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Sovereign debt with heterogeneous creditors☆

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ABSTRACT

We develop a sovereign debt model with heterogeneous creditors (private and official) where the probability of default depends on both the level and the composition of debt. Higher exposure to official lenders improves incentives to repay due to more severe sanctions but it is also costly because it lowers the value of the sovereign's default option. The model can account for the co-existence of private and official lending, the time variation in their shares in total debt as well as the low rates charged on both. It also produces intertwined default and debt-composition choices.

1. Introduction

The recent sovereign debt crisis in the Eurozone has exhibited diverse patterns regarding the composition of sovereign debt: Greece completely switched financing from private to official (other Eurozone members and the IMF) funds that carried a low interest rate. Italy did not receive any direct official loans and continued to rely on more expensive private funds. Other debt distressed countries, namely Ireland, Portugal and Spain, experienced a change in the composition of new funding towards cheaper official sources but nevertheless continued borrowing from private credit markets.

The canonical sovereign debt model Eaton and Gersovitz (1981); Arellano (2008) contains homogeneous creditors. It is thus ill suited to analyze the determinants of debt composition and to shed light on portfolio and default choices in sovereign crisis episodes like the recent European one. In this paper we extend the standard model by introducing creditor heterogeneity. We show that this extension has interesting implications not only for the debt composition but also for default choices.

Creditor heterogeneity may take various forms. For instance, creditors may differ with respect to the type and extent of monitoring activities they engage in; the characteristics of their debt contracts such as conditionality schemes or policy requirements; and so on. In our view, these differences can largely be encapsulated by a single factor, namely the severity of the costs that the sovereign suffers when defaulting against a particular class of creditors. We assume that one class of creditors, namely official, is endowed with stronger "enforcement power" relative to another class, namely

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private creditors. We elaborate on the justification for this assumption below.[1]

The more severe sanctions imply a lower probability of default on official funds and thus lower default risk premia and interest rates. This feature can explain the low interest rates charged on large official loans. But the low rates do not represent a "free lunch" for the borrower, otherwise borrowers would always prefer official to private credit. There is a countervailing force as official loans reduce ex-post policy flexibility: More severe default costs imply that debt is repaid in some states of the world (say, during a protracted, severe recession) in which the sovereign would have opted for default were the debt owned by private creditors instead.[2] The resulting trade-off shapes the sovereign's portfolio choice.

What does the availability of "cheap" official funds imply for the riskiness of private loans and the sovereign's demand for them? Holding total debt constant, a switch from private to official funds may make *private* loans more or less safe. The former outcome arises when higher default costs associated with official funds also apply to the private portion of total debt. Such an extension of "protection" can result either directly from the existence of pari passu provisions in debt contracts;[3] or indirectly, from the characteristics of default costs, for instance from the existence of fixed costs. In either case, private funds acquire the risk characteristics of official funds; they are priced accordingly; and borrowing from official sources can crowd in private loans. The opposite outcome—crowding out of private loans—may result when higher default costs associated with official funds *reduce* the cost of defaulting against private loans.

Holding private debt constant, an increase in the amount of official credit increases total liabilities. When higher total debt raises the probability of default against all creditors (such dilution is a standard property of the canonical sovereign debt model), then private loans become riskier. But private loans may also become safer when official credit serves to enhance the debtor country's repayment capacity, for instance if its provision requires the adoption of structural reforms[4] and the resulting "collateral creation" effects are strong enough to also benefit private creditors.[5]

Extension of protection and collateral creation effects appear to have been operative throughout the Eurozone debt crisis. Both policy statements and the fact that the dispensation of official credit has been accompanied by a significant compression of private sovereign loan spreads (even in Greece in the period prior to the last elections) suggest that private claims were perceived to have been placed under official protection.[6] They also indicate that markets have expected pressure by official creditors for debtor countries to undertake measures that enhance their repayment ability, such as downsizing the public sector, liberalizing markets etc.

Countries involved in debt crisis often differ in terms of the level of debt overhang. We show that such differences have important implications for the interaction between a country's decision to default and its choice of debt composition. First, long-term debt overhang may induce a sovereign to collude with prospective official creditors in order to wipe out *outstanding* privately held long-term debt, rendering freshly issued official loans safer and cheaper and thus benefitting both official creditors and the sovereign. While this implication is well known in the literature as it applies to any situation with fresh financing by a new group of creditors, it is accentuated in our model by the superior enforcement power of official creditors. The borrower has a stronger incentive to default because in addition to eliminating the debt overhang, he also gets the chance to borrow at more favorable terms than if all classes of creditors had equal enforcement power. That is, inexpensive official funding in the presence of debt overhang simply aggravates default incentives. This seems consistent with the Greek default experience. The second and more novel implication is that, under pari passu, a sovereign with large future obligations to private creditors who chooses *not* to default against them in the present will also try to stay clear of official loans in order to maintain the—large—option value of renouncing the private claims in the future.

Consequently, these two features, namely, more severe sanctions for default against official creditors and differences in the stock of outstanding privately held long-term debt allow the standard sovereign debt model to generate several interesting implications. First, that involvement of official creditors may suppress interest rates on sovereign debt issued to private creditors, even for heavily indebted countries. Second, that countries that have large borrowing needs will favor borrowing from official creditors, in particular when they also face acute credibility problems. And third, the combination of the two features can help shed light on the nexus of default and debt-composition patterns. The model implies that a country with high debt overhang is more likely to switch financing from private towards official sources of funding if she defaults. And to continue favoring private funds when she does not default. More specifically, the model has the implication that in the absence of default, the share of official funds in fresh borrowing depends negatively on the stock of outstanding long-term debt.

We derive the optimal debt portfolio by focusing exclusively on the demand side, that is, we assume that the supply of official and private credit is perfectly elastic. The introduction of supply considerations would not add much value given the focus of this paper (debt composition). Assuming, for instance, that the supply of official credit is increasing in the risk adjusted rate only implies that the equilibrium share of official in total debt as well as the amount of total debt falls; the qualitative properties of the model remain the

[1] That the identity of the creditor can make a difference for the cost of default and hence the riskiness of the loan seems undisputed. For instance, there is a widely shared view supported by anecdotal evidence that loans by Mafia carry lower risk and thus a low interest rate because the incentive to repay such loans is much stronger than the incentive to repay other creditors due to Mafia's more extensive set of enforcement tools. Note that the alternative to a Mafia loan typically is no loan at all, that is a loan with a prohibitively high rate. Gambetta (1996) discusses how Mafia's protection and guarantees of safe conduct substitute for lack of trust in society.

[2] See Zame (1993) for a discussion of the insurance benefits of *implicitly* state contingent debt. Under incomplete markets, a country may trade a higher interest rate on its debt for the option to declare default in states where debt repayment would have been very costly. The desire by sovereigns to maintain a wider default option may explain both Spain's resistance to accept official loans as well as Greece's recent attempts under the previous government to switch away from inexpensive official to much more expensive private sources of funds.

[3] Zettelmeyer et al. (2013, p. 539) report that the new bonds issued by Greece after the 2012 default include pari passu clauses and are subject to "a co-financing agreement that created an exact symmetry between Greece's debt service to the new bondholders and its debt service to the EFSF related to the EFSF notes and bills that it had received for the purposes of the debt exchange. In the event of a shortfall in payments by Greece, a common paying agent committed to distributing this shortfall pro rata between the EFSF and the bondholders. Hence, the co-financing agreement made it difficult for Greece to default on its bondholders without also defaulting on the EFSF."

[4] The establishment of a credit relationship with official creditors has often been associated with measures that create or expand collateral, such as monitoring and conditionality.

[5] These benefits for private creditors may arise independently of whether default costs take the form of pure social costs suffered by the sovereign, or resources seized by the creditors. See the discussion in Appendix A .

[6] Anxious to avoid a crowding out of private funding, official lenders conceded that safeguards should be put in place to impede ex-post discrimination against their private counterparts. Consistent with this intention, the Greek debt exchange in Spring 2012 put private and official lenders (the EFSF) on an equal footing, see Zettelmeyer et al. (2013, p. 539). The Wall Street Journal (June 29, 2012, Investors Cheer Europe Deal) reports that Angela Merkel's agreement "to make ESM loans to Spain equal to Spanish bonds in creditors' pecking order was largely a recognition by Germany that this was necessary to protect Spain's ability to sell bonds ...." In another but related context, the recent New York court decision in the dispute between Argentina and Elliott Management regarding Argentina's default in 2002 has undermined the ex post preferred creditor status of certain lenders and provided a boost for pari passu.

same. Similarly, assuming a binding upper bound on official credit may affect the *total* optimal amount of debt issued[7] but trivializes the choice of debt composition.

1.1. Related literature

The literature on the composition of sovereign debt by type of creditor is scant. Boz (2011) reviews the literature on IMF lending, summarizes empirical evidence and presents a quantitative model of a sovereign that may borrow from private lenders and the IMF. In her model, private lending is subject to default risk while IMF lending is default risk free. The cost of IMF funds exceeds the risk free rate by an exogenous surcharge. The model predicts modest, countercyclical and intermittent IMF lending with *complete* crowding out of private funding. The last feature makes the model unsuitable for analyzing episodes where private and official new lending co-exist (such as the Eurozone debt crisis).[8]

Bolton and Jeanne (2011) analyze the interaction between multiple sovereigns of different credit quality and the banking system in a financially integrated area. They argue that a country issuing 'safe haven' government debt may derive rents from exploiting its position as monopolistic supplier of this safe asset. The model proposed here can also allow for non-competitive rents, but in contrast to Bolton and Jeanne (2011) , it could have lenders rather than the borrower extract them. Niepelt (2014) analyzes the composition of sovereign debt across maturities rather than lenders, as considered here. Tirole (2015) distinguishes between ex-post bailouts that aim at avoiding collateral damage and ex-ante risk-sharing (for example joint-and-several liability) among sovereigns.

Broner et al. (2013) develop a model without official creditors but with multiple classes of private creditors (domestic vs. foreign). Focusing on the effect of potential discrimination against foreigners on the crowding out of physical investment Broner et al. (2013) argue that the probability of default is lower for domestically held sovereign debt.[9]

Finally, the literature on "catalytic finance" links private lending to the lending of official creditors in the context of debt crises characterized by creditor *coordination* failures. A key point of this theory Rochet and Vives (2004); Corsetti et al. (2006); Morris and Shin (2006) is that the provision of official funds (say, by the IMF) can enhance the incentives for governments to undertake costly adjustment effort. This improves repayment capacity and may induce private creditors to supply more funds. Official funds thus act as a catalyst that solves a coordination failure problem. Our model shares with the catalytic finance literature the property that more official credit may increase the cost of default on privately held debt either by increasing the amount of—pledgeable—collateral or by placing—indirectly—private credit under the more effective official protection. But in our model, provision of official funds mitigates a commitment rather than a coordination problem. This difference implies that the entry of official creditors may not suffice to make official and private credit strategic complements even when the debtor's cost of default increases with official credit.

1.2. Outline

The rest of the paper is organized as follows. Section 2 provides empirical support for the main premises of the model. The model is set up in Section 3 and the equilibrium is characterized in Section 4. Section 5 analyzes the consequences of debt overhang for default and debt-composition decisions. Section 6 concludes. Appendices contain generalizations of the basic model as well as additional simulation results.

2. Empirical foundations of the model

The key new element in our model relative to the standard sovereign debt setup concerns the existence of differential enforcement powers across groups of creditors. Such differential powers may arise from various factors. One is that the credit relationship may be part of a broader set of relations between the borrower and the lender, as it is the case with participation in the same club. Consider, for instance, the relationship between Greece and the other members of the Eurozone. A Greek default on official loans from those countries could trigger retaliation and lower Greece's benefits from club membership in the European Monetary Union (EMU) or even the European Union (EU): structural fund payments and other transfers might be cut; Greece might be forced to leave the Eurozone; official lenders might be tempted to adopt policies that are less favorable to Greek interests; support for Greek foreign policy positions might wither; and so on.

As the ongoing crisis constitutes the first instance in which certain members of the Eurozone have borrowed large amounts from other members, and since no outright default against official creditors has occurred we cannot yet know whether official lenders would be in a position to inflict sanctions[10] of the type described above; and if they were, whether they would actually choose to do so. The drama surrounding recent negotiations between Greece and the other Eurozone countries about a third Greek bailout, for instance German finance minister Schaeuble proposing Grexit, suggests that they would. Of course, what matters for the behavior of agents in our model—and hence for the properties of equilibrium—is the *perception* of the existence and likely use of such sanctioning powers, rather than the use itself.[11] There is ample evidence from the public debate in Europe and statements by policy makers during the Eurozone crisis for a widely shared belief that superior sanctioning powers do exist and official lenders would be willing to use them.

For example, in Germany which provides most of the official financing, the statements by politicians, the debates in parliament and the public reaction all conjure the impression that the loans were perceived to face a low probability of default. In fact, such a perception was a sina qua non for large German loan provision at low rates to be politically feasible in the first place, given voters' expressed antipathy to transfers towards Greece. This perception was also founded in the knowledge that a default by Greece on debt held by official creditors amounts to violating EU treaties and breaking national laws, leaving Greece in uncharted and treacherous political territory regarding its future within the EU.[12] Naturally, time consistency is an issue as it would also be costly for Germany to impose sanctions ex post. But repeat business within the club (e.g., lending to Portugal, Ireland and Spain) renders reputation considerations important, and not imposing sanctions following a Greek default

[7] It may reduce it, as for instance in the analytical example in Section 4.4 .

[8] Boz's model also has the unsatisfactory property that accessing the IMF lending facilities changes preferences by assumption by triggering an increase in the sovereign's discount factor.

[9] This corresponds to the assumption that the default costs are higher when default occurs against foreign than against domestic holders of debt.

[10] Superior power certainly existed during the times when mighty countries would use military force to enforce repayment (for instance, when the British navy bombarded Athens).

[11] Naturally, in a model with asynchronous borrowing and default decisions of multiple borrowers, default by one country could reveal the existence of such powers and affect perceptions in those countries that have not made a default decision yet.

[12] The German government spokesman Steffen Seibert argued that the countries of the Eurozone could not accept a reduction in the value of their loans to Greece because this would contradict EU treaties as well as national legislation in Germany and other countries that prohibits member countries to assume the debts of other countries (Kathimerini, November 27, 2012).

could undermine Germany's credibility.[13]Note also that in order to ensure broad political support for enforcement ex post, Germany has required club-wide participation in the official lending operations and is now (in August 2015) pushing for renewed IMF-involvement in the third Greek bailout package.

Similar perceptions about the additional, severe cost of Greek default as a consequence of Eurozone loans were also held in Greece with all major parties voicing support for Greece's honoring its debt obligations to its official creditors.[14] The recent votes in the Greek parliament supporting overwhelmingly the measures proposed by the Troika also reflect this perception.

3. The model

The economy lasts for two periods, $t = 1, 2$. It is inhabited by a representative agent, a government and foreign investors. In period t the representative agent receives an exogenous endowment, y_t, which may be stochastic. The agent has time- and state-additive preferences over consumption with strictly increasing and concave felicity function $u(\cdot)$ and discount factor $\delta \in (0,1)$ but neither saves nor borrows. The economy starts with zero net assets vis-a-vis the rest of the world (in the next section, we allow for debt overhang of different maturities). Foreign investors are risk neutral, require a risk free gross interest rate $\beta^{-1} > 1$ and hold all government debt (since taxpayers do not save).[15] They consist of competitive private lenders and of official lenders.

Official lenders may or may not be competitive. Either as a consequence of this, or due to differences in the cost of funds across classes of lenders, the interest rates charged on official and private loans could differ.[16] In the benchmark model we have opted to assume that both types of lenders charge the same interest rate. It is straightforward to solve the model under the alternative assumption that the two interest rates differ either in an exogenous or an endogenous manner, giving rise to a price markup (a subsidy) or markdown (a service charge). We derive the implications of interest rate differentials in the working paper version of this article Dellas and Niepelt (2013) . A price markup offers a simple and immediate explanation of why sovereigns do not seek official funds during normal times.

The government is benevolent and maximizes the welfare of the representative agent. Without loss of generality, public spending other than debt repayment is normalized to zero. In period $t = 1$, the government issues zero-coupon, one-period debt, b, of which b^e is purchased by official and $b - b^e$ by private creditors at the price q. Crucially, the government cannot commit its successors (or future selves). Short-sales are ruled out. In period $t = 2$, the government sets a common repayment rate on both debt tranches, see the discussion above and in the introduction. This common repayment rate could reflect an explicit pari passu provision, the structure of sanctions or other fundamentals. The debt service is financed by taxes.

[13] Steffen Seibert has argued that debt forgiveness would lead to a huge loss of credibility for Germany and could encourage other countries with debt problems to ask for similar treatment (Kathimerini, November 27, 2012).

[14] One should not misunderstand the outcome of the July 5th, 2015 referendum as an indication of popular support for a Greek default. The government suggested a "no"-vote as a means of extracting more favorable terms from creditors.

[15] The assumption that the sets of taxpayers and investors do not "overlap" simplifies the analysis and does not matter for the main results.

[16] For example, the interest rate charged by the IMF exceeds the risk free rate by a "service" charge and the rate initially charged by European lenders to Greece similarly contained a price markdown. Such a charge could act as a deterrent to borrowing from official sources during normal times when market rates are close to the risk free rate. But during debt crises, when interest rates on private markets increase relative to the risk free rate, it could increase the incentive to borrow from official sources to suppress default spreads.

Let the repayment rate on debt maturing in period $t = 2$ be denoted by r_2 with $r_2 = 1$ representing full and $r_2 < 1$ partial repayment (default). Suppose that the cost of default is given by a loss of output whose size depends on the realization of a random variable, $\tilde{L}_2$, that is known before the default decision is made. The cdf of $\tilde{L}_2$ is a function of the state variables in period $t = 2$, $\tilde{F}(\tilde{L}_2; y_2, b, b^e)$; that is, the level of debt, its composition as well as the level of output may matter for the distribution function $\tilde{F}$ and thus for the risk of default.

This implies that changes in b and/or b^e have two conceptually distinct effects on default risk and debt prices. First, the usual dilution effect: For given values of the other state variables, an increase in b raises the probability of default because the government defaults when $\tilde{L}_2$ is small relative to b, see the discussion below. And second, a novel "credibility" effect: If a change in the quantity of debt or its composition alters the distribution function $\tilde{F}$, then it also alters the probability of default for a given amount of maturing debt. For example, if an increase in b^e, holding b constant shifts probability mass from low to high realizations of $\tilde{L}_2$ then it also increases the likelihood of repayment. The pari passu assumption is not important for the existence of dilution and credibility effects but it simplifies the analysis. In Appendix A, we present the general case. There, we also show that the distinction between dilution and credibility effects arises independently of whether default is associated with pure social losses—as we assume here—or with transfers to lenders.

We employ a simple specification of $\tilde{F}$ according to which an increase in b^e shifts the distribution function $\tilde{F}$ "to the right" and we also assume that the state variables do not matter for the shape of $\tilde{F}$. Formally, we let $\tilde{L}_2 \equiv L_2 + \phi(b^e), \phi' > 0$, and assume that L_2 is distributed according to a cdf F which is *not* a function of the state variables. The cost of default thus contains two components: a random one, L_2; and a deterministic one, $\phi(b^e)$, that increases with the sovereign's exposure to official credit.

The objective function of the government in period $t = 1$ takes the form

$$G_1(b, b^e) = u(y_1 + qb) + \delta \mathbb{E}_1 G_2(b, b^e) \tag{1}$$

where

$$G_2(b, b^e) = \max_{r_2} u(y_2 - br_2 - \mathbf{1}_{[r_2<1]}(L_2 + \phi(b^e)) \tag{2}$$

and $\mathbf{1}_{[x]}$ is the indicator function that takes the value of one when choice x has been made and zero otherwise.

Since private creditors are competitive and risk neutral they price their loans at $q = \beta \mathbb{E}_1 r_2$.

4. Equilibrium

4.1. Choice of repayment rate

We characterize equilibrium by backward induction starting with the choice of the repayment rate, r_2. Due to the specification of the default costs, the marginal cost of lowering r_2 is zero when $r_2 < 1$, so the optimal repayment rate equals either zero or unity. In particular,

$$r_2 = \begin{cases} 1 & \text{if} \quad L_2 \geq b - \phi(b^e) \\ 0 & \text{if} \quad L_2 < b - \phi(b^e) \end{cases} . \tag{3}$$

Condition (3) states that the government chooses to default when the resulting income losses, $L_2 + \phi(b^e)$, are smaller than the amount of debt due. The equilibrium price of funds is then

$$q = \beta \mathbb{E}_1 r_2 = \beta(1 - F(b - \phi(b^e))) \tag{4}$$

where F is the cdf of L_2. The price q is decreasing in the quantity of debt issued, b.

4.2. Choice of debt: private lenders

Issuing an extra unit of debt to private lenders has two effects on the funds obtained. First, it raises funds from this marginal unit of debt in proportion to its price, that is, one unit of b raises q units of funds in the present. And second, by lowering the price of debt (see Eq. (4)), it reduces the amount of funds raised from inframarginal units of debt. This latter effect is the direct consequence of the government's lack of commitment and reflects the endogeneity of subsequent repayment decisions. The total effect is given by the slope of the "debt-Laffer" curve as

$$\frac{\partial(qb)}{\partial b} = q + \frac{\partial q}{\partial b} b = \beta[(1-F) - fb] \tag{5}$$

where $f = F'$ is the pdf of L_2.

Funding is maximized at the top of the debt-Laffer curve, that is at the point where $\frac{\partial qb}{\partial b} = 0$. A perfectly myopic government ($\delta = 0$) would opt for the level of debt that corresponds to the maximum of the debt-Laffer curve. A non-myopic government ($\delta > 0$), in contrast, selects a lower level of debt because it also cares about future consumption.

Let λ and μ denote the multipliers associated with the short-sale constraints $b^e \geq 0$ and $b \geq b^e$, respectively. The effect of a marginal increase in debt issued to private lenders on the government's objective is given by

$$\frac{\partial G_1}{\partial b} = u'(c_1)\frac{\partial(qb)}{\partial b} + \delta\frac{\partial \mathbb{E}_1 G_2}{\partial b} + \mu$$

which can be expressed as[17]

$$(1-F)(\beta u'(c_1) - \delta\mathbb{E}_1 u'(y_2 - b)) - u'(c_1)b\beta f + \mu. \tag{6}$$

The first part of this marginal effect represents the consumption smoothing benefit from the marginal unit of debt. It differs from the corresponding expression in the case without default risk because the price of debt equals $\beta(1-F)$ rather than β and because debt repayment occurs with probability $1 - F$ rather than always.[18] The second part of the marginal effect arises because the repayment probability depends on the quantity issued: Each extra unit of debt lowers the price of all inframarginal units or, equivalently, raises the interest rate on them. This increase in the interest rate—which would be absent in a model with commitment—makes first period consumption more expensive. As a consequence, the equilibrium amount of debt issued (conditional on b^e) tends to be smaller than under commitment. The final part of the marginal effect, the multiplier μ, is strictly positive if the short-sale constraint $b \geq b^e$ is binding, and equals zero otherwise.

It may seem surprising that the negative welfare effect associated with the reduction of funds raised from inframarginal units of debt (the second part discussed above) is not balanced by a positive welfare effect from the reduced repayment probability of these inframarginal units in the future. In fact, this effect is present. However, it does not appear in (6) because it is equal in absolute value to another welfare effect of opposite sign, reflecting the increased risk of future social losses in the wake of default.[19] It is these social losses that are at the source of the reduced incentive (relative to the commitment case) for the government to issue debt. Niepelt (2014) contains a detailed discussion in the context of a model with multiple maturities.

4.3. Choice of debt: official lenders

Issuing debt to official lenders while holding total debt constant (that is, substituting official for private debt) raises the output losses of the borrowing country in case of default, thus reducing default risk. The resulting increase in the price of debt q has a positive effect on the amount of funds procured. Formally,

$$\frac{\partial(qb)}{\partial b^e} = b\frac{\partial q}{\partial b^e} = b\beta f\phi'$$

where $\phi' = d\phi/db^e$. The effect of substituting official for private funds on the government's objective is given by

$$\frac{\partial G_1}{\partial b^e} = u'(c_1)\frac{\partial(qb)}{\partial b^e} + \delta\frac{\partial \mathbb{E}_1 G_2}{\partial b^e} + \lambda - \mu$$

where the multipliers reflect the two short-sale constraints. This can be re-written as[20]

$$\phi'\left(u'(c_1)\beta f b - \delta\mathbb{E}_1\int_0^{b-\phi(b^e)} u'(y_2 - L_2 - \phi(b^e))dF(L_2)\right) + \lambda - \mu. \tag{7}$$

The first part of this marginal effect reflects the benefit from higher credibility. Due to more severe sanctions, a larger share of official debt generates stronger repayment incentives and hence lowers the default risk. For given b, this raises q and the amount of funds that can be obtained in the present, allowing the country to consume more in that period. The second part of the marginal effect reflects the cost of reduced flexibility. A larger share of official debt translates into additional income losses if default occurs. For given expected income losses, their consequence is greater when marginal utility in the default states is high, say because of a recession. Conditional on the distribution F, the cost of reduced flexibility therefore is a more important concern when income y_2 is negatively correlated with L_2, that is, if default states are states with low consumption. Consequently, since private credit offers better insurance (due to the associated enhanced default option) it will be favored by insurance considerations in the optimal portfolio choice.

4.4. Discussion

The marginal conditions derived above make clear that the optimal choice of the quantity and composition of sovereign debt depends on factors such as the intensity of the borrowing needs, as manifested in the ratio β/δ and the steepness of the output profile; the distribution function of output losses, F; preferences; and the enforcement technology, ϕ. The properties of the equilibrium can be

[17] We use the fact that

$$\begin{aligned}\frac{\partial \mathbb{E}_1 G_2}{\partial b} &= \frac{\partial}{\partial b}\int_0^{b-\phi(b^e)} \mathbb{E}_1[u(y_2 - L_2 - \phi(b^e))|L_2]\,dF(L_2) + \frac{\partial}{\partial b}\int_{b-\phi(b^e)}^{\infty} \mathbb{E}_1[u(y_2 - b)]dF(L_2) \\ &= \mathbb{E}_1[u(y_2 - b)]f - \mathbb{E}_1[u(y_2 - b)]f - (1-F)\mathbb{E}_1[u'(y_2 - b)].\end{aligned}$$

[18] With risk free debt, the marginal effect would reduce to $\beta u'(c_1) - \delta\mathbb{E}_1 u'(y_2 - b)$.

[19] Higher debt issuance increases subsequent default risk and thus, the risk of future output losses when default occurs. The corresponding first-order welfare effects that operate through the continuation value are zero. This is a consequence of an envelope condition—the successor government is indifferent at the margin between bearing the costs of debt repayment or suffering the income losses from default (see footnote 17).

[20] Note that $\partial\mathbb{E}_1 G_2/\partial b^e = -\phi'\mathbb{E}_1\left[\int_0^{b-\phi(b^e)} u'(y_2 - L_2 - \phi(b^e))dF(L_2)\right]$ (see footnote 17).

characterized analytically only for a limited set of specifications, for instance linear utility, linear default costs ϕ, and uniform F. By its very nature, however, the linear-uniform specification produces corner solutions. While it is adequate for certain purposes (for instance, to capture an important aspect of the interaction between debt overhang and the default and debt-composition decisions, see the next section), the linear-uniform specification is not useful for analyzing portfolio aspects. In order to study them we employ a concave (logarithmic) utility function and—for tractability—a discrete pdf for L_2 as well as a linear enforcement function, ϕ. Naturally, the key qualitative properties of the equilibrium are not affected by this choice. We discuss the continuous pdf case in Appendix B.

In particular, suppose that L_2 takes the value of zero with probability $1-\pi$ and the value of $\bar{L}_2$ with probability π. Let also $\phi(b^e)=\phi b^e, \phi>0$. Note that in the case of only private credit, the maximum loan that can be extended at a non-zero price is $\bar{b}=\bar{L}_2$ and its price $q=\beta\pi$. For official credit only, the maximum loan is $\bar{b}^e=\frac{\bar{L}_2}{1-\phi}$ and its price is $q=\beta\pi$. When both private and official funds co-exist, then the debt portfolio must satisfy the repayment condition (3), $b\le\bar{L}_2+\phi b^e$, both tranches are priced at $q=\beta\pi$, and the total quantity of debt lies between $\bar{b}$ and $\bar{b}^e$. That is, in all cases, the country defaults if the realization of L_2 is low and pays back when it is high.

The Langrangean associated with the government's optimization problem then is

$$\mathcal{L}=\ln(y_1+\beta\pi b)+\delta\pi\ln(y_2-b)+\delta(1-\pi)\ln(y_2-\phi b^e)+\nu(\bar{L}_2+\phi b^e-b)$$

(subject to the short-sales constraints) where ν denotes the multiplier associated with the repayment constraint. In order to guarantee that the commitment problem is operative, we assume that the optimal amount of state contingent debt under commitment, $\tilde{b}=\frac{\beta y_2-\delta y_1}{\beta(1+\delta\pi)}$ exceeds $\bar{b}$. If this is the case, $\nu>0$ and the optimal values for b, b^e solve the equations

$$\frac{\beta\pi}{y_1+\beta\pi b}-\frac{\delta\pi}{y_2-b}-\frac{\delta(1-\pi)}{y_2+\bar{L}_2-b}=0 \tag{8}$$

$$b^e-\frac{b-\bar{L}_2}{\phi}=0. \tag{9}$$

An interior solution with strictly positive b and b^e exists if the left-hand-side of Eq. (8) is less than zero at $b=\bar{L}_2$ and greater than zero at $b=\bar{L}_2$. For a numerical example, use $\beta=0.9, \delta=0.5, \pi=0.6, y_1=1, y_2=1.5, \bar{L}_2=0.4, \phi=0.3$. This produces $b=0.47$ and $b^e=0.23$. Noting that $\bar{b}=0.4$ and $\bar{b}^e=0.57$, this example reveals the key determinants of the debt portfolio issued by a sovereign, and in particular the trade-off between obtaining more funds now at the expense of decreasing future flexibility. In our example, the sovereign turns to official creditors in order to improve his repayment credibility and thus alleviate his borrowing constraint. But because official loans are costlier in the case of default, flexibility motives make the sovereign show restraint when tapping official funds. The trade-off between alleviating the thirst for current funds and constraining the default option is resolved at a level of total debt (0.47) that exceeds the level that would have maximized future flexibility ($b\le0.40, b^e=0$) but falls short of the one that maximizes current funds (0.57).

How does this trade-off vary with the parameters of the model? Let us consider how the share of official funds in total funds as well as the total amount of debt varies with the intensity of the borrowing needs (the value of δ) of the sovereign. Using Eq. (8) it can be shown that as δ decreases, the optimal quantity of debt, b, increases; and using Eq. (9), that the share of official loans to total loans ($s=b^e/b$) is decreasing in δ also. The former property is standard. The latter derives from the former: The sovereign can satisfy his greater appetite for current funds by issuing more official debt as this flattens the debt-Laffer curve and lowers borrowing costs. While this comes at the cost of reducing the option value of future default, a lower δ means a smaller concern for such costs. This general property of the model obtains independently of whether the price of debt is a continuous or discontinuous function of debt (as it would be with a continuous pdf or it is in this example, respectively), see Appendix B. The key implication, namely that sovereigns tend to favor official over private credit when borrowing costs on private markets are high, accords well with the empirical evidence.

Similar comparative static exercises can be conducted with regard to the enforcement power of the official creditors, ϕ; the expected rate of growth, y_2/y_1; and the probability of high realizations of sanction costs, π. Consider the sign of $ds/d\phi$. Eq. (8) implies that total debt is independent of ϕ. Using this fact in Eq. (9) we have for the share of official loans to total loans that $ds/d\phi<0$. That is, the higher ϕ the less b^e is needed in order to support a given total level of debt and thus the lower its share. Similarly, the effect of higher expected growth on s is positive. This is due to the fact that higher future relative to current output increases desired borrowing and this higher borrowing can only materialize by going to official creditors. Also a lower π makes official debt less desirable because it makes it more likely that a sovereign will default and thus suffer the higher costs associated with official funds. That is, $ds/d\pi>0$.

Through the paper we have emphasized the key trade off faced in the choice of official credit: Receiving more funds now but at the expense of facing reduced flexibility in the future due to the diminished default option. In order to highlight the superior *insurance* properties of private funds that arise from their enhanced default option it is necessary to introduce variation in economic activity in the second period. We thus make output in the second period stochastic and consider the effects of a mean preserving spread on total debt and the portfolio choice. For instance, assume that output in the second period can take two values, 1.3 and 1.5 with probability 0.5. The optimal portfolio choice now is $b=0.455$ and $b^e=0.15$. Higher uncertainty about future economic conditions thus decreases total borrowing and, more importantly from the point of view of the key mechanism in this paper, it leads to a substitution of private for official funds. Increasing further the probability of the bad state has even more drastic effects on the demand for public debt. For instance, if output in the second period takes the value of 1.3 with probability 0.8 and 2.3 with probability 0.2 then $b=0.41$ and $b^e=0.03$.

5. Debt overhang

We now explore the implications of debt overhang. We assume that the economy starts in period $t=1$ with inherited quantities of privately held sovereign debt b_1 and b_2 that are due in periods $t=1$ and $t=2$ respectively. Besides choosing a portfolio of debt instruments in period $t=1$, there is an additional decision, namely, whether to honor outstanding debt or not. We assume that default on b_1 is accompanied by default on b_2. Such "acceleration" represents standard practice in actual defaults involving multi-period debt. As we demonstrate below, acceleration has important consequences for the interaction of default and debt composition decisions.

We assume that default on privately held debt does *not* trigger exclusion from credit markets and only carries the cost discussed earlier. But since a default in period $t=1$ wipes out both debt maturing in the present and in the future, the default decision requires the comparison of expected utility streams rather than simply the comparison of the current default cost and the amount of debt due in that period. In general, there exists a critical value $\hat{L}_1$ such that the sovereign will choose to default on both b_1 and b_2 in period $t = 1$ whenever $L_1 < \hat{L}_1$; and to honor debt repayment otherwise.

In order to illuminate the role of debt overhang for the interaction of default and debt composition decisions, we use a specification of the model that favors corner solutions in the optimal choice of the debt instrument. This specification produces a switch from one source of funding to another and brings out the interaction between the two decisions most clearly. We assume linear utility with $u'(c) = 1$; linear sanctions with $\phi(b^e) = \phi b^e, 0 \leq \phi < 1$; and a uniform cdf for L_2 so that the probability of default in the second period, F, is given by $F = f \cdot (b - \phi b^e)$ if there has been default in the first period (that also wiped out b_2) and $F = f \cdot (b + b_2 - \phi b^e)$ otherwise, where $f = F'$. Naturally, under the linear specification of preferences, the insurance aspect of alternative debt instruments is missing and thus does not play any role in the portfolio selection; only expected values matter. Nonetheless, we already know how curvature in the utility function matters for the optimal portfolio choice: namely, it favors private credit. It is thus straightforward to make the necessary quantitative adjustments to the results reported below.

In period $t = 1$, the government's objective function is

$$G_1(b, b^e, r_1) = y_1 - b_1 r_1 - \mathbf{1}_{[r_1 < 1]} L_1 + \beta(1 - f \cdot (b + b_2 r_1 - \phi b^e))\, b + \delta \mathbb{E}_1 G_2(b, b^e),$$

$$G_2(b, b^e) = \int_0^{b + b_2 r_1 - \phi b^e} - (L_2 + \phi b^e) f dL_2 - (1 - f \cdot (b + b_2 r_1 - \phi b^e)) \times (b + b_2 r_1). \tag{10}$$

The optimal choice of b and b^e is determined by

$$\frac{\partial G_1}{\partial b} = (1 - F)(\beta - \delta) - \beta f b, \tag{11}$$

$$\frac{\partial G_1}{\partial b^e} = \phi(\beta f b - \delta F). \tag{12}$$

Holding b^e constant, G_1 is concave in b. The determinant of the Hessian is negative, so the Hessian is indefinite. This implies that any interior critical point of (10) constitutes a saddle point and consequently the equilibrium is in a corner. We consider the two corner equilibria—one with private debt and the other with official debt—in turn. The third equilibrium with zero debt is of no interest and is ruled out by assuming that δ/β is sufficiently small.

If all new sovereign debt is exclusively funded from private sources then the equilibrium level of debt is given by

$$b^{\mathrm{PR}} = \frac{1}{f} \frac{\beta - \delta}{2\beta - \delta} (1 - f b_2 r_1).$$

Less debt—by a factor of $(1 - f b_2 r_1)$—is issued relative to the case without long-term debt overhang. This is due to the fact that outstanding long-term debt already places the country higher up on the debt-Laffer curve, making default more likely, and thus new debt issuance more costly and less attractive.

When all new debt is financed by official sources then the equilibrium debt level is given by

$$b^{\mathrm{OF}} = \frac{1}{f} \frac{\beta - \delta}{2\beta - \delta(1 - \phi)} \frac{1}{1 - \phi} (1 - f b_2 r_1) - \frac{1}{f} \frac{\delta \phi f b_2 r_1}{2\beta - \delta(1 - \phi)} \frac{1}{1 - \phi}.$$

As in the case with private financing only, outstanding long-term liabilities reduce the incentive to issue new debt because they

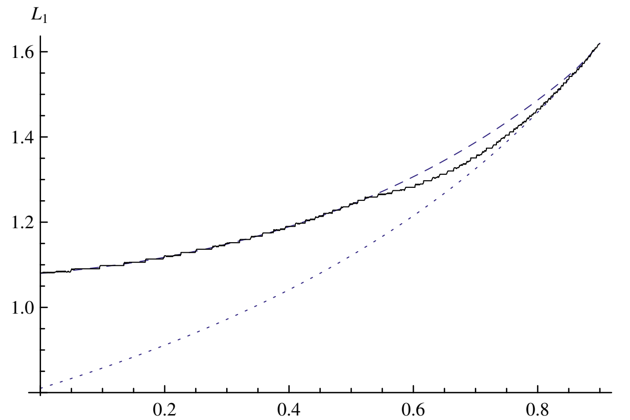

Fig. 1. $\hat{L}_1^{\mathrm{PR}}$ (dotted), $\hat{L}_1^{\mathrm{OF}}$ (dashed), $\hat{L}_1$ (solid).

place the borrowing country higher up on the debt-Laffer curve. This effect is reflected in the wedge $(1 - f b_2 r_1)$. But long-term debt overhang makes official financing particularly unattractive. This is because debt overhang already makes default more likely, so any debt instrument that carries high costs of default becomes less attractive. Consequently, optimal new debt issuance is even lower, a fact captured by the right-most term in the expression for b^{OF}.

Let us now turn to the default decision in the first period and its interaction with the debt composition decision. It is instructive to start with a situation where these two are independent of each other because there is only one source of funds available to the sovereign. Fig. 1 illustrates the default decision in this case. Ignore for the time being the solid line. The default threshold line $\hat{L}_1^{\mathrm{PR}}$ applies to the case when only private funds are available; and the threshold $\hat{L}_1^{\mathrm{OF}}$ applies when only official funds are available. Default occurs for realizations of default costs in period $t = 1$ below the relevant loci. For $b_2 = 0$, the default thresholds are independent of δ and the two loci coincide and are flat at the level b_1. For $b_2 > 0$, as in the example illustrated in the figure (where $b_1 = 0, b_2 = 2$), the loci have a positive slope.[21] More to the point, the figure shows that default in period $t = 1$ is always more likely when only official rather than private funds are available for refinancing (the default threshold locus in the latter case lies below that in the former case).

The solid line in Fig. 1 represents the equilibrium default threshold $\hat{L}_1$ when *both* sources of funds are available for refinancing, and the sovereign can select optimally which source to tap (recall that the equilibrium is always in a corner). It coincides with the default threshold $\hat{L}_1^{\mathrm{OF}}$ (the upper, dashed line), whenever the government *chooses* to borrow from official sources *independently* of the realization of L_1; and with the default threshold $\hat{L}_1^{\mathrm{PR}}$ (the lower, dotted line) whenever the government *chooses* to borrow from private sources *independently* of the realization of L_1. For instance, consider the situation with a low δ. In this case, the sovereign is hungry for funds in the present and cares little about maintaining flexibility in the future. Official funding can support a larger loan and represents the best choice in this case independent of the default decision in the current period (whether L_1 is high or low).

When the solid line coincides with either of the two threshold loci, the financing and default decisions are independent of each other; otherwise the two decisions are connected. Fig. 2 clarifies this interaction by depicting how the default decision correlates with the debt choice. Let us focus on the solid lines for the time being. This

[21] Lowering delta implies that the borrower attaches a lower weight to the benefit of eliminating debt due in the future while leaving the cost from current default unaffected.

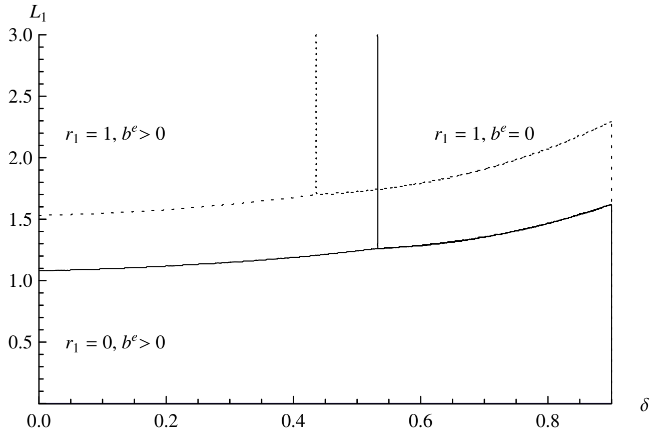

Fig. 2. Default and official lending regions for $b_2 = 2$ (solid) and $b_2 = 3$ (dashed).

line has both horizontal and vertical segments. The horizontal segment corresponds to the solid line in Fig. 1 so it marks the default decision: For realizations of L_1 above it there is no default and for realizations of L_1 below it there is default. The vertical lines represent the threshold for the choice of type of debt. To the right, the government chooses private debt and to the left it opts for official. The main point to notice here is that the two vertical segments do not lie on top of each other. That is, the debt choice depends on the default decision, with default in the first period favoring official and no default favoring private debt. The distance between the two vertical segments gives the range of δ over which default makes a difference for the type of debt instrument selected. For instance, for $\delta = 0.65$, the sovereign selects official if he defaults in the current period but private if he does not default.

The dotted lines correspond to a higher level of long-term debt overhang. Comparison of the dotted to the solid lines helps highlight the role of overhang for the default-debt-composition nexus. The horizontal segment of the dotted lines lies above its solid lines counterpart, which simply signifies that the higher the amount of debt overhang the more likely a default. The more interesting part, though, concerns the distance between the two vertical segments. As debt overhang increases, so does this distance. But this distance represents the parameter range over which the default and debt composition choices interact. The more to the left a vertical upper segment lies (the larger the debt overhang) the more likely it is that a sovereign opts for private debt if he does not default, even when δ is low.

The preceding analysis has studied the interaction between default and portfolio decisions under the assumption that the change (or, the cross country differences) in desired borrowing arises from a change (or, cross country differences) in the discount rate. It is straightforward to use the model to study this interaction in models where the change in borrowing arises from changes in current economic conditions relative to future ones (a current recession), an exogenous change in the risk free rate and so on.

6. Concluding remarks

We have extended the standard sovereign debt model to include multiple creditors with differential enforcement powers. Our model has several implications regarding a country's debt portfolio choice. First, the larger a country's financing needs, the more likely the use of official credit. Second, the higher the value of the default option (say, because of the possibility of bad economic states in the future) the less likely that the sovereign will seek official funds. The role played by the sovereign's level of credibility for this choice is more nuanced. While a high level of credibility favors private, low credibility may favor either private or official funds depending on the country's borrowing needs. This is due to the fact that the trade-off between the present benefits of official funds and their future costs varies with these needs. And third, the model implies that interest rates on debt issued to private creditors may decline when official creditors enter the scene even when total debt increases. This is because interest rates on either type of credit depend not only on the total amount of credit but also on its composition.

In the presence also of long-term, private debt overhang and under explicit or implicit—through the structure of sanctions—pari passu the model produces intertwined default and debt composition decisions. In particular, it predicts that countries with large long term obligations that choose to default will show a preference for official credit as a source of new funds. But those that do not default will show reluctance to draw official funds. The model also predicts that official would-be creditors may encourage a sovereign to default on outstanding private debt before they provide fresh funds.

An important advantage of our model is its simplicity. In particular, it generates rich cross-country variation of government choices as well as within country correlations without any need for political economy considerations.[22] An additional advantage is its generality: It can easily be applied to the study of other credit relationships that do not involve sovereign debt as long as the relationship contains classes of creditors that differ in terms of the punishment they can inflict on delinquent debtors.

Two extensions of the analysis seem worth pursuing. First, the examination of the properties of the model (for instance, the optimal ratio of private to total debt) in the absence of pari passu. And second, the exploration of normative issues, such as how the pari passu provision affects the welfare of the borrower.

Appendix A. Dilution versus credibility

For a general treatment of dilution and credibility effects, consider an economy with a set $\mathcal{I} = \{1, \ldots, I\}$ of international creditors who are competitive and require an expected gross return β^{-1}. The state at the beginning of period t includes (x_t, b_t) as well as time if the horizon is finite. Vector $b_t = \{b_{1t}, b_{2t}, ..b_{It}\}$ represents the quantities of zero coupon bonds held by the I creditors. Vector x_t includes the exogenous income, y_t, as well as the realization of a scalar random variable, L_t, that determines the consequences of a default. Income y_t follows an exogenous law of motion and L_t is distributed according to the cdf $F(\cdot\,; y_t, b_t)$ which may depend on the other states, in particular the debt portfolio b_t. This could be due to the fact that a default on specific creditor groups is more or less costly for the domestic economy (or, interpreted more generally, the ruling government). Or it may reflect the fact that specific creditor groups induce the government to implement policies that increase the stock of collateral or the bargaining power of creditors in a renegotiation.[23]

The sovereign chooses creditor specific repayment rates and fresh debt issuance, $(r_t, b_{t+1}) \in \mathcal{R} \times \mathcal{B}$ after observing the state where $\mathcal{R} = [0, 1]^I$ and $\mathcal{B} \subseteq \mathbb{R}^I_+$. Scalar $g(x_t, b_t, r_t) \geq 0$ with $g(x_t, b_t, 1) = 0$ represents the adverse consequences of default for the domestic economy which take the form of temporary income losses.[24] The default costs

[22] Ardagna and Caselli (2014) offer an informal, political economy perspective on the Greek default.

[23] Structural reforms under an IMF program may be interpreted in this light.

[24] The assumption of temporary income losses in the wake of a default is consistent with empirical evidence (see for example Panizza et al., 2009). The setup can be extended to capture other negative consequences of default. For example, default induced exclusion from financial markets (potentially sustaining trigger strategies) can be modeled by augmenting x_t with a state variable that summarizes the history of default choices and letting the choice set for debt issuance be a function of the state and the repayment rate, $\mathcal{B}(x_t, r_t)$.

$g(x_t, b_t, r_t)$ may amount to social losses (as is typically assumed in the sovereign debt literature) or to transfers. Social losses are present if the default costs reduce the consumption possibilities in the domestic economy without corresponding gain for creditors. If the costs are associated with gains for creditors, for example because default triggers a renegotiation that eventually gives rise to a compensation payment or because default implies a loss of collateral, then the costs have a transfer component. Let $\rho_t^i(x_t, b_t, r_t)b_t^i$ denote the compensation payment or collateral receipt by creditor i after a default. The polar case with complete social losses corresponds to vector $\rho_t(x_t, b_t, r_t) = 0$ for all (x_t, b_t, r_t); the opposite polar case with no social losses corresponds to $g(x_t, b_t, r_t) = \rho_t(x_t, b_t, r_t) \cdot b_t$ for all (x_t, b_t, r_t). In equilibrium, creditors earn the required rate of return such that

$$q_t(x_t, b_t, r_t, b_{t+1}) = \beta \mathbb{E}[r_{t+1}(x_{t+1}, b_{t+1}) + \rho_{t+1}(x_{t+1}, b_{t+1}, r_{t+1}(x_{t+1}, b_{t+1}))|\iota_t] \tag{13}$$

for all (x_t, b_t) where ι_t denotes the information set at the end of period t, including (x_t, b_t, r_t, b_{t+1}). Since the sovereign chooses b_{t+1} independently of r_t (and investors are forward looking), $r_t(\cdot)$ satisfies

$$r_t(x_t, b_t) = \arg\max_{r_t \in \mathcal{R}} -g(x_t, b_t, r_t) - b_t \cdot r_t \tag{14}$$

for all (x_t, b_t).

To simplify the notation we assume that the income losses in the wake of a default, $g(x_t, b_t, r_t)$, have a fixed cost character; that is, reducing the repayment rate on a debt tranche from a starting value strictly smaller than unity has no effect on $g(x_t, b_t, r_t)$ if the repayment rates on all other debt tranches are held constant. In the case with complete social losses, $\rho_t(x_t, b_t, r_t) = 0$, this implies that the equilibrium repayment rates either equal zero or unity.

A.1. Dilution versus credibility effects

A change in the quantity of debt or its composition has two types of effects on default choices (and thus, debt prices) and the government's value. Both types only arise when the government lacks commitment. First, "dilution effects." They arise because changes in the debt stock or composition alter the ex-post optimal default or rollover decisions of subsequent governments. Dilution effects are discussed at length in the sovereign and corporate debt literature. Second, "credibility (enhancing/reducing) effects." They arise if the distribution function of the random variable L_t varies with the state, in particular with b_t.

Formally, let $\hat{L}^i(y_t, b_t)$ denote the threshold value of the random variable L_t at or above which the repayment rate on tranche b_t^i equals unity (we assume that this threshold value is unique, for example because $g(x_t, b_t, r_t)$ is the sum of tranche specific default costs). The government then repays tranche b_t^i in full with probability $1 - F(\hat{L}^i(y_t, b_t); y_t, b_t)$, and it defaults with the complementary probability. A change of b_t, from $b_t = \varphi$ to $b_t = \psi$ say, triggers dilution effects by altering the default threshold value from $\hat{L}^i(y_t, \varphi)$ to $\hat{L}^i(y_t, \psi)$ and the compensation\index{The dilution effect can again be decomposed into two components, one related to the change of default costs} payment/collateral receipt in default states from $\rho_t^i(x_t, \varphi, r_t(x_t, \varphi))$ to $\rho_t^i(x_t, \psi, r_t(x_t, \psi))$; and it triggers credibility effects by altering the probability distribution of L_t from $F(\cdot; y_t, \varphi)$ to $F(\cdot; y_t, \psi)$.

A.2. Dilution and credibility effects on debt returns

Assume that $F(\cdot; y_t, b_t)$ is differentiable with respect to L_t, with pdf $f(\cdot; y_t, b_t)$; and that this density function (as well as the other equilibrium objects) is differentiable with respect to b_t. The conditionally expected return on debt tranche b_t^i then equals

$$\int_0^{\hat{L}^i(y_t,b_t)} \rho_t^i(x_t, b_t, r_t(x_t, b_t)) dF(L_t; y_t, b_t) + \int_{\hat{L}^i(y_t,b_t)}^{\infty} 1 dF(L_t; y_t, b_t).$$

The term on the left-hand side represents the probability of default times the conditional expectation of the "settlement" in this case; the term on the right-hand side represents the probability of full repayment. The effect of a marginal change of debt structure on the expected return can be expressed as

$$\begin{aligned} &\int_0^{\hat{L}^i(y_t,b_t)} \frac{d\rho_t^i(x_t, b_t, r_t(x_t, b_t))}{db_t^j} dF(L_t; y_t, b_t) \\ &\quad + \frac{\partial \hat{L}^i(y_t, b_t)}{\partial b_t^j} f(\hat{L}^i; y_t, b_t)(\rho_t^i(x_t, b_t, r_t(x_t, b_t))_{|\hat{L}^i} - 1) \\ &\quad + \int_0^{\hat{L}^i(y_t,b_t)} \left(\rho_t^i(x_t, b_t, r_t(x_t, b_t)) - 1\right) \frac{\partial f(L_t; y_t, b_t)}{\partial b_t^j} dL_t. \end{aligned} \tag{15}$$

The first two terms in (15) represent the conventional dilution effects and the third term the credibility effect due to the altered distribution function of L_t.

The first dilution effect, corresponding to the first term in (15), reflects modified debt returns in default states. It is only present if $\rho_t^i(x_t, b_t, r_t(x_t, b_t))$ varies with b_t and thus in particular, if $\rho_t^i(x_t, b_t, r_t(x_t, b_t)) > 0$.[25] The second type of dilution effect, corresponding to the second term in (15), arises due to reduced probability of repayment in full. When L_t equals $\hat{L}^i(y_t, b_t)$ a marginal increase of b_t^j implies a lower repayment rate r_t^i. When default triggers complete social losses the repayment rate drops to zero. Otherwise it falls by a smaller amount. Absent any social losses from default the second type of dilution effect is not present.

The third term in (15) represents the credibility effect. Suppose that a marginal increase of a debt tranche increases the probability of a specific realization of $L_t < \hat{L}^i(y_t, b_t)$ for which the sovereign does not repay b_t^i in full, and it decreases the probability of a realization with full repayment. Depending on the return in the default state, $\rho_t^i(x_t, b_t, r_t(x_t, b_t))$, this change of probability mass lowers the expected repayment rate on the debt tranche by more or less. The third term in (15) comprises the sum of all effects due to changes in probability mass.

With complete social losses, $\rho_t(x_t, b_t, r_t(x_t, b_t)) \equiv 0$, the credibility effect simplifies considerably under the assumption that a change in b_t "shifts" the density functions F and f in the sense that there exists a differentiable function $\alpha(\cdot)$ such that $F(L_t; y_t, b_t) \equiv$

[25] Suppose that lenders are on an equal footing such that returns in default states are identical across debt tranches,

$$\rho_t^i(x_t, b_t, r_t(x_t, b_t)) = \rho_t^j(x_t, b_t, r_t(x_t, b_t)) = \frac{\theta_t(x_t)}{\sum_{k \in \mathcal{I}} b_t^k}, i, j \in \mathcal{I},$$

for some function $\theta_t(\cdot)$. An increase in the quantity of one debt tranche then reduces the return on all tranches in all default states. Suppose instead that the first lender $(i = 1)$ is senior, receiving a return $\rho_t^1(x_t, b_t, r_t(x_t, b_t)) = \min[\theta_t(x_t)/b_t^1, 1]$, while junior creditors receive

$$\rho_t^i(x_t, b_t, r_t(x_t, b_t)) = \frac{\max[\theta_t(x_t) - b_t^1, 0]}{\sum_{k \in \mathcal{I} \backslash 1} b_t^k}, \quad i \in \mathcal{I} \backslash 1,$$

in default states. A larger volume of senior debt now reduces the return on junior loans (not on all loans as in the case with equal footing) unless default on the latter can be ruled out; and a larger volume of debt also reduces the return on the tranche itself (as in the case with equal footing) in default states.

$F(L_t - \alpha(b_t); y_t, 0)$ for all $b_t \geq 0$. Exchanging the order of integration and differentiationwe then have

$$\int_0^{\hat{L}^i(y_t,b_t)} (0-1)\frac{\partial f(L_t; y_t, b_t)}{\partial b_t^j} dL_t = -\int_0^{\hat{L}^i(y_t,b_t)} \frac{\partial f(L_t - \alpha(b_t); y_t, 0)}{\partial b_t^j} dL_t =$$
$$= -\frac{\partial F(\hat{L}^i(y_t,b_t) - \alpha(b_t); y_t, 0)}{\partial b_t^j}|_{\hat{L}^i} = \frac{\partial \alpha(b_t)}{\partial b_t^j} f(\hat{L}^i(y_t,b_t) - \alpha(b_t); y_t, 0). \quad (16)$$

Intuitively, a shift of the probability density function in combination with a fixed default threshold is equivalent to an unchanged probability density function in combination with a reduced default threshold. A parallel result holds if L_t has a discrete distribution. We use this result in the body of the paper.

A.3. Dilution and credibility effects on the continuation value

Consider the implications of debt structure b_t for expected utility in the repayment period. To simplify the notation, we assume that the cost function $g(x_t, b_t, r_t)$ is such that the default decision is perfectly correlated across debt tranches, with the default threshold denoted by $\hat{L}(y_t, b_t)$; we also disregard the utility flow from subsequent periods. Expected utility conditional on y_t then equals

$$\begin{aligned}
&F(\hat{L}(y_t,b_t); y_t, b_t) \times \mathbb{E}[u(y_t - g(x_t, b_t, 0) + b_{t+1}(x_t, b_t) \\
&\times q_t(x_t, b_t, 0, b_{t+1}(x_t, b_t)))|\iota_{t-1}, y_t, L_t < \hat{L}(y_t, b_t)] \\
&+ (1 - F(\hat{L}(y_t,b_t); y_t, b_t))\, u(y_t - b_t \cdot 1 + b_{t+1}(x_t, b_t) \\
&\times q_t(x_t, b_t, 1, b_{t+1}(x_t, b_t))).
\end{aligned}$$

Dropping some arguments of the utility function as well as conditioning variables for legibility, the dilution effect due to a change of debt structure from $b_t = \varphi$ to $b_t = \psi$ equals

$$\begin{aligned}
&F(\hat{L}(y_t,\psi); y_t, \varphi)\mathbb{E}_\varphi[u(-g(x_t, \psi, 0))|L_t < \hat{L}(y_t, \psi)] \\
&+ (1 - F(\hat{L}(y_t,\psi); y_t, \varphi))u(-\psi \cdot 1) \\
&- F(\hat{L}(y_t,\varphi); y_t, \varphi)\mathbb{E}_\varphi[u(-g(x_t, \varphi, 0))|L_t < \hat{L}(y_t, \varphi)] \\
&+ (1 - F(\hat{L}(y_t,\varphi); y_t, \varphi))u(-\varphi \cdot 1)
\end{aligned}$$

and the credibility effect is given by

$$\begin{aligned}
&F(\hat{L}(y_t,\psi); y_t, \psi)\mathbb{E}_\psi[u(-g(x_t, \psi, 0))|L_t < \hat{L}(y_t, \psi)] \\
&+ (1 - F(\hat{L}(y_t,\psi); y_t, \psi))u(-\psi \cdot 1) \\
&- F(\hat{L}(y_t,\psi); y_t, \varphi)\mathbb{E}_\varphi[u(-g(x_t, \psi, 0))|L_t < \hat{L}(y_t, \psi)] \\
&+ (1 - F(\hat{L}(y_t,\psi); y_t, \varphi))u(-\psi \cdot 1).
\end{aligned}$$

The dilution effect can again be decomposed into two components, one related to the change of default costs, $g(x_t, \psi, 0)$ versus $g(x_t, \varphi, 0)$, and the other related to the change of default threshold, $\hat{L}(y_t, \psi)$ versus $\hat{L}(y_t, \varphi)$. The credibility effect operates through a reallocation of probability mass between default and no-default states. It is absent if the probability measures before and after the change of debt structure coincides, $F(\cdot; y_t, \psi) = F(\cdot; y_t, \varphi)$.

Suppose that default costs constitute complete social losses and consider a change of debt structure from $b_t = \varphi$ to $b_t = \psi$ that reallocates probability mass among realizations of L_t that all fall short of the default threshold (before and after the change), $\hat{L}(y_t, b_t)$. Such a change does not affect the expected return on debt and thus, debt prices or domestic consumption in the period when the debt is issued; but it does affect the expected utility of domestic consumption in the repayment period if $g(x_t, \psi, 0) \neq g(x_t, \varphi, 0)$.

Appendix B. Optimal debt portfolio with a continuous PDF

One expects that the main determinant of the probability of default, namely the shape of the cdf of L would play a critical role for the properties of equilibrium. In Section 4.4 we abstracted from this feature for the sake of tractability. In this section, we study the role played by a continuous cdf. We consider a quadratic distribution function because the shape of this distribution corresponds well to that of the discrete case considered in Section 4.4, something that expedites and facilitates the interpretation of the results.

Fig. 3 shows three parametric examples from this distribution, A, B, and C. Table 1 reports the equilibrium debt portfolio and default risk for each of these examples, computed numerically by solving the marginal conditions with regard to b and b^e, Eqs. (8)–(9). Except for the distribution function, the values of the parameters of the model are the same as those in Section 4.4.

The shifting of mass from high to low realizations of L (moving from case A to case C) increases the probability of default. This has two opposing effects on the desirability of official funds. On the one hand, official funds become more desirable because they help partly offset the increase in default risk and thus support more borrowing in the present. On the other hand, they become less desirable because they increase the cost suffered by the borrower in case of default, and the probability of default has increased due to the shift in mass. Which effect proves stronger is in general ambiguous, depending on the curvature of the utility function, the ratios δ/β and y_2/y_1, the properties of the cdf and so on. For the family of distribution functions depicted in Fig. 3, the latter effect dominates, so the share of official funds in total funds decreases when moving from case A to case C. We saw a similar pattern in the analysis of debt overhang. There it was the debt overhang that made the credit relationship with official creditors perilous, here it is the exogenous change in the cdf of L.

Table 2 illustrates the role played by the size of borrowing needs (the value of δ) for the optimal composition of debt. For a given cdf of L, the sovereign chooses the private corner when borrowing needs are sufficiently small ($\delta = 0.55$). As they increase, the sovereign starts drawing funds also from official creditors ($\delta = 0.50$) and eventually switches completely to official funds when these needs become sufficiently high ($\delta = 0.4$).

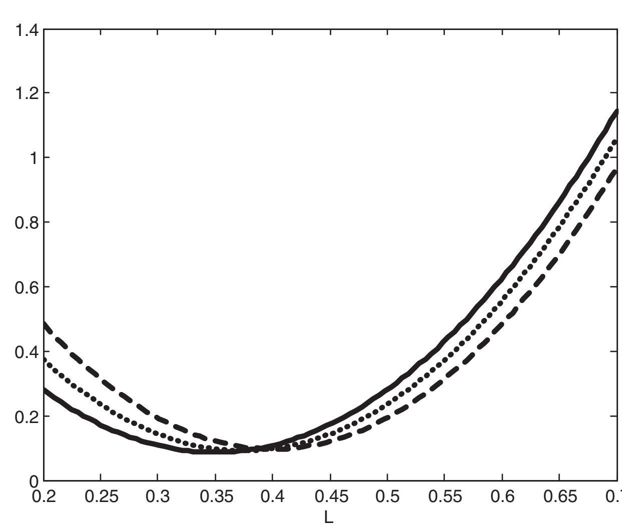

Fig. 3. Quadratic pdf: Case A (solid), case B (dotted), case C (dashed)

Table 1
Optimal debt portfolio: effect of the distribution function.

Case	A	B	C
b	0.583	0.577	0.546
b^e	0.421	0.236	0.034
$F(b-\phi b^e)$	0.184	0.228	0.277

Table 2
Optimal debt portfolio (case B): Effect of borrowing needs.

δ	0.55	0.50	0.40
b	0.500	0.577	0.690
b^e	0.000	0.236	0.690

References

Ardagna, S., Caselli, F., 2014. The political economy of the Greek debt crisis: a tale of two bailouts. Am. Econ. J. Macroecon. 6 (4), 291–323.

Arellano, C., 2008. Default risk and income fluctuations in emerging economies. Am. Econ. Rev. 98 (3), 690–712.

Bolton, P., Jeanne, O., 2011. Sovereign default risk and bank fragility in financially integrated economies. Working Paper 16899 NBER. Cambridge, Massachusetts.

Boz, E., 2011. Sovereign default, private sector creditors, and the IFIs. J. Int. Econ. 83 (1), 70–82.

Broner, F.A., Erce, A., Martin, A., Ventura, J., 2013. Sovereign debt markets in turbulent times: creditor discrimination and crowding out effects. Working Paper 19676 NBER. Cambridge, Massachusetts.

Corsetti, G., Guimaraes, B., Roubini, N., 2006. International lending of last resort and moral hazard: a model of IMF's catalytic finance. J. Monet. Econ. 53 (3), 441–471.

Dellas, H., Niepelt, D., 2013. .Credibility for sale. Discussion Paper 9562, CEPR.

Eaton, J., Gersovitz, M., 1981. Debt with potential repudiation: theoretical and empirical analysis. Rev. Econ. Stud. 48 (2), 289–309.

Gambetta, D., 1996. The Sicilian Mafia: The Business of Private Protection. Harvard University Press, Cambridge, Massachusetts.

Morris, S., Shin, H.S., 2006. Catalytic finance: when does it work?. J. Int. Econ. 70 (1), 161–177.

Niepelt, D., 2014. Debt maturity without commitment. J. Monet. Econ. 68, 37–54. S.

Panizza, U., Sturzenegger, F., Zettelmeyer, J., 2009. The economics and law of sovereign debt and default. J. Econ. Lit. 47 (3), 651–698.

Rochet, J.C., Vives, X., 2004. Coordination failures and the lender of last resort: was Bagehot right after all?. J. Eur. Econ. Assoc. 2 (6), 1116–1147.

Tirole, J., 2015. Country solidarity in sovereign crises. Am. Econ. Rev. 105 (8), 2333–2363.

Zame, W.R., 1993. Efficiency and the role of default when security markets are incomplete. Am. Econ. Rev. 83 (5), 1142–1164.

Zettelmeyer, J., Trebesch, C., Gulati, M., 2013. The Greek debt restructuring: an autopsy. Econ. Policy 513–563.