrm{d}\iota_{t,t+2}} = q_{t,t+2}(s_t, r_t, \iota_t) + \underbrace{\iota_{t,t+1}\frac{\mathrm{d}q_{t,t+1}(s_t, r_t, \iota_t)}{\mathrm{d}\iota_{t,t+2}}}_{\mathcal{R}_{t,ls}} + \underbrace{\iota_{t,t+2}\frac{\mathrm{d}q_{t,t+2}(s_t, r_t, \iota_t)}{\mathrm{d}\iota_{t,t+2}}}_{\mathcal{R}_{t,ll}}. \tag{16}$$

For brevity, we denote revenue effects on infra-marginal units of debt by $\mathcal{R}_{t,\cdot}$; for example, $\mathcal{R}_{t,sl}$ denotes the revenue effects on infra-marginal units of long-term debt caused by a marginal increase of short-term debt issuance.

If society's discount factor equals zero, $\delta=0$, then the optimising government simply maximises the deficit (it selects the peak of the "debt-Laffer surface"). If δ is strictly positive, in contrast, then the government's choice of debt issuance takes both the effect on the deficit and on the continuation value into account.

Consider first short-term debt. A marginal increase in $\iota_{t,t+1}$ raises the government's value by

$$u'(c_t)\left(q_{t,t+1}(s_t, r_t, \iota_t) + \mathcal{R}_{t,ss} + \mathcal{R}_{t,sl}\right) + \delta\frac{\partial \mathbb{E}_t[G_{t+1}(s_{t+1})]}{\partial b_{t+1,t+1}} \tag{17}$$

[12] If L_{t+1} is sufficiently large for a default in period $t+1$ to be avoided then debt issuance is independent of the particular realisation of L_{t+1} and expectations can be conditioned on $L_{t+1}=\infty$.

[13] In general, the objective function need not be concave in debt issuance because higher debt issuance reduces the probability of repayment in the future and because it implies increasingly smaller revenue effects on infra-marginal units of debt if the price function is convex.

which can be expressed as

$$u'(c_t)(\mathcal{R}_{t,ss}+\mathcal{R}_{t,sl})+\mathbb{E}_t[(1-F_{t+1})(\beta u'(c_t)-\delta u'(S_{t+1}))|L_{t+1}=\infty]. \tag{18}$$

This marginal effect consists of two parts: A consumption smoothing benefit, represented by the term on the right-hand side and reflecting the fact that issuance of the marginal unit of debt at price $\beta\mathbb{E}_t[(1-F_{t+1})]$ allows to smooth consumption across the current period and the repayment states in the subsequent period; and an effect represented by the term on the left-hand side and reflecting the welfare consequences of the revenue effects on infra-marginal units of debt. A direct consequence of lack of commitment, the latter effect arises because a government's choice of debt issuance alters the subsequent government's choice of repayment rate and debt rollover and thus, current prices and deficit.[14]

Note from (18) that the marginal effect of debt issuance on the continuation value equals the expected utility loss from repaying the marginal unit of debt—it does not reflect the welfare consequences of the fact that the repayment probability of marginal and infra-marginal units is slightly reduced. To understand this result, note that less likely repayment of infra-marginal units goes hand in hand with more likely social losses in the wake of a default. The welfare effects associated with these two changes exactly cancel each other since the subsequent government is indifferent at the margin between repaying or defaulting, and due to the congruence of the subsequent government's objective function and the current government's continuation value function. The presence of *social* rather than private costs of default is of first-order importance for this result and thus, for the government's rollover choice. We discuss this in more detail in the online appendix.

Consider next long-term debt. A marginal increase in $\iota_{t,t+2}$ raises the government's value by

$$u'(c_t)\left(q_{t,t+2}(s_t,r_t,\iota_t)+\mathcal{R}_{t,ls}+\mathcal{R}_{t,ll}\right)+\delta\frac{\partial\mathbb{E}_t[G_{t+1}(s_{t+1})]}{\partial b_{t+1,t+2}}. \tag{19}$$

If short-term debt issuance in the subsequent period is interior then the government is indifferent between redeeming long-term debt after one period or holding it to maturity. The marginal effect of issuing long-term debt held to maturity then can be expressed as the marginal effect of issuing long-term debt that is redeemed after one period at price $\beta\mathbb{E}_{t+1}[1-F_{t+2}]$. This latter marginal effect is given by

$$\begin{aligned}&u'(c_t)(\mathcal{R}_{t,ls}+\mathcal{R}_{t,ll})+\mathbb{E}_t\left[(1-F_{t+1})q_{t+1,t+2}(S_{t+1})(\beta u'(c_t)-\delta u'(S_{t+1}))|L_{t+1}=\infty\right]\\&\quad+(1-\pi)\mathbb{E}_t\left[\int_0^{b_{t+1,t+1}}q_{t+1,t+2}(S_{t+1})(\beta u'(c_t)-\delta u'(S_{t+1}))dF(L_{t+1})|\xi_{t+1}=0\right].\end{aligned} \tag{20}$$

The marginal effect of long-term debt issuance in (20) differs threefold from the effect of short-term debt issuance in (18). First, long- and short-term debt issuance generate different revenue effects, represented by $\mathcal{R}_{t,ls}+\mathcal{R}_{t,ll}$ versus $\mathcal{R}_{t,ss}+\mathcal{R}_{t,sl}$, respectively. Second and third, the consumption smoothing terms differ across maturities, for two reasons. On the one hand, short-term debt allows to smooth consumption across the current period and the *repayment* states in the subsequent period while long-term debt allows to smooth consumption across the current period and future states *without cross-default*. On the other hand, the return per unit of long-term debt in states without cross-default is given by the price of newly-issued short-term debt while the no-default return on a unit of short-term debt equals unity.

As is apparent from (18) and (20), the government's preferred maturity structure generally is determinate. This contrasts with the situation under commitment. If the government could commit its successors to honour maturing debt at face value, all revenue effects on infra-marginal units in the previous expressions would vanish, and (18) and (20) would reduce to the consumption smoothing benefits

$$q_{t,t+1}u'(c_t)-\delta\mathbb{E}_t[u'(c_{t+1})], \tag{21}$$

$$q_{t,t+2}u'(c_t)-\delta\mathbb{E}_t[u'(c_{t+1})q_{t+1,t+2}], \tag{22}$$

respectively. Under the maintained assumption of risk neutrality on the part of investors, the absence of default risk would imply $q_{t,t+1}=q_{t+1,t+2}=\beta$, $q_{t,t+2}=\beta^2$ and thus, equality of the two marginal effects and indeterminacy of the portfolio choice. To restore determinacy in a setting with commitment, the price of default-free outstanding debt, $q_{t+1,t+2}$, would need to be state contingent, for example due to an endogenous asset pricing kernel of investors (Angeletos, 2002; Nosbusch, 2008), and taxpayers would need to be risk averse. In the model considered here, the equilibrium maturity structure is determinate although the asset pricing kernel of investors is not stochastic and even if taxpayers are not risk averse.

To gain an alternative perspective on the debt-issuance trade-off faced by the government, consider the welfare effect of a marginal increase in short-term debt issuance by one unit accompanied by a reduction of long-term debt issuance by z_t^{-1} units where $z_t\equiv\mathbb{E}_t[q_{t+1,t+2}(S_{t+1})|L_{t+1}=\infty]$. From (18) and (20), this portfolio adjustment yields

$$u'(c_t)\left(\mathcal{R}_{t,ss}+\mathcal{R}_{t,sl}-\frac{\mathcal{R}_{t,ls}+\mathcal{R}_{t,ll}}{z_t}\right)-\mathbb{C}\mathrm{ov}_t\left[q_{t+1,t+2}(S_{t+1}),(1-F_{t+1})(\beta u'(c_t)-\delta u'(S_{t+1}))|L_{t+1}=\infty\right]/z_t$$

[14] The interests of the current and the subsequent government may be misaligned not only if debt is issued but also if long-term debt is prematurely redeemed ($\iota_{t,t+1}<0$). Premature redemption reduces the likelihood of a default by the subsequent government and thereby raises the price at which the current government buys back the bonds.

$$-\frac{1-\pi}{z_t}\mathbb{E}_t\left[\int_0^{b_{t+1,t+1}} q_{t+1,t+2}(S_{t+1})(\beta u'(c_t)-\delta u'(S_{t+1}))dF(L_{t+1})|\xi_{t+1}=0\right]. \tag{23}$$

The welfare effect in (23) derives from three sources. First, if $\pi<1$, the adjustment affects the expected portfolio value in the subsequent period. Across repayment states in the subsequent period the rebalanced portfolio's expected returns are unchanged, due to the choice of z_t. Across states with a default but no cross-default, however, the portfolio adjustment lowers the expected market value of the portfolio (because in these states, the return on short-term debt is zero while the return on long-term debt is positive) and accordingly, the associated benefit from consumption smoothing. This is represented by the last term in the expression.

Second, while the adjustment leaves the average portfolio value across repayment states unchanged, it alters the covariance between the portfolio value and marginal utility. If the price of short-term debt in the subsequent period tends to fall if marginal utility is high (as in the special cases considered below), then the market value of outstanding long-term debt co-varies negatively with marginal utility. Issuing long-term debt then provides insurance benefits and the portfolio adjustment reduces the government's value. This is represented by the second term in the expression. Finally, the first term in the expression accounts for differences in the revenue effects across maturities.

5. Analytical results

To characterize the equilibrium maturity structure in more detail, we consider several special cases before turning to numerical simulations of the general model.

5.1. No cross-default

We first abstract from the possibility of cross-default, $\pi=0$. Moreover, we make two assumptions throughout the subsection. First, we assume that the hazard function $H(L)\equiv f(L)/(1-F(L))$ satisfies a regularity condition:

$$\text{For all } L\geq 0,\ \text{(i) } H'(L)\geq 0 \text{ and (ii) } 2H'(L)^2-H(L)H''(L)\geq 0,\ \text{for example because the hazard function is concave.}$$

Second, we assume that marginal utility is constant or a function of output.

The assumption about marginal utility is motivated by tractability considerations. If marginal utility is independent of the stock of maturing debt and the income losses after default, then so is equilibrium debt issuance, $\iota_{t,t+1}(y_t,b_{t,t+1})$ and $\iota_{t,t+2}(y_t,b_{t,t+1})$, and this allows to characterise the equilibrium maturity structure in closed form. Intuitively, equilibrium debt issuance then reflects the consumption smoothing benefit abstracting from the effects of L_t or $b_{t,t}$ on disposable income while it fully reflects the revenue effects on infra-marginal units of debt and the fact that outstanding debt depresses the price of newly-issued debt.

The assumption about marginal utility is satisfied if taxpayers are risk neutral. In fact, Propositions 1 and 2 below do apply under the assumption of risk neutrality. However, risk neutrality renders the consumption smoothing benefit time and state invariant and therefore it is not a useful assumption when we wish to characterise equilibrium in cyclical or stochastic environments as we do in Propositions 3 and 4. For that reason, we allow marginal utility to be a function $\mu(\cdot)$ of output that is, we let $u'(c_t)=\mu(y_t)$ with $\mu(y^h)\leq\mu(y^l)$ for $y^l<y^h$. This specification can be rationalised as approximating the case of a strictly concave utility function and processes for output and default costs such that variations in output have a much stronger effect on marginal utility than induced variations in government policy.[15] In the numerical simulations discussed in Section 6, we find that the analytical results are robust to relaxing the assumption about the marginal utility function.

The regularity condition on the hazard function implies that the marginal effects of debt issuance on the government's value are monotone such that equilibrium analysis can be based on first-order conditions. Many distribution functions typically used in economic applications satisfy the regularity condition.[16]

Due to the absence of cross-default and since $b_{t+1,t+2}=\iota_{t,t+2}$, the equilibrium price functions satisfy

$$q_{t,t+1}(s_t,r_t,\iota_t)=\beta(1-F(b_{t,t+1}+\iota_{t,t+1})), \tag{24}$$

$$q_{t,t+2}(s_t,r_t,\iota_t)=\beta^2\mathbb{E}_t[1-F(\iota_{t,t+2}+\iota_{t+1,t+2}(y_{t+1},\iota_{t,t+2}))]. \tag{25}$$

[15] If income losses after default are "small" then so is equilibrium debt. If, moreover, output variation is "large" then consumption and marginal utility fluctuations mainly are driven by output fluctuations. Strictly speaking, the preferences stipulated in this subsection do not constitute a special case of the preferences assumed throughout the rest of the paper. To be consistent, the utility function in the general model should be specified as $u(c_t,\phi_t)$ rather than $u(c_t)$ where ϕ_t is an exogenous preference shifter; in this subsection $\phi_t=y_t$.

[16] Examples of distribution functions with increasing hazard functions include uniform, normal, exponential, logistic, extreme value, Laplace, power, Weibull, gamma, chi-squared, chi, or beta distributions (see, e.g., Bagnoli and Bergstrom, 2005. If L_t is distributed according to an exponential distribution, $F(L)=1-\exp(-\lambda L)$, then the hazard function is constant, $H(L)=\lambda$. If L_t is distributed according to a Weibull distribution, $F(L)=1-\exp(-L^\lambda)$, $\lambda>1$, then the hazard function is strictly increasing, $H(L)=\lambda L^{\lambda-1}$; moreover, for $1\leq\lambda\leq 2$, the hazard function is concave, and for all $\lambda>1$, $H'(L)^2-H(L)H''(L)>0$.

The revenue effects on infra-marginal units therefore are given by

$$\mathcal{R}_{t,ss} = -\iota_{t,t+1}\beta f(b_{t+1,t+1}), \tag{26}$$

$$\mathcal{R}_{t,ll} = -\iota_{t,t+2}\beta^2\mathbb{E}_t\left[f(b_{t+2,t+2})\left(1+\frac{\partial\iota_{t+1,t+2}(y_{t+1},\iota_{t,t+2})}{\partial b_{t+1,t+2}}\right)\right] \tag{27}$$

while $\mathcal{R}_{t,sl}=\mathcal{R}_{t,ls}=0$. Note that $\mathcal{R}_{t,ss}$ reflects the endogenous repayment rate while $\mathcal{R}_{t,ll}$ reflects the fact that both the repayment rate and debt issuance are endogenous. In this sense, the revenue effects on short-term debt reflect one and those on long-term debt two manifestations of lack of commitment.

From (18) and (20), the marginal effects of $\iota_{t,t+1}$ and $\iota_{t,t+2}$, respectively, are

$$\mu(y_t)\mathcal{R}_{t,ss}+(1-F(b_{t+1,t+1}))\mathbb{E}_t\left[\beta\mu(y_t)-\delta\mu(y_{t+1})\right], \tag{28}$$

$$\mu(y_t)\mathcal{R}_{t,ll}+\mathbb{E}_t\left[\beta(1-F(b_{t+2,t+2}))(\beta\mu(y_t)-\delta\mu(y_{t+1}))\right]. \tag{29}$$

Under the regularity condition on the hazard function, these two marginal effects equal zero in equilibrium and the equilibrium maturity structure balances for each maturity the consumption smoothing benefit from the marginal unit and the costs due to revenue effects on infra-marginal units. The policy prescription embodied in these optimality conditions parallels Barro's (1979) "tax-smoothing" prescription. In Barro's (1979) model, welfare losses of taxation are convex. Welfare maximising taxation therefore requires a smoothing of tax distortions, relative to tax revenues, across the available tax instruments. In the model considered here, welfare reducing revenue effects are convex. Optimal policy therefore amounts to smoothing these revenue effects, relative to the consumption smoothing benefit from the marginal unit, across the available maturities. In this sense, "maturity smoothing" parallels "tax smoothing."

Rearranging the two first-order conditions yields

$$\iota_{t,t+1} = \frac{1-F(b_{t+1,t+1})}{f(b_{t+1,t+1})}\mathbb{E}_t\left[1-\frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}\right], \tag{30}$$

$$\iota_{t,t+2} = \frac{\mathbb{E}_t\left[(1-F(b_{t+2,t+2}))\left(1-\frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}\right)\right]}{\mathbb{E}_t\left[f(b_{t+2,t+2})\left(1+\frac{\partial\iota_{t+1,t+2}(y_{t+1},\iota_{t,t+2})}{\partial b_{t+1,t+2}}\right)\right]}. \tag{31}$$

Eq. (30) defines a policy function $\iota_{t,t+1}(\cdot)$ of output and outstanding debt that is positive (for δ sufficiently smaller than β) as well as decreasing and convex in its second argument. Higher outstanding debt reduces the equilibrium level of short-term debt issuance because it increases the negative revenue effects on infra-marginal units. This effect is absent only if the hazard function is constant as is the case with exponentially distributed income losses (considered below). Otherwise, the effect is present but weakens as the quantity of outstanding debt increases. Using (30), Eq. (31) defines a positive policy function $\iota_{t,t+2}(\cdot)$ of output. Summarising, we have the following preliminary result:

Lemma 1. *Suppose that $\pi=0$, marginal utility is constant or a function of output, and the regularity assumption on the hazard function is satisfied. Then, there exists an equilibrium in which the policy functions $\iota_{t,t+1}(s_t)$ and $\iota_{t,t+2}(s_t)$ do not depend on $b_{t,t}$ or L_t (or ξ_t). The maturity structure in this equilibrium is unique with $\iota_{t,t+1}(s_t), \iota_{t,t+2}(s_t)>0$.*

The equilibrium characterised in the lemma is the only equilibrium that arises in a finite horizon economy (potentially, with the number of periods approaching infinity). This follows from a straightforward backward induction argument. In the discussion, we focus on this type of equilibrium.

Consider the case of a constant hazard function, $H(L)=H$, and suppose that output follows a deterministic process. With a constant hazard function, the stock of outstanding debt does not affect equilibrium short-term debt issuance in Eq. (30) and $\mathcal{R}_{t,ll}$ simplifies to $-\iota_{t,t+2}\beta^2 f(b_{t+2,t+2})$. The equilibrium conditions (30) and (31) therefore reduce to

$$\iota_{t,t+1}H = 1-\frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}, \tag{32}$$

$$\iota_{t,t+2}H = 1-\frac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}, \tag{33}$$

implying that the equilibrium maturity structure is fully balanced at all times. Intuitively, with a constant hazard function, the structure of the revenue effects on infra-marginal units of debt is symmetric across maturities. Absent output risk, the welfare costs due to these revenue effects relative to the consumption smoothing benefits from the marginal unit also are symmetric across maturities. Maturity smoothing therefore prescribes a balanced maturity structure:

Proposition 1. *Suppose that $\pi=0$ and marginal utility is constant or a function of output. If the hazard function is constant and y_t is deterministic, then the equilibrium maturity structure is fully balanced. Debt issuance is high when marginal utility relative to marginal utility in the subsequent period is high.*

The finding of a fully balanced maturity structure hinges on the feature that the revenue effects on infra-marginal units relative to the revenue on the marginal unit are symmetric across maturities. With a strictly increasing hazard function, this symmetry breaks down because long-term debt issuance affects short-term debt issuance in the subsequent period. Consider an environment where the consumption smoothing benefit $\mathcal{C}_t \equiv 1-\delta\mu(y_{t+1})/\beta\mu(y_t)$ is time invariant at value $\mathcal{C}$ such that short- and long-term debt issuance is time invariant as well at values $\imath_{\text{shrt}}$ and $\imath_{\text{long}}$, respectively. The equilibrium conditions then read

$$\imath_{\text{shrt}} H(\imath_{\text{shrt}}+\imath_{\text{long}}) = \mathcal{C}, \tag{34}$$

$$\imath_{\text{long}} H(\imath_{\text{shrt}}+\imath_{\text{long}})\left(1+\imath'_{\text{shrt}}(\imath_{\text{long}})\right) = \mathcal{C}. \tag{35}$$

Since with a strictly increasing hazard function, short-term debt issuance responds negatively to the quantity of outstanding debt ($\imath'_{\text{shrt}}(\imath_{\text{long}})<0$, from the first equation), the two conditions imply that the equilibrium maturity structure is tilted towards the long end, $0<\imath_{\text{shrt}}<\imath_{\text{long}}$. Moreover, due to the convexity of $\imath_{\text{shrt}}(\imath_{\text{long}})$, the tilt towards long-term debt becomes smaller as the total amount of debt increases. Higher debt-to-GDP ratios therefore go hand in hand with a shortening of the maturity structure, in line with the evidence (Rodrik and Velasco, 1999). Intuitively, if short-term debt issuance responds negatively to the quantity of outstanding debt then long-term debt issuance increases the quantity of debt to mature two periods later by less than one-to-one. Accordingly, long-term debt issuance has a comparatively small price impact, rendering it relatively “cheap” from the government's perspective.

Proposition 2. *Suppose that $\pi=0$, marginal utility is constant or a function of output, and the regularity assumption on the hazard function is satisfied. If the hazard function is strictly increasing and the consumption smoothing benefit is time invariant, then the equilibrium maturity structure is tilted towards the long end. Higher debt-to-GDP ratios go hand in hand with a shorter maturity structure.*

Consider next an environment where the consumption smoothing benefit varies over time in the sense that $\mathcal{C}_t$ is equal in all periods except for period $T-1$ when $\mathcal{C}_{T-1}$ is smaller.[17] This scenario describes a “crisis” where the economy plunges from a peak in period $T-1$ into a trough in period T before returning to trend growth. From the previous result, one would expect lower total debt issuance in period $T-1$ (due to the weaker borrowing motive in that period) to be accompanied by a lengthening of the maturity structure. This should be followed by a shortening of the maturity structure at the trough in period T. In other words, one would expect a shortening of the maturity structure as the economy moves from the “pre-crisis period” into the “crisis period,” in line with the evidence (Broner et al., 2013). A general result along these lines can indeed be proved under the assumption that the hazard function is proportional (representing Weibull distributed income losses):

Proposition 3. *Suppose that $\pi=0$ and marginal utility is a function of output. If $H(L)=2L$ and $\mathcal{C}_{T-1}$ is slightly reduced relative to $\mathcal{C}_t=\mathcal{C}$ in the other periods, then the equilibrium maturity structure lengthens in period $T-1$ and shortens in period T.*

Turning to the equilibrium implications of output risk, it is instructive to rewrite (31) as

$$\imath_{t,t+2} = \frac{\mathbb{E}_t[1-F(b_{t+2,t+2})]\mathbb{E}_t\left[1-\dfrac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}\right]+\mathbb{C}\text{ov}_t\left[1-F(b_{t+2,t+2}),1-\dfrac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}\right]}{\mathbb{E}_t\left[f(b_{t+2,t+2})\left(1+\dfrac{\partial\imath_{t+1,t+2}(y_{t+1},\imath_{t,t+2})}{\partial b_{t+1,t+2}}\right)\right]}. \tag{36}$$

The first term on the right-hand side of the equality represents the average consumption smoothing benefit from the marginal unit of long-term debt relative to the cost due to revenue effects on infra-marginal units. The second term represents an insurance benefit due to the covariance between the price of outstanding debt in the subsequent period and marginal utility. Short-term debt does not provide this insurance benefit since its repayment rate does not co-vary with output and marginal utility. The insurance benefit of long-term debt is positive if marginal utility co-varies negatively with the price of outstanding long-term debt and thus, if it co-varies positively with short-term debt issuance. From (30), this is indeed the case if output is mean reverting in the sense that a realisation of $\mu(y_t)$ above its mean implies that the consumption smoothing benefit $\mathcal{C}_t$ exceeds its mean as well.

Consider the case of a constant hazard function, $H(L)=\lambda$. Condition (31) then reduces to

$$\imath_{t,t+2} = \lambda^{-1}\frac{\mathbb{E}_t\left[\exp(-\lambda(\imath_{t+1,t+2}(y_{t+1})))\left(1-\dfrac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}\right)\right]}{\mathbb{E}_t[\exp(-\lambda(\imath_{t+1,t+2}(y_{t+1})))]} \tag{37}$$

$$\imath_{t,t+2} = \imath_{t,t+1}+\lambda^{-1}\frac{\mathbb{C}\text{ov}_t\left[\exp(-\lambda(\imath_{t+1,t+2}(y_{t+1}))),\left(1-\dfrac{\delta\mu(y_{t+1})}{\beta\mu(y_t)}\right)\right]}{\mathbb{E}_t[\exp(-\lambda(\imath_{t+1,t+2}(y_{t+1})))]}, \tag{38}$$

[17] An alternative source of time variation of the maturity structure relates to changes of the hazard function over time. A priori, it is not clear how such changes should correlate with output and marginal utility.

where the second equality follows from (30). Since the covariance term is positive, long-term debt issuance exceeds short-term debt issuance:

Proposition 4. *Suppose that $\pi=0$, marginal utility is a function of output, and the regularity assumption on the hazard function is satisfied. If output is stochastic and mean reverting, then long-term debt provides insurance benefits. If, moreover, the hazard function is constant, then the equilibrium maturity structure is tilted towards the long end.*

5.2. Cross-default

The analysis so far has identified advantages of long-term debt issuance, related to the provision of insurance and the attenuation of subsequent short-term debt issuance. Cross-default risk introduces additional factors which might give rise to advantages of short-term debt issuance. Intuitively, these factors arise because long-term debt is exposed to default risk both in the short and the long run while short-term debt is not. This has consequences for the revenue effects on infra-marginal units. In a knife-edge case, these consequences are sufficiently pronounced to render short-term debt strictly preferable over long-term debt. The assumptions underlying the knife-edge case include maximal cross-default risk ($\pi=1$) and constant marginal utility.

Recall from (23) the welfare effect of a marginal increase in $\iota_{t,t+1}$ by one unit accompanied by a reduction of $\iota_{t,t+2}$ by z_t^{-1} units where $z_t \equiv \mathbb{E}_t[q_{t+1,t+2}(S_{t+1})|L_{t+1}=\infty]$. As discussed previously, this portfolio adjustment triggers changes in the revenue effects, alters the covariance between the portfolio value and marginal utility, and affects the expected portfolio value in the subsequent period. We abstract from the second and third effect which tend to work in favour of long-term debt, since we assume that marginal utility is constant and $\pi=1$. As a consequence, the welfare effect of the portfolio adjustment exclusively reflects the differential revenue effects

$$\mathcal{R}_{t,ss}+\mathcal{R}_{t,sl}-\frac{\mathcal{R}_{t,ls}+\mathcal{R}_{t,ll}}{z_t}. \tag{39}$$

With constant marginal utility, changes in output or the stock of maturing debt do not alter $\alpha_t(y_t, b_{t,t}, b_{t,t+1})$ that is, the only relevant argument of the function $\alpha_t(\cdot)$ is outstanding debt. Letting α'_{t+1} denote the derivative of this function in period $t+1$, the revenue effects are given by (from (10) and (11))

$$\mathcal{R}_{t,ss}=-\iota_{t,t+1}\beta f_{t+1}^{\xi=1}, \tag{40}$$

$$\mathcal{R}_{t,sl}=-\iota_{t,t+2}\beta f_{t+1}^{\xi=1} q_{t+1,t+2}(S_{t+1})|_{r_{t+1}=1}, \tag{41}$$

$$\mathcal{R}_{t,ls}=-\iota_{t,t+1}\beta f_{t+1}^{\xi=1}\alpha'_{t+1}, \tag{42}$$

$$\mathcal{R}_{t,ll}=-\iota_{t,t+2}\beta(f_{t+1}^{\xi=1} q_{t+1,t+2}(S_{t+1})|_{r_{t+1}=1}\alpha'_{t+1}+\Delta), \tag{43}$$

where $\Delta>0$ is proportional to the increase in long-term default risk conditional on no default in the short term. Risk neutrality implies $\alpha_t(b_{t,t+1})=q_{t,t+1}(S_t)|_{r_t=1}\cdot b_{t,t+1}$ and thus, $\alpha'_{t+1}=z_t$ (from the perspective of period t, the price $q_{t+1,t+2}(S_{t+1})|_{r_{t+1}=1}$ is not stochastic since policy does not vary with output). Consequently,

$$\mathcal{R}_{t,ss}+\mathcal{R}_{t,sl}-\frac{\mathcal{R}_{t,ls}+\mathcal{R}_{t,ll}}{z_t}=\iota_{t,t+2}\beta\Delta/z_t. \tag{44}$$

We conclude that for any positive $\iota_{t,t+2}$, long-term debt generates larger adverse revenue effects relative to the revenue raised on the marginal unit than short-term debt, due to the effect of long-term debt issuance on conditional default risk in the long-term. Short-term debt therefore strictly dominates long-term debt.

Proposition 5. *Suppose that $\pi=1$ and marginal utility is constant. Then, the equilibrium maturity structure is concentrated on the short end.*

We emphasise the knife-edge character of Proposition 5. If the probability of cross-default falls short of unity the welfare effect of the portfolio adjustment in (23) includes an additional negative term, as discussed before, and debt issuance triggers additional revenue effects. For both reasons, the strict dominance result no longer applies. Moreover, if marginal utility is not constant, an insurance benefit of long-term debt is present (with the effect of also undermining the strict dominance result), as discussed earlier; the revenue effects include covariance terms; and the function $\alpha_t(\cdot)$ depends on output and maturing debt in addition to outstanding debt which has consequences for the revenue effects and implies that α'_{t+1} may be smaller than z_t.

6. Numerical results

Next, we solve the general model numerically and simulate it. Our analysis is divided into two parts. In the first part (discussed in Section 6.2), we replicate the results derived in the previous section and check whether they serve as useful guides even when the restrictive assumption about the marginal utility function that we imposed in parts of Section 5.1 is relaxed. We find that this is indeed the case. In the second part (discussed in Section 6.3), we simulate the model subject to a baseline calibration with risk averse taxpayers, risky output and cross-default risk.

6.1. Calibration

We assume that taxpayers' preferences are of the CIES type with the inverse of the inter temporal elasticity of substitution varying between a minimal value of 0.01 (approximating the linear utility case analysed in parts of Section 5) and a maximal value of 5; that income losses in the wake of a default are distributed according to a Weibull distribution with parameter $\lambda = 2$ (the simplest distribution with a strictly increasing hazard function); and that the annual risk free interest rate equals 2 percent. One period in the model corresponds to 3 years in the data implying a (plausible) long-term debt maturity of six years. Simulation statistics are based on sequences of 20,000 observations.

Exogenous output is assumed either to be constant or to fluctuate between a high, average and low value, y^h, $\overline{y}$ and y^l, respectively, with a standard deviation $\text{std}(y_t)$ of 3.5 percent, corresponding to the standard deviation of de-trended three-yearly (and also annual) Argentinian GDP data. Mean exogenous output $\overline{y}$ as well as the parameter δ are calibrated by equalising a debt-service-to-GDP ratio of 5.5 percent Arellano, 2008 and debt-to-GDP ratio of 40 percent with their model counterparts under the assumptions of Proposition 2:

$$\begin{aligned} 0.055\overline{y} &= \imath_{\text{shrt}} + \imath_{\text{long}} - (\beta \imath_{\text{shrt}} + \beta^2 \imath_{\text{long}})(1 - F(\imath_{\text{shrt}} + \imath_{\text{long}})), \\ 0.400\overline{y} &= \imath_{\text{shrt}} + \imath_{\text{long}}. \end{aligned} \tag{45}$$

In the simulations with risk, the exogenous output process is generated by the transition matrix

$$\Pi = \begin{pmatrix} 0.50 & 0.50 & 0.00 \\ 0.25 & 0.50 & 0.25 \\ 0.00 & 0.50 & 0.50 \end{pmatrix}, \tag{46}$$

implying unconditional probabilities of high, average and low exogenous output of 25, 50 and 25 percent, respectively. In the simulations without risk, $y_t = \overline{y}$ in all periods. Table 1 summarises the calibration.

Under the calibration assumptions, condition (45) implies

$$\frac{\imath_{\text{shrt}}}{\overline{y}} = 0.16, \quad \frac{\imath_{\text{long}}}{\overline{y}} = 0.23, \quad q_{\text{shrt}} = 0.89, \quad q_{\text{long}} = 0.84. \tag{47}$$

In the simulation with $\sigma = 2$ as well as output and cross-default risk that we discuss further below the debt quotas are smaller.

We approximate the state space by a grid and to find the policy functions, we use an iterative algorithm that searches over this grid. Unless otherwise noted, the shock sequence for income losses in the wake of a default is maintained across all simulations. The shock sequences for the exogenous output and the cross-default shock vary across simulations, depending on the specific scenario considered.

Table 1
Calibration.

Parameter/function		Value/form	Source
$u(c)$	Utility function		
	– $\sigma \neq 1$	$(c^{1-\sigma})/(1-\sigma)$	Standard assumption
	– $\sigma = 1$	$\log(c)$	Standard assumption
σ	Utility fct curvature		
	– Section 6.2	0.01, 1, 2, 5	
	– Section 6.3	2	Arellano (2008)
π	Probability $\xi_t = 1$		
	– Section 6.2	0, 1	
	– Section 6.3	0.8	Assumption
$F(L)$	Distribution function	$1-\exp(-L^2)$	Assumption
β^{-1}	Risk free interest rate	$0.98^{-3} = 0.9412^{-1}$	Standard assumption
$\overline{y}$	Mean output	0.5715	$F(L), \beta$, condition (45)
δ	Discount factor	0.9005	$F(L), \beta$, condition (45)
	Additional parameters in simulations with output risk		
Π	Trans matrix output	See text	Assumption
$\text{std}(y_t)$	Standard dev output	0.0350	Argentinian data
y^h	High output	0.5998	$\Pi, \text{std}(y_t), \overline{y}$
y^l	Low output	0.5432	$\Pi, \text{std}(y_t), \overline{y}$

6.2. Replication of analytical results and extensions

Table 2 reports the average values of ι_t, $q_{t,t+1}$, $q_{t,t+2}$, α_t and r_t in simulations without output risk and with either no or maximal cross-default risk, $\pi = 0$ or $\pi = 1$ respectively. For each case, we analyse the role of risk aversion by varying σ between 0.01, 1, 2 and 5.

Consider first the results for the case without cross-default risk (reported on the left-hand side of Table 2). In line with the theoretical argument, $\alpha_t = 0$. Moreover, in the essentially risk neutral case ($\sigma = 0.01$), long-term debt issuance exceeds short-term debt issuance, corroborating Proposition 2. With stronger curvature of the utility function, short-term debt issuance falls and long-term debt issuance continues to exceed short-term debt issuance, suggesting that the finding in Proposition 2 is robust to relaxing the assumption about the marginal utility function. Stronger curvature of the utility function also tends to decrease the sum of short- and long-term debt issuance and thus, the amount of debt coming due in a typical period. As a consequence, it goes hand in hand with a lower default frequency. The simulated default frequency equals 5.4 percent in the essentially risk neutral case and 3.8 percent when $\sigma = 5$.

Consider next the case with maximal cross-default risk (the results are reported on the right-hand side of Table 2). With essentially risk neutral preferences, long-term debt issuance equals zero, corroborating Proposition 5. Accordingly, outstanding debt and thus α_t equal zero as well. But as anticipated in the discussion of Proposition 5, this extreme result is not robust. For higher values of σ, long-term debt issuance is strictly positive and so is α_t. In fact, the maturity structure is tilted towards the long end, as in the case without cross-default risk. Cross-default risk is associated with smaller debt positions. Moreover, stronger curvature of the utility function tends to decrease total debt issuance. This is reflected in a slight reduction of default risk (2.1 percent when $\sigma = 0.01$ versus 1.8 percent when $\sigma = 5$).

Table 3 shows that the presence of output risk does not alter the previous findings in important ways. Independently of cross-default risk, long-term debt issuance tends to exceed short-term debt issuance except when $\pi = 1$ and σ is small ($\sigma = 0.01$ and now also $\sigma = 1$). When $\pi = 0$, stronger curvature of the utility function goes hand in hand with lower default risk; when $\pi = 1$, there is essentially no effect of σ on the average default rate.

In conclusion, these findings corroborate those propositions—Propositions 2 and 5—which can directly be checked based on numerical simulations. Moreover, the findings confirm the knife-edge character of the extreme result in Proposition 5 and they suggest that the forces working in favour of a relatively long maturity structure—emphasised in Propositions 2 and 4—tend to be active independently of the strong assumption about the marginal utility function that we adopted in parts of

Table 2
Simulations without output risk: first moments of selected variables.

π	0				1			
σ	0.01	1	2	5	0.01	1	2	5
$\iota_{t,t+1}$	0.0896	0.0643	0.0454	0.0256	0.1462	0.0442	0.0433	0.0172
$\iota_{t,t+2}$	0.1462	0.1312	0.1544	0.1715	0	0.0556	0.0556	0.0634
$q_{t,t+1}$	0.8903	0.9058	0.9043	0.9051	0.9213	0.9203	0.9205	0.9232
$q_{t,t+2}$	0.8416	0.8610	0.8624	0.8585	0.8671	0.8660	0.8662	0.8687
α_t	0	0	0	0	0	0.0512	0.0511	0.0588
r_t	0.9456	0.9623	0.9601	0.9613	0.9798	0.9783	0.9787	0.9818

Note: π and σ denote the probability of a cross-default shock and the curvature of the utility function, respectively. $\iota_{t,t+1}$, $\iota_{t,t+2}$, $q_{t,t+1}$, $q_{t,t+2}$, α_t and r_t denote short- and long-term debt issuance, the prices of short- and long-term debt, the default threshold in excess of the stock of maturing debt, and the repayment rate, respectively.

Table 3
Simulations with output risk: first moments of selected variables.

π	0				1			
σ	0.01	1	2	5	0.01	1	2	5
$\iota_{t,t+1}$	0.0853	0.0673	0.0494	0.0397	0.1462	0.0561	0.0501	0.0345
$\iota_{t,t+2}$	0.1627	0.1465	0.1607	0.1647	0	0.0527	0.0536	0.0556
$q_{t,t+1}$	0.8851	0.8984	0.8998	0.9016	0.9213	0.9183	0.9193	0.9218
$q_{t,t+2}$	0.8349	0.8566	0.8521	0.8548	0.8671	0.8643	0.8652	0.8674
α_t	0	0	0	0	0	0.0484	0.0493	0.0512
r_t	0.9404	0.9545	0.9555	0.9571	0.9798	0.9763	0.9771	0.9796

Note: π and σ denote the probability of a cross-default shock and the curvature of the utility function, respectively. $\iota_{t,t+1}$, $\iota_{t,t+2}$, $q_{t,t+1}$, $q_{t,t+2}$, α_t and r_t denote short- and long-term debt issuance, the prices of short- and long-term debt, the default threshold in excess of the stock of maturing debt, and the repayment rate, respectively.

Section 5.1. All in all, this suggests that the version of the model that can be solved in closed form incorporates the most important mechanisms that are active in the general framework.

6.3. Simulation subject to baseline calibration

We numerically solve and simulate the model under the assumption of risky output (transition matrix given by Π), high cross-default risk ($\pi = 0.8$) and substantial risk aversion ($\sigma = 2$).

Fig. 1 displays the equilibrium policy functions for short- and long-term debt issuance, $\iota_{t,t+1}$ and $\iota_{t,t+2}$ respectively, for different exogenous output levels. The top left panel shows short-term debt issuance (reported as a fraction of average exogenous output) as function of debt coming due (also reported as a quota) where the quantity of outstanding debt is held constant at its average value. For each exogenous output level, the function is increasing, reflecting the desire to smooth consumption: debt issuance is higher in periods when a lot of debt has to be repaid. Furthermore, a comparison of the three curves in the panel indicates that short-term debt issuance is higher when exogenous output is lower, also reflecting the consumption smoothing motive. If the equilibrium repayment rate equals zero and the country suffers default induced income losses L_t as a consequence, then the same policy functions apply, evaluated at $b_{t,t} = L_t$ and, in case of cross default, $b_{t,t+1} = 0$.

The top right panel of the figure shows short-term debt issuance as function of outstanding debt (normalised by average exogenous output, and holding maturing debt fixed at its average value). The function is decreasing, in line with the theoretical result in Proposition 2. The negative slope reflects the fact that higher outstanding debt increases the negative revenue effects on infra-marginal units, and it constitutes one of the reasons for the equilibrium maturity structure to be tilted towards the long end.

The two bottom panels of the figure display long-term debt issuance as function of maturing (left) or outstanding debt (right). As with short-term debt issuance, lower exogenous output leads to higher debt issuance, for consumption smoothing reasons. Long-term debt issuance is increasing in the stock of maturing debt, unlike in Section 5 where by assumption, maturing debt does not affect marginal utility. In contrast, long-term debt issuance as function of outstanding debt essentially is constant, as in Section 5.

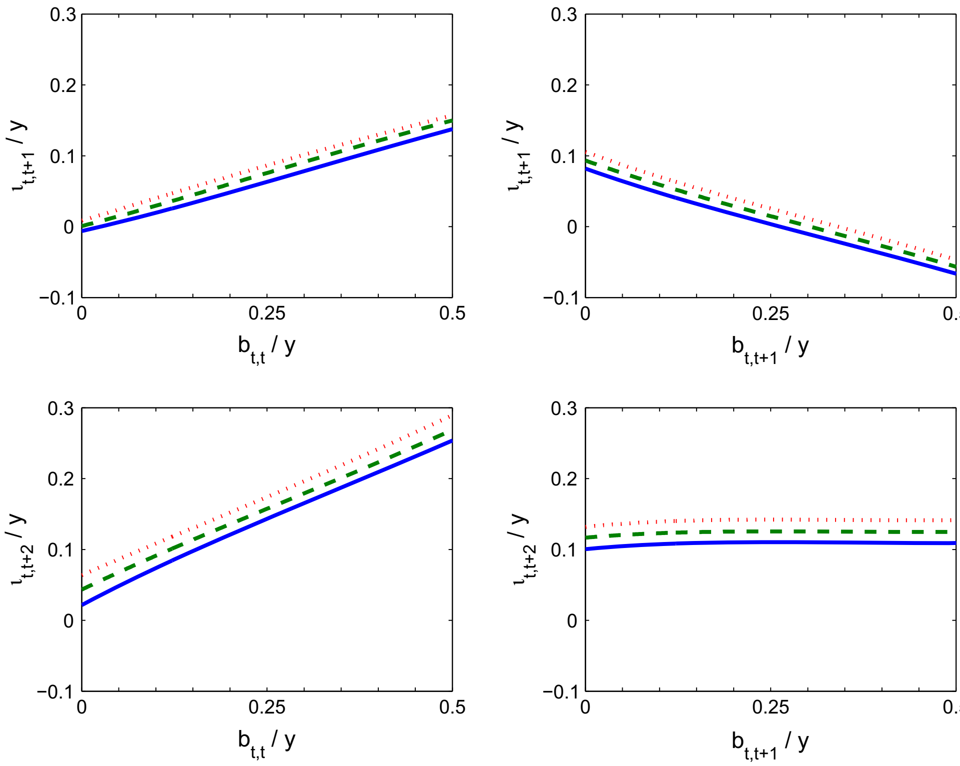

Fig. 1. Policy functions for short- and long-term debt issuance. *Note*: The top panels depict short-term debt issuance as function of maturing debt, evaluated at the average value of outstanding debt (left); and as function of outstanding debt, evaluated at the average value of maturing debt (right). The solid, dashed and dotted curves apply when the realisation of exogenous output is high, average and low, respectively. The bottom panels depict long-term debt issuance under the same conventions. All values are normalised by average exogenous output. Debt issuance in case of a default triggering costs L_t equals debt issuance with no default and with $b_{t,t} = L_t$ and, in case of cross default, $b_{t,t+1} = 0$.

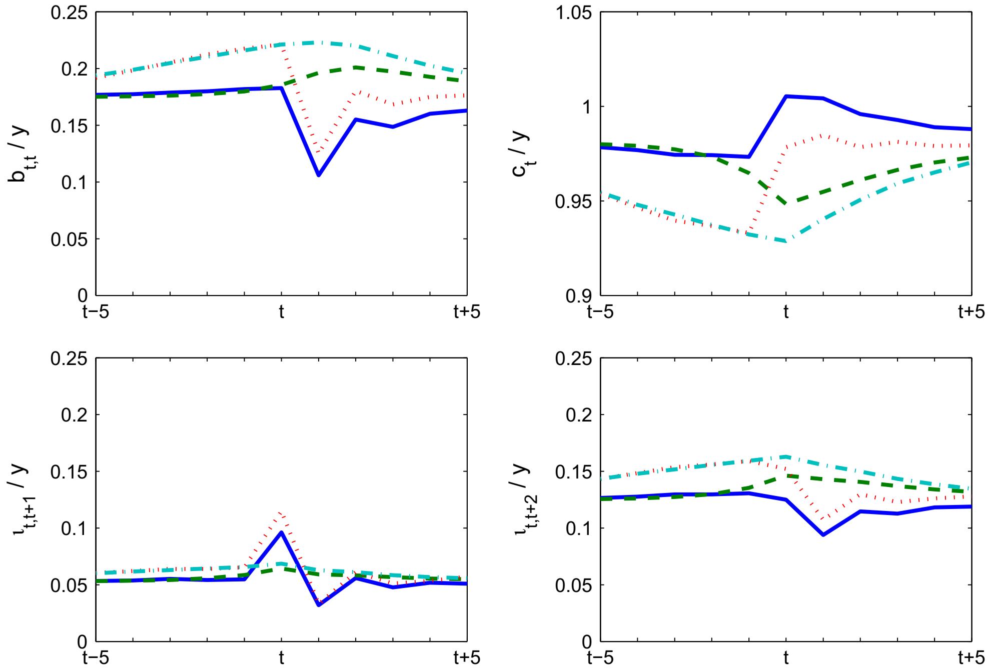

Fig. 2. Typical dynamics around specific events. *Note*: The curves display typical dynamics around specific events which are centered in period *t*. The solid curves depict default events; the dashed lines low output events; the dotted curves "crisis" events; and the dash-dotted curves "depression" events. See the text for detailed explanations.

The policy function for the choice of repayment rate is encoded in the function $\alpha_t(s_t)$ (not displayed) because the equilibrium repayment rate equals unity if $L_t - b_{t,t} \geq \alpha_t(s_t)\xi_t$, and zero otherwise. As a function of $b_{t,t+1}$ over the equilibrium domain, $\alpha_t(s_t)$ is approximately linear with a slope smaller than unity (and approximately equal to the price of short-term debt, as expected); it does not significantly vary with the exogenous output level.

Fig. 2 summarises the model predictions by reporting typical dynamics around specific events. To construct the plot for an event x say, we select all periods in the simulated time series in which x holds true. We define the typical value of a variable in an x event as the average value of that variable in all selected periods and in the plot, we report this typical value as the time t value. We similarly define the typical values of the variable during each of the five periods before and after the event x and we report them as the time $t-5$ to $t+5$ values in the plot.

We consider four specific events. The first one is a default, $r_t = 0$, which occurs in about 2.5 percent of the periods. In equilibrium, defaults are sparse and they are typically accompanied by a low realisation of L_t and a cross-default shock, $\xi_t = 1$. Exogenous output tends to be slightly lower before the event. The solid curves in Fig. 2 display the typical equilibrium dynamics of maturing debt, consumption and debt issuance around such an event (all normalised by average exogenous output). Maturing debt hoovers around its unconditional mean before a default but is slightly increasing; after the default, of course, it is much lower. Short- and long-term debt issuance also are relatively constant before the default event. In period t, their ratio increases and debt prices (not displayed) increase as well, more strongly so for long-term debt. Once default occurs, maturing debt is wiped out—and outstanding debt typically as well, due to the cross-default shock. Issuing fresh debt thus becomes inexpensive. This holds particularly true for short-term debt because the cross default on outstanding long-term debt removes the usual downward pressure on the price of short-term debt. In the period after the default, only short-term debt must be repaid and the stock of outstanding debt is only slightly lower than usual. Debt issuance, and particularly short-term one, therefore falls below its unconditional mean before converging back to its long run average. Finally, consumption falls before the default and increases thereafter, reflecting the high debt issuance at favourable prices as well as the low default cost.[18]

The second event is a low output realisation, $y_t = y^l$, which occurs in about 25 percent of the periods (the dashed curves in the figure illustrate the equilibrium dynamics). The transition matrix Π implies that this event tends to be preceded by a declining exogenous output path and to be followed by an increasing path. By assumption, both ξ_t and L_t are uncorrelated with y_t; low output periods therefore do not feature default unusually often but defaults are more frequent a few periods after a low output event. For consumption smoothing reasons, debt issuance increases towards the event and decreases afterwards. The maturity structure shortens, debt prices fall and more so for the price of long-term debt. The V-shaped path of exogenous output is mirrored by a similarly shaped equilibrium path of consumption.

[18] The consumption spike after a default would disappear if one assumed that default triggered financial autarky.

The dotted curves illustrate the typical paths around the third event, a period of very high indebtedness (the sum of maturing and outstanding debt exceeds its ninth decile) joint with a default, which occurs rarely, in only 0.25 percent of the periods. We refer to such an event as a “crisis.” Crisis events tend to have low realisations of L_t and a cross-default shock, $\xi_t = 1$, and they tend to follow a series of lower than average exogenous output realisations. (The last property distinguishes the crisis event from the default event.) As a consequence of this series of low output realisations as well as the induced higher debt issuance before the crisis, maturing debt is higher than usual and higher than in a default event. Once default occurs, the debt and price dynamics are similar across the two events except for the fact that debt issuance and maturing debt are higher after the crisis than the default event. This reflects the slower recovery of consumption after the crisis, mirroring the slower recovery of exogenous output.

Finally, the dash-dotted curves represent the dynamics around a “depression” event with high debt levels (defined as for the crisis event) and a low output realisation. Such events occur in roughly 3.2 percent of the periods. A depression event features default slightly more often than usual. Like the crisis event, it is characterised by a preceding spell of low output realisations that are responsible for the high debt levels. Since a default occurs only rarely, these high debt levels revert back to their long run averages only slowly although output recovers exactly as after the low output event. As a consequence, large amounts of debt are rolled over, but consumption remains depressed for an extended period. In the depression event, the buildup of debt is accompanied by a shortening of the maturity structure and a fall of debt prices, more so for long-term debt.

In summary, the simulations robustly generate the prediction that the maturity structure shortens during times of default or low output although the strength of this effect varies across the scenarios considered. The findings therefore corroborate the analytical result in Proposition 3 and qualitatively correspond with the data. The model predicted unconditional default probability (which did not serve as a target in the calibration) equals 2.5 percent, a realistic value. Since the model predicts counter cyclical debt issuance it predicts counter cyclical spreads as well (although at too low levels), consistent with the data (Arellano and Ramanarayanan, 2012; Broner et al., 2013), and it does so without assuming any correlation between output and default induced income losses.

7. Conclusions

Lack of commitment paired with social losses of default implies that debt issuance affects the amount of debt coming due unequally across maturities. It gives rise to an optimal maturity structure that smoothes the revenue effects on infra-marginal units of debt, relative to the consumption smoothing benefits of the marginal unit, across maturities. Maturity smoothing implies a lengthening of the maturity structure in response to risk and a shortening around times of crisis and low output.

These results follow under the standard premise that debt contracts stipulate non-contingent payments, presumably reflecting information and incentive constraints that prevent sovereign borrowers from entering into more sophisticated financial arrangements. The paper does not address the reasons for such constraints, nor does it question other central tenets in the sovereign debt literature.

The model presented in the paper relaxes many specific assumptions that have been employed in the sovereign debt literature to replicate features in the data. For example, to match bond prices or their cyclicality, output losses in the wake of a default have been assumed to be correlated with output, or to be complemented by a temporary, exogenous exclusion from financial markets. Without doubt, these or similar ad-hoc assumptions could also “help” in the current context. But we consider the model's simple and transparent structure a major asset, specifically in light of its ability to replicate key features of the data even without such specific assumptions.

Finally, the paper is silent about the choice of maturity structure in countries whose debt is perceived to be default-risk free.[19] But as indicated by sovereign bond ratings and spreads, the set of such countries has recently been shrinking quite dramatically. This suggests that credibility problems of the type considered in the paper are likely to bear on the maturity structure in a wide range of developing and developed economies.

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[19] Debt structure in those countries may be affected by liquidity concerns. The UK Debt Management Office “argues that cost is not the only factor. There is a virtue in being predictable, and in keeping all sections of the bond market supplied with debt to trade” (*The Economist*, “Losing interest,” June 14th 2008). See also Greenwood et al. (2010).

Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jmoneco.2014.08.006.

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