Macroeconomic Analysis

Dirk Niepelt
Study Center Gerzensee; University of Bern

This version: February 14, 2019

Chapter 11

The Government

We have so far abstracted from the public sector. Now we introduce a government whose policy instruments affect budget constraints, change incentives, and require resources. Our equilibrium concept remains unchanged—households and firms optimize and markets clear—but requires refinement: We assume that agents in the private sector take the government's policy as given; we require that the government also satisfies a budget constraint; and we account for the government's resource use when specifying market clearing conditions. A policy is *feasible* when it implements an equilibrium.

We analyze the macroeconomic implications of fiscal policy including taxation, government consumption, debt, and social security. Thereafter, we identify conditions under which policy changes do not alter the equilibrium allocation. Finally, we study how monetary and fiscal policy jointly affect inflation and output.

11.1 Taxation and Government Consumption

Consider the representative agent economy analyzed in section 3.1, augmented by a government sector. The government levies income taxes on labor, at rate τ_t^w, and on financial assets or capital, at rate τ_t^k.[1] The budget constraint of a household reads

$$a_{t+1} = a_t R_t (1 - \tau_t^k) + w_t (1 - \tau_t^w) - c_t$$

and household optimization thus implies the Euler equation

$$u'(c_t) = \beta R_{t+1} (1 - \tau_{t+1}^k) u'(c_{t+1}).$$

The tax on capital income reduces the net return on household saving (or borrowing). A higher τ_{t+1}^k therefore induces the same type of income and substitution effects as a lower R_{t+1} (see subsection 2.1.1), and it discourages saving. In contrast, the labor

[1] To simplify the notation, we assume that the capital income tax is levied on the gross rather than the net return that is, the principal is taxed as well. This is without loss of generality; each tax rate τ_t^k is associated with a tax rate on net capital income.

income tax does not induce a substitution effect since labor is supplied inelastically; it only reduces household wealth.

Taxes finance government consumption, g_t. We assume that household preferences are separable between private and government consumption (or that households do not value g_t) such that the household's first-order conditions are unaffected by g_t. For now, we also abstract from government deficits or surpluses that is, we assume that the government runs a *balanced budget* policy. This implies the government budget constraint

$$g_t = a_t R_t \tau_t^k + w_t \tau_t^w.$$

Substituting into the household's constraint yields

$$a_{t+1} = a_t R_t + w_t - c_t - g_t,$$

indicating that the tax financed government consumption reduces the household's disposable income.

Combining the budget constraints of households, firms, and the government and imposing the market clearing and equilibrium conditions discussed in section 3.1 yields the core equilibrium conditions

$$k_{t+1} = k_t(1-\delta) + f(k_t, 1) - c_t - g_t, \quad (11.1)$$

$$u'(c_t) = \beta(1 + f_K(k_{t+1}, 1) - \delta)(1 - \tau_{t+1}^k)u'(c_{t+1}). \quad (11.2)$$

Compared to the core conditions in the model without government, (3.8) and (3.9), the resource constraint (or GDP identity) now accounts for government consumption, and the tax rate on capital income enters the Euler equation.

In steady state,

$$\begin{aligned} c &= f(k, 1) - \delta k - g, \\ 1 &= \beta(1 + f_K(k, 1) - \delta)(1 - \tau^k). \end{aligned}$$

Since replacement investment, δk, and total consumption, $c + g$, sum to output, private consumption falls when government consumption rises, conditional on k_t. Moreover, since the after-tax return on saving equals β^{-1}, a tax on capital income reduces the capital stock, unlike a labor income tax. Accordingly, steady-state private consumption is maximal (conditional on g) if the government only levies labor income taxes.

Off steady state, capital income taxation also generates inferior outcomes. To see this, we compare the equilibrium allocation with the allocation in a Robinson Crusoe economy. Robinson Crusoe's program corresponds to the program studied in subsection 3.1.6 except that the additional term $-g_t$ enters the resource constraint. The optimality conditions then are given by (11.1) and (11.2) subject to $\tau_{t+1}^k = 0$. We conclude that the decentralized equilibrium allocation is Pareto optimal if and only if capital income taxes equal zero at all times.

Capital income taxes *distort* the allocation because they drive a wedge, $1 - \tau_{t+1}^k$, between the private and the social marginal rates of transformation. An individual household takes tax rates as given and equalizes the marginal rate of substitution between consumption at dates t and $t+1$ with the private marginal rate of transformation, $R_{t+1}(1-\tau^k_{t+1})$. This privately optimal choice does not account for the fact that the tax induced substitution towards consumption at date t has a social cost because individual tax avoidance requires a higher equilibrium tax rate for everybody else. From a societal point of view, the marginal rate of transformation equals R_{t+1} even if the return on saving partly is appropriated by the government. The substitution effect associated with the tax wedge therefore causes a *deadweight loss*.

The substitution effect also implies that the tax revenue increases less than proportionally with the tax rate: As the tax rate rises, the *tax base* shrinks. The fall in the tax base may (eventually) be so pronounced that the revenue declines although the rate further increases. Tax revenue as a function of the tax rate thus may be inverse-U shaped, displaying a *Laffer curve* relationship.

In the Robinson Crusoe economy deadweight losses are absent since the decision maker internalizes that government consumption needs to be funded. Deadweight losses also are absent in the decentralized equilibrium of an economy with *lump-sum taxes* that is, taxes whose amount the household cannot affect. With lump-sum taxes, T_t, the household's budget constraint reads

$$a_{t+1} = a_t R_t + w_t - c_t - T_t$$

and the equilibrium conditions coincide with the conditions in the economy with labor but no capital income taxes (since $T_t = g_t$ in equilibrium). This is not surprising; with exogenous labor supply, a tax on labor income does not induce substitution effects and therefore is a lump-sum tax.

These findings generalize. Not only does government consumption reduce household wealth, but its financing by means of taxes that induce substitution effects gives rise to welfare reducing tax distortions. Abstracting from distributive considerations, taxes that induce substitution effects (here, capital income taxes) therefore generate Pareto inferior outcomes than taxes that do not induce such effects (here, labor income taxes).

In endogenous growth models of the type considered in subsection 6.2.2, a tax induced reduction in the after-tax interest rate can lower the economy's equilibrium *growth rate*, with potentially large welfare consequences.

11.2 Government Debt and Social Security

Next, we introduce *government debt* as a source of funding. We abstract from distortions and assume that taxes only are levied on labor income, at rate τ_t. Allowing for population growth at the gross rate ν, the resource constraint is given by

$$\nu k_{t+1} = f(k_t, 1) + k_t(1-\delta) - c_t - g_t,$$

where capital as well as private and government consumption are expressed in per-worker terms.

Government debt allows to intertemporally decouple tax collections and government spending. The *dynamic government budget constraint* reads

$$\nu b_{t+1} = b_t R_t + g_t - \tau_t w_t,$$

where b_t denotes the stock of government debt per worker. The constraint states that a *primary deficit*, $g_t - \tau_t w_t > 0$, must be financed by debt issuance in excess of debt service (repayment of principal plus interest); when the government runs a *primary surplus*, in contrast, then debt is repaid on net. Equivalently, a *deficit*, $b_t(R_t - 1) + g_t - \tau_t w_t > 0$, increases the government's indebtedness, and a *surplus* reduces it. Note that we do not distinguish between the interest rates on government debt and capital. Market clearing requires that households are indifferent between the two assets and thus, absent risk, that the interest rates coincide.

Along a balanced growth path with constant per-capita values, the government budget constraint reads

$$\nu b = bR + g - \tau w.$$

Absent population growth, taxes equal government consumption and interest payments on debt. With strictly positive population growth, in contrast, taxes fall short of these spending items because new debt is issued in each period.

Let $q_0 \equiv 1$ and $q_t \equiv (R_1 \cdots R_t)^{-1}$. The no-Ponzi-game condition with equality, $\lim_{t\to\infty} q_t \nu^{t+1} b_{t+1} = 0$, and the dynamic budget constraints imply the *intertemporal government budget constraint*

$$0 = b_0 R_0 + \sum_{t=0}^{\infty} q_t \nu^t (g_t - \tau_t w_t).$$

It states that the value of government consumption spending and the value of tax revenues balance intertemporally, correcting for initial government indebtedness.

11.2.1 Government Debt with a Representative Agent

Consider first an economy with a representative household that maximizes $\sum_{t=0}^{\infty} \beta^t \nu^t u(c_t)$ subject to its dynamic budget constraint and a no-Ponzi-game condition. The Euler equation, dynamic and intertemporal budget constraints are given by

$$\begin{aligned} u'(c_t) &= \beta R_{t+1} u'(c_{t+1}), \\ \nu a_{t+1} &= a_t R_t + w_t(1-\tau_t) - c_t, \\ 0 &= a_0 R_0 + \sum_{t=0}^{\infty} q_t \nu^t (w_t(1-\tau_t) - c_t). \end{aligned}$$

Suppose the government initially balances its budget in each period such that $b_t = 0$ and $\tau_t w_t = g_t$, implying $k_t = a_t$. Consider a change of policy that alters the timing of tax collections while holding its present value (at the initial interest rates) constant. For example, suppose that the government reduces tax collections at date t by Δ per capita and increases taxes at date $t+1$ by $R_{t+1}\Delta/\nu$ per capita. The government also issues debt Δ per capita at date t and fully services it in the subsequent period out of the additional tax revenue.

Capital accumulation, consumption, interest rates and wages are not affected by this policy change. To establish this result, we conjecture that wages and interest rates indeed remain unchanged and verify that the initial capital and consumption sequences continue to satisfy all equilibrium conditions. Under the conjecture, the representative household's budget set is unaffected by the policy change because the wealth effects of the tax cut at date t and the tax hike at date $t+1$ cancel each other out,

$$-\Delta + \frac{\Delta R_{t+1}}{\nu}\frac{\nu}{R_{t+1}} = 0.$$

The initial consumption sequence therefore remains optimal for the household. From the household's dynamic budget constraint, this implies that the household increases saving when the government runs a deficit, by exactly the same amount. Since $a_{t+1} = b_{t+1} + k_{t+1}$, capital accumulation and thus, wages and interest rates therefore remain unchanged. Since the government's budget constraints are satisfied as well the conjecture is verified. We conclude that the equilibrium allocation remains unaltered although the tax and government debt paths change.

This result is an instance of the *Ricardian equivalence* proposition. The proposition states that for a given government consumption sequence (and thus, present value of taxes) the timing of tax collections does not affect the equilibrium allocation. Note that the proposition makes a statement about changes in government financing, not government consumption. The proposition holds under three key conditions, all of which are satisfied in the environment studied here. First, households and the government save or borrow at the same interest rates. Second, the policy change does not shift the tax burden from one group to another. And third, taxes are non-distorting. These conditions guarantee that a change of government financing does not alter budget sets in the private sector.

11.2.2 Government Debt with Overlapping Generations

Consider next an economy with two-period lived overlapping generations. In general equilibrium,

$$\begin{aligned}
u'(c_{1,t}) &= \beta R_{t+1} u'(c_{2,t+1}), \\
\nu(b_{t+1} + k_{t+1}) &= w_t(1-\tau_t) - c_{1,t}, \\
c_{2,t+1} &= \nu(b_{t+1} + k_{t+1}) R_{t+1},
\end{aligned}$$

where $c_{1,t}$ and $c_{2,t}$ denote consumption of a young and old household at date t, respectively. Note that we have imposed the market clearing condition, $a_{t+1} = b_{t+1} + k_{t+1}$.

In this economy, Ricardian equivalence does not hold. Reducing taxes at date t and increasing them at date $t+1$ shifts the tax burden from workers in cohort t to those in the subsequent cohort. The debt that the government issues to finance its deficit constitutes net wealth for the workers who acquire it in the sense that they do not have to contribute future resources to servicing it. Because of the tax cut's positive wealth effect on cohort t, the cohort increases saving by less than the amount of the tax cut, raising consumption.

Capital accumulation therefore slows down: Government debt *crowds out* capital. As a consequence, interest rates rise and wages fall in the subsequent period. Cohort t does not only benefit from a lighter tax burden but also from a higher return on saving while cohort $t+1$ bears a heavier tax burden and receives lower wages.

Recall that along a balanced growth path, the government budget constraint reads $\nu b = bR + g - \tau w$. If the economy is inefficient, $R < \nu$, then the revenue raised from additional debt issuance exceeds interest payments and the government may purchase goods without ever collecting taxes, simply by holding the debt-to-worker ratio constant ($g > 0, \tau = 0, b > 0$; note that this violates the no-Ponzi-game condition). Debt is welfare increasing in this case because it reduces capital over accumulation. Moreover, debt is a *bubble*: While it never generates dividends (tax revenues) it is rolled over forever at a positive price.

11.2.3 Pay-As-You-Go Social Security with Overlapping Generations

Maintaining the overlapping generations structure, consider finally a *pay-as-you-go social security system* in which workers contribute resources that finance contemporaneous benefits for retirees. Each old household at date t receives a transfer, $T_t\nu$, that is fully financed by labor income taxes levied at rate τ_t^s, such that $T_t = w_t\tau_t^s$. Abstracting from debt and government consumption equilibrium is characterized by the conditions

$$\begin{aligned} u'(c_{1,t}) &= \beta R_{t+1} u'(c_{2,t+1}), \\ \nu k_{t+1} &= w_t(1-\tau_t^s) - c_{1,t}, \\ c_{2,t+1} &= \nu k_{t+1} R_{t+1} + T_{t+1}\nu. \end{aligned}$$

For any feasible social security policy $\{\tau_t^s, T_t\}_{t\geq 0}$, there exists an *equivalent* tax-and-debt policy $\{\tau_t, b_{t+1}\}_{t\geq 0}$ of the type considered in subsection 11.2.2 that implements the same equilibrium allocation. To see this, note first that under the social security policy, the intertemporal budget constraint of a household in cohort t is given by

$$c_{1,t} + \frac{c_{2,t+1}}{R_{t+1}} = w_t(1-\tau_t^s) + \frac{T_{t+1}\nu}{R_{t+1}},$$

while under the tax-and-debt policy, the constraint reads

$$c_{1,t} + \frac{c_{2,t+1}}{R_{t+1}} = w_t(1-\tau_t).$$

Given the equilibrium prices supported by the social security policy, the budget sets characterized by the two constraints are identical if the present value of taxes net of transfers under the social security policy, $w_t \tau_t^s - T_{t+1}\nu/R_{t+1}$, equals taxes under the tax-and-debt policy, $w_t \tau_t$. Since $T_{t+1} = w_{t+1}\tau_{t+1}^s$, this implies the equivalence condition

$$\tau_t = \tau_t^s - \frac{w_{t+1}\tau_{t+1}^s \nu}{w_t R_{t+1}},$$

which maps the sequence of social security tax rates into a sequence of tax rates.

A second equivalence condition follows from the requirement that the dynamic budget constraints of the government (or households) be satisfied. The requirement that two policies pay the same amount of funds to the old at date t,

$$\nu b_t R_t = \nu w_t \tau_t^s,$$

relates the sequence of social security tax rates to a debt sequence. Since neither policy affects the resource constraint or the factor price conditions we conclude that the two equivalence conditions map any feasible social security policy into a tax-and-debt policy that implements the same equilibrium allocation and prices. Absent restrictions on the available tax and transfer instruments, similar mappings can be derived in environments where social security taxes are distorting, households long-lived and heterogenous within a cohort, or outcomes stochastic.

Intuitively, under the social security policy, households save little because they receive transfers in old age. Under the equivalent tax-and-debt policy, they save more because they pay lower taxes when young but do not receive transfers when old. The difference in saving exactly corresponds to the debt the government issues under the tax-and-debt policy. In light of this equivalence, one refers to the present value of the already committed future social security benefits as the *implicit debt* of the pay-as-you-go financed social security system.

Since implicit debt associated with a social security policy or other government program and explicit debt entail the same financial commitments, focusing on the latter and disregarding the former can be misleading. Explicit debt (net of government assets) does not comprehensively measure the fiscal burden a policy imposes on future generations as these generations also have to contribute resources to service the implicit debt. Generational accounts, in contrast, do provide a comprehensive measure. The *generational account* of a group is the present value of the group's remaining lifetime taxes net of received transfers. From the government's intertemporal budget constraint, the sum of all generational accounts equals the present value of current and future government consumption plus the outstanding government debt.

Suppose that at date $t = 0$ a pay-as-you-go social security system $\{\tau_t^s, T_t\}_{t \geq 0}$ is introduced. The effect on the budget set of an old household at date $t = 0$ and on the budget set of a member of cohort $t \geq 0$, respectively, are given by

$$w_0 \tau_0^s \nu \quad \text{and} \quad -w_t \tau_t^s + \frac{w_{t+1}\tau_{t+1}^s \nu}{R_{t+1}}.$$

The first generation receiving social security benefits clearly is made better off. Whether subsequent generations benefit or loose depends on whether the equilibrium is efficient or not. Along an inefficient balanced growth path ($R < \nu$ such that $-w\tau^s +$

$w\tau^s\nu/R > 0$) subsequent generations also benefit; along an efficient path, they are harmed. Using the relations derived earlier, we can equivalently represent the introduction of the social security system as a policy that finances a transfer to the old at date $t = 0$ out of taxes and debt, which subsequent cohorts service over time, see table 11.1.

	Pay-as-you-go		Explicit debt
Effect on household budget at date t			
lifetime net taxes:	τ_t	$=$	τ_t
$+$ taxes on young households	τ_t^s	$>$	τ_t
$-$ discounted old age benefits	$\frac{T_{t+1}\nu_{t+1}}{R_{t+1}}$	$>$	0
Effect on government budget at date t			
cash flow, $t = 0$:	0	$=$	0
$+$ total cash inflow	$N_0\tau_0^s$	$=$	$N_0\tau_0^s$
$+$ taxes on young households	$N_0\tau_0^s$	$>$	$N_0\tau_0$
$+$ debt issued	0	$<$	$N_0 b_1 \nu_1$
$-$ total cash outflow	$N_0 T_0$	$=$	$N_0\theta_0$
$-$ transfer to old households	$N_0 T_0$	$=$	$N_0\theta_0$
cash flow, $t > 0$:	0	$=$	0
$+$ total cash inflow	$N_t\tau_t^s$	$=$	$N_t\tau_t^s$
$+$ taxes on young households	$N_t\tau_t^s$	$>$	$N_t\tau_t$
$+$ debt issued	0	$<$	$N_t b_{t+1}\nu_{t+1}$
$-$ total cash outflow	$N_t T_t$	$=$	$N_t b_t R_t$
$-$ transfer to old households	$N_t T_t$	$>$	0
$-$ debt service	0	$<$	$N_t b_t R_t$

Notes: N_t denotes the size of cohort t and $\nu_{t+1} \equiv N_{t+1}/N_t$ the possibly time-varying growth rate. Wages are normalized to one. Equivalence then requires $\nu_{t+1}b_{t+1} = \tau_t^s - \tau_t$ and $T_{t+1} = b_{t+1}R_{t+1}$. In the economy with government debt, the transfer to the initial old, θ_0, corresponds with the transfer paid under the pay-as-you-go system. A "+" or "−" indicates positive or negative contributions.

Table 11.1: Equivalence of implicit and explicit government debt: Pay-as-you-go social security and explicit government debt.

In stochastic environments, a history-contingent social security policy (or equivalent tax-and-debt policy with history-contingent returns on government debt) can contribute to inter generational risk sharing. This may be valuable because, absent such policies, overlapping generations cannot implement all ex-ante beneficial insurance arrangements (see section 4.4). A social security policy that provides annuities may also contribute to intra generational risk sharing by insuring longevity risk.

A pay-as-you-go social security system contrasts with a *fully funded* system with individual accounts where households contribute resources in young age and consume the return on their contributions in old age. In equilibrium, the contributions fund capital accumulation. Changes in the contribution rate do not have macroeconomic effects as long as households can undo them by adjusting their savings outside of the system. That is, mandatory saving in a fully funded system is irrelevant as long as the desired saving of households exceeds the mandatory saving.

11.3 Equivalence of Policies

The Ricardian equivalence proposition discussed in subsection 11.2.1 describes *equivalence classes* of fiscal policies whose members implement the same equilibrium allocation that is, the same sequences for consumption, capital, wages, and interest rates but not necessarily for financial assets like government debt. Our discussion of equivalent pay-as-you-go social security and tax-and-debt policies in subsection 11.2.3 identified another type of equivalence classes. We now unify these discussions and consider additional applications.

11.3.1 General Equivalence Result

Let μ denote the state at the initial date and let φ denote a policy. Equivalence classes relate pairs of policies and states. A pair (μ, φ) and another pair $(\bar{\mu}, \bar{\varphi})$ belong to the same equivalence class if and only if both pairs implement the same equilibrium allocation.[2]

A direct approach to establishing that (μ, φ) and $(\bar{\mu}, \bar{\varphi})$ belong to the same equivalence class relies on characterizing the equilibrium allocations implemented by each pair (if they exist) and showing that they are identical. An indirect approach relies on establishing that the *choice sets* of households and firms are not affected by the change of policy. Suppose a pair (μ, φ) implements an equilibrium and suppose that another pair, $(\bar{\mu}, \bar{\varphi})$, satisfies the following conditions:

i. μ and $\bar{\mu}$ encode identical production possibilities, and restrictions on inputs and/or outputs of firms are identical across policies;

ii. households' choice sets are identical if evaluated at the equilibrium prices;

iii. at the equilibrium allocation and prices, $(\bar{\mu}, \bar{\varphi})$ satisfies the government's dynamic budget constraints.

Then, the two pairs belong to the same equivalence class.

This can be seen as follows: Conjecture that equilibrium prices under (μ, φ) and $(\bar{\mu}, \bar{\varphi})$ are the same. With household choice sets unchanged, household demand functions are unaltered since preferences do not depend on policy. With constraints on

[2]For simplicity, we disregard issues related to multiplicity of equilibria.

production unaffected, firm net supply functions are unaltered. The original household and firm choices (except possibly for financial assets) thus remain optimal and clear markets. Private sector choices and the government's new policy also satisfy all budget constraints. Given that the equilibrium allocation under (μ, φ) and $(\bar{\mu}, \bar{\varphi})$ is the same, the conjecture is verified.

11.3.2 Applications

The reasoning underlying the general equivalence result parallels the arguments that we made to establish Ricardian equivalence as well as the equivalence of pay-as-you-go social security and tax-and-debt policies. There, the choice set of a household is the set of affordable consumption allocations over the household's lifetime, and condition i. is trivially satisfied because the initial capital stock which corresponds to μ is held constant. But the result holds much more broadly as the following examples show.

Heterogeneity

Suppose that taxes are non-distorting but households are heterogeneous within cohorts. An equivalence class (conditional on some initial state) then consists of policies that satisfy the government budget constraints and impose on each household a given household specific present value of taxes.

Tax Distortions

Suppose that households value consumption and leisure such that labor income taxes are distorting. The choice set of a household then is given by the set of affordable consumption and leisure combinations. An equivalence class (conditional on the initial state) consists of policies that satisfy the government budget constraints and impose on each household a given household specific lifetime *tax function* which specifies the present value of taxes as a function of the household's choices. For example, one tax policy in such an equivalence class might tax labor income at date t at rate τ_t^w, while another policy in the same class might tax labor income at date t at rate $\tau_t^w R_{t+1}$ but collect the tax only in the subsequent period. Since both policies have the same effect on the household's choice set an equilibrium allocation implemented by the former policy also constitutes an equilibrium allocation under the latter. As with standard Ricardian equivalence, however, the two policies are associated with different levels of government debt.

Multiple Tax Instruments

Suppose that the government taxes consumption expenditures at rate τ_t^c, capital income at rate τ_t^k, and labor income at rate τ_t^w. The household's dynamic budget constraint reads

$$a_{t+1} = a_t R_t (1 - \tau_t^k) + w_t (1 - x_t)(1 - \tau_t^w) - c_t (1 + \tau_t^c),$$

where x_t denotes leisure. Iterating the dynamic budget constraint and imposing a no-Ponzi-game condition yields the intertemporal budget constraint

$$a_0 R_0 (1-\tau_0^k) + \sum_{t=0}^{\infty} q_t \kappa_t \left(w_t (1-x_t)(1-\tau_t^w) - c_t (1+\tau_t^c) \right) = 0,$$

where we define $\kappa_0 \equiv 1$ and $\kappa_t \equiv ((1-\tau_1^k)\cdots(1-\tau_t^k))^{-1}, t > 0$. Letting $\xi_t \equiv (1+\tau_t^c)/(1+\tau_0^c)$, we can rewrite this as

$$\frac{a_0 R_0 (1-\tau_0^k)}{1+\tau_0^c} + \sum_{t=0}^{\infty} q_t \kappa_t \xi_t \left(w_t (1-x_t) \frac{1-\tau_t^w}{1+\tau_t^c} - c_t \right) = 0.$$

From the household's perspective, the price of leisure relative to consumption equals $w_t(1-\tau_t^w)/(1+\tau_t^c)$ and the price of consumption at date $t+1$ relative to consumption at date t equals $q_{t+1}\kappa_{t+1}\xi_{t+1}/(q_t\kappa_t\xi_t) = (1+\tau_{t+1}^c)/(R_{t+1}(1-\tau_{t+1}^k)(1+\tau_t^c))$. That is, the *tax wedges*

$$\frac{1-\tau_t^w}{1+\tau_t^c} \text{ and } \frac{1+\tau_{t+1}^c}{(1-\tau_{t+1}^k)(1+\tau_t^c)}$$

distort the consumption-leisure and consumption-saving choices, respectively.

If $a_0 R_0 = 0$ then the three tax rates affect the household's budget set only through the two tax wedges. Feasible tax sequences generating the same wedge sequences constitute an equivalence class in this case. For example, a feasible tax policy employing all three tax instruments is equivalent to another policy that only relies on a specific combination of capital and labor income taxes.

If $a_0 R_0 \neq 0$ then the budget set also depends on $(1-\tau_0^k)/(1+\tau_0^c)$. A (non-distorting) tax levied on date-$t=0$ financial wealth, $\tau_0^k > 0$, can be replicated by a change of consumption and labor income taxes. To see this, suppose for simplicity that the initial policy imposes no taxes except for capital income taxes at date $t=0$, $\tau_0^k > 0$. This is equivalent to a policy with no capital income taxes but a positive consumption tax at date $t=0$, which satisfies $1+\tau_0^c = (1-\tau_0^k)^{-1}$; positive consumption taxes in all other periods, $\tau_t^c = \tau_0^c$, to keep intertemporal wedges unchanged; and subsidies for labor supply in all periods, $\tau_t^w = -\tau_t^c$, to keep relative prices between consumption and leisure unchanged.

11.4 Fiscal-Monetary Policy Interaction

When a government issues nominal liabilities such as nominal debt and central bank money then the government's budget constraint includes fiscal and monetary policy instruments as well as the price level. Accordingly, fiscal and monetary policy must be coordinated in equilibrium and their interplay affects inflation.

11.4.1 Consolidated Government Budget Constraint

Suppose that the government redeems *real* and *nominal debt*, $b_t(\epsilon^{t-1})$ and $B_t(\epsilon^{t-1})$ respectively; and issues new debt as well as additional money balances, $b_{t+1}(\epsilon^t)$, $B_{t+1}(\epsilon^t)$, and $M_{t+1}(\epsilon^t) - M_t(\epsilon^{t-1})$. Liabilities issued at date t and maturing at date $t+1$ are indexed by the history ϵ^t. Real or *inflation indexed debt* pays the potentially history-contingent gross rate of return $R_{t+1}(\epsilon^{t+1})$, expressed in real terms; nominal debt pays the gross rate of return $I_{t+1}(\epsilon^{t+1})$, expressed in nominal terms, which translates into the real rate of return $I_{t+1}(\epsilon^{t+1})\Pi_{t+1}^{-1}(\epsilon^{t+1})$, where $\Pi_{t+1}(\epsilon^{t+1})$ denotes gross inflation. Throughout, debt positions should be interpreted as net debt positions of the government.

Let $\{m_{t+1}(\epsilon^{t+1})\}_{t\geq 0}$ denote the stochastic discount factor and $P_t(\epsilon^t)$ the aggregate price level. Standard asset pricing (see equation (5.1) in section 5.3 and equation (9.2) in section 9.1) implies that the equilibrium price of real and nominal debt equals unity and $P_t^{-1}(\epsilon^t)$, respectively. To see the latter, rewrite the Euler equation for nominal debt as

$$1 = \mathbb{E}_t \left[m_{t+1}(\epsilon^{t+1}) \frac{I_{t+1}(\epsilon^{t+1})}{\Pi_{t+1}(\epsilon^{t+1})} \right] = \mathbb{E}_t \left[m_{t+1}(\epsilon^{t+1}) \frac{I_{t+1}(\epsilon^{t+1})/P_{t+1}(\epsilon^{t+1})}{1/P_t(\epsilon^t)} \right]$$

and note that $I_{t+1}(\epsilon^{t+1})/P_{t+1}(\epsilon^{t+1})$ is the real payoff of nominal debt. It follows that the equilibrium price of nominal debt equals $1/P_t(\epsilon^t)$.

Also, when issuing one unit of money, the government receives $1/P_t(\epsilon^t)$ units of the good in exchange. The *consolidated government budget constraint* therefore reads

$$\begin{aligned} & b_t(\epsilon^{t-1})R_t(\epsilon^t) + \frac{B_t(\epsilon^{t-1})I_t(\epsilon^t)}{P_t(\epsilon^t)} = \\ & \qquad \tau_t(\epsilon^t) - g_t(\epsilon^t) + b_{t+1}(\epsilon^t) + \frac{B_{t+1}(\epsilon^t)}{P_t(\epsilon^t)} + \frac{M_{t+1}(\epsilon^t) - M_t(\epsilon^{t-1})}{P_t(\epsilon^t)}, \end{aligned} \tag{11.3}$$

where $\tau_t(\epsilon^t) - g_t(\epsilon^t)$ denotes the primary surplus. Condition (11.3) states that the government funds maturing debt including interest (on the left-hand side) with its primary surplus and the revenue from new debt and money issuance (on the right-hand side).[3] Note that the real value of maturing government debt may be history-contingent for two reasons: Because of contingent interest rates or—with nominal debt—due to a stochastic price level.

Solving the dynamic budget constraint forward (using the Euler equation) and imposing a no-Ponzi-game condition yields the *intertemporal government budget constraint*

$$\begin{aligned} & b_t(\epsilon^{t-1})R_t(\epsilon^t) + \frac{B_t(\epsilon^{t-1})I_t(\epsilon^t)}{P_t(\epsilon^t)} = \\ & \sum_{j=0}^{\infty} \mathbb{E}_t \Big[(m_{t+1}(\epsilon^{t+1}) \cdots m_{t+j}(\epsilon^{t+j})) \times \\ & \qquad \left(\tau_{t+j}(\epsilon^{t+j}) - g_{t+j}(\epsilon^{t+j}) + \frac{M_{t+1+j}(\epsilon^{t+j}) - M_{t+j}(\epsilon^{t+j-1})}{P_{t+j}(\epsilon^{t+j})} \right) \Big]. \end{aligned} \tag{11.4}$$

[3] We assume that all debt is short-term that is, it matures after one period. With longer-term debt, the right-hand side of condition (11.3) would include changes in debt positions.

Condition (11.4) states that the market value of outstanding debt equals the present value of current and future primary surpluses including *seignorage* revenues, where seignorage is defined as the resources the government collects in exchange for the money it issues.

When we rewrite the dynamic budget constraint as

$$b_t(\epsilon^{t-1})R_t(\epsilon^t) + \frac{B_t(\epsilon^{t-1})I_t(\epsilon^t)}{P_t(\epsilon^t)} + \frac{M_t(\epsilon^{t-1})}{P_t(\epsilon^t)} = \\ \tau_t(\epsilon^t) - g_t(\epsilon^t) + b_{t+1}(\epsilon^t) + \frac{B_{t+1}(\epsilon^t)}{P_t(\epsilon^t)} + \frac{M_{t+1}(\epsilon^t)}{P_t(\epsilon^t)},$$

the left-hand side comprises a broader measure of government liabilities that includes both debt and outstanding money balances. Solving this equation forward yields

$$b_t(\epsilon^{t-1})R_t(\epsilon^t) + \frac{B_t(\epsilon^{t-1})I_t(\epsilon^t)}{P_t(\epsilon^t)} + \frac{M_t(\epsilon^{t-1})}{P_t(\epsilon^t)} = \\ \sum_{j=0}^{\infty} \mathbb{E}_t \Big[(m_{t+1}(\epsilon^{t+1}) \cdots m_{t+j}(\epsilon^{t+j})) \times \\ \left(\tau_{t+j}(\epsilon^{t+j}) - g_{t+j}(\epsilon^{t+j}) + \frac{m_{t+1+j}(\epsilon^{t+1+j})\ M_{t+1+j}(\epsilon^{t+j})\ i_{t+1+j}(\epsilon^{t+1+j})}{P_{t+1+j}(\epsilon^{t+1+j})} \right) \Big] .$$

The last term on the right-hand side represents an alternative measure of seignorage, namely the cost reduction for the government due to the fact that money, unlike debt, does not pay interest. Owing money rather than debt reduces the government's interest payments at date $t+j+1$ by $M_{t+1+j}(\epsilon^{t+j})i_{t+1+j}(\epsilon^{t+1+j})$; the real value as of date $t+j$ of this reduction is the reduction times $m_{t+1+j}(\epsilon^{t+1+j})/P_{t+1+j}(\epsilon^{t+1+j})$. Note that the cost reduction enters the budget constraint in parallel to a tax revenue.

If money paid interest (see subsection 12.3.1) then no such seignorage term would be present.

11.4.2 Seignorage Needs as Driver of Inflation

Consider a deterministic economy without debt and with fixed taxes and government consumption, $g - \tau > 0$. For an equilibrium to exist, seignorage revenue then must be sufficient to balance the budget. Specifically, equation (11.3) requires that

$$g - \tau = \frac{M_{t+1} - M_t}{P_t} = \frac{M_{t+1} - M_t}{M_t} \frac{P_{t-1}}{P_t} \frac{M_t}{P_{t-1}}.$$

The right-hand side of the equation indicates that seignorage is proportional to the money growth rate; inverse inflation; and the private sector's money demand, M_t/P_{t-1}. The latter dependence implies that the government is constrained in its ability to raise seignorage revenue.

Assume that M_t grows at the constant gross rate γ_M and the private sector's demand for real balances depends negatively on the nominal interest rate and thus (from the Fisher equation), expected inflation. Expected and actual inflation are equal to each other and to the money growth rate, reflecting a quantity theory relationship. The budget constraint can then be expressed as

$$g - \tau = \frac{\gamma_M - 1}{\gamma_M} \cdot \text{money demand}(\gamma_M).$$

The right-hand side of this equation is the product of an increasing and a decreasing function of γ_M: On the one hand, higher money growth increases the *inflation tax* that households pay when acquiring additional money in order to keep real balances constant; but on the other hand, households reduce real balances and thus, the base of the inflation tax in response to higher inflation and nominal interest rates. The two counteracting effects give rise to a *seignorage Laffer curve*—a hump shaped relationship between the inflation-tax rate, γ_M, and the seignorage revenue. Except for too high levels of seignorage revenue, there exist at least two money growth rates that generate that revenue, a low and a high one.

The turnpike model analyzed in subsection 9.2.2 provides micro foundations for an inflation elastic money demand function. Consider an equilibrium in which non-constrained households buy the bubble $a = M_{t+1}/P_t$ and sell it at value M_{t+1}/P_{t+1} in the subsequent period; the gross return on the bubble thus equals the inverse gross inflation rate, Π^{-1}. The difference between bubble purchases (by non-constrained households) and sales (by constrained households) in a period, $a(1 - \Pi^{-1})$, corresponds to the new bubble sales by the government. To satisfy the government budget constraint, these sales have to equal the primary deficit,

$$g - \tau = a(1 - \Pi^{-1}) = a\pi/\Pi,$$

where π denotes the net inflation rate.

Let $\bar{c} \equiv \bar{w} - a$ and $\underline{c} \equiv \underline{w} + a\Pi^{-1}$ denote equilibrium consumption of a household with high and low endowment, respectively. The resource constraint is given by $\bar{c} + \underline{c} = \bar{w} + \underline{w} - g$; the Euler equation of a household investing in the bubble reads

$$u'(\bar{c}) = \beta\Pi^{-1}u'(\underline{c});$$

and the borrowing constraint implies the condition $1 \geq u'(\bar{c})/u'(\underline{c})$ which is necessarily satisfied when $a \geq 0$ and $g - \tau > 0$.

For a given inflation rate the Euler equation pins down the demand for real balances, a, and thus seignorage revenue, $a\pi/\Pi$. Note that inflation affects seignorage revenue twofold. On the one hand, it gives rise to income and substitution effects on the demand for the bubble (see subsection 2.1.2); with logarithmic utility and $\underline{w} = 0$, these effects cancel. On the other hand, higher inflation increases the tax on bubble holdings as well as new bubble sales. When the bubble demand is sufficiently elastic seigniorage is a hump shaped function of the inflation rate.

11.4.3 Inflation Effects of Government Financing

Consider an endowment economy inhabited by an infinitely lived representative household that owns a history-contingent endowment sequence, $\{w_t(\epsilon^t)\}_{t\geq 0}$; and a government with history-contingent resource requirement $\{g_t(\epsilon^t)\}_{t\geq 0}$. Households may not consume their own endowments, and consumption goods can only be sold to, and bought from, other households against cash (see subsection 9.3.3). The government also must use cash to purchase goods. The economy's resource constraint reads $w_t(\epsilon^t) = c_t(\epsilon^t) + g_t(\epsilon^t)$. The households' and government's cash-in-advance constraints (which bind because of positive interest rates) are given by $c_t(\epsilon^t) = M^h_{t+1}(\epsilon^t)/P_t(\epsilon^t)$ and $g_t(\epsilon^t) = M^g_{t+1}(\epsilon^t)/P_t(\epsilon^t)$, respectively. Let $M_{t+1}(\epsilon^t) \equiv M^h_{t+1}(\epsilon^t) + M^g_{t+1}(\epsilon^t)$. Securities are traded and money holdings chosen after the state of nature is realized, before cash transactions take place.

The household's dynamic budget constraint reads

$$\tau_t(\epsilon^t) + b_{t+1}(\epsilon^t) + \frac{B_{t+1}(\epsilon^t)}{P_t(\epsilon^t)} + \frac{M^h_{t+1}(\epsilon^t)}{P_t(\epsilon^t)} = \\ b_t(\epsilon^{t-1})R_t(\epsilon^t) + \frac{B_t(\epsilon^{t-1})I_t(\epsilon^t)}{P_t(\epsilon^t)} + \frac{M^h_t(\epsilon^{t-1})}{P_t(\epsilon^t)} + \frac{w_{t-1}(\epsilon^{t-1}) - c_{t-1}(\epsilon^{t-1})}{\Pi_t(\epsilon^t)},$$

where the last term on the right-hand side represents the real value of cash inflows from endowment sales, net of cash outflows for consumption purchases in the previous period. Since the cash-in-advance constraint binds, this collapses to

$$\tau_t(\epsilon^t) + b_{t+1}(\epsilon^t) + \frac{B_{t+1}(\epsilon^t)}{P_t(\epsilon^t)} + c_t(\epsilon^t) = \\ b_t(\epsilon^{t-1})R_t(\epsilon^t) + \frac{B_t(\epsilon^{t-1})I_t(\epsilon^t)}{P_t(\epsilon^t)} + w_{t-1}(\epsilon^{t-1})\Pi_t^{-1}(\epsilon^t).$$

Since the revenue from endowment sales accrues in cash that must be carried into the next period, inflation or even low deflation ($\Pi_t^{-1}(\epsilon^t) < R_t(\epsilon^t)$) acts as a (non-distorting) tax on sales.

The initial state in the economy is $\mu = (M^h_0, M^g_0, b_0R_0, B_0I_0)$ and a policy is given by

$$\varphi \equiv \{\tau_t(\epsilon^t), g_t(\epsilon^t), b_{t+1}(\epsilon^t), R_{t+1}(\epsilon^{t+1}), B_{t+1}(\epsilon^t), I_{t+1}(\epsilon^{t+1}), M_{t+1}(\epsilon^t)\}_{t\geq 0}.$$

The endogenous variables are $\{c_t(\epsilon^t), P_t(\epsilon^t), m_{t+1}(\epsilon^{t+1}), M^h_{t+1}(\epsilon^t), M^g_{t+1}(\epsilon^t)\}_{t\geq 0}$. Equilibrium conditional on (μ, φ) requires

$$\begin{aligned}
c_t(\epsilon^t) &= w_t(\epsilon^t) - g_t(\epsilon^t), \\
m_{t+1}(\epsilon^{t+1}) &= \beta \frac{u'(w_{t+1}(\epsilon^{t+1}) - g_{t+1}(\epsilon^{t+1}))}{u'(w_t(\epsilon^t) - g_t(\epsilon^t))}, \\
\frac{M^h_{t+1}(\epsilon^t)}{M^g_{t+1}(\epsilon^t)} &= \frac{w_t(\epsilon^t) - g_t(\epsilon^t)}{g_t(\epsilon^t)}, \\
1 &= \mathbb{E}_t \left[m_{t+1}(\epsilon^{t+1}) R_{t+1}(\epsilon^{t+1}) \right], \\
1 &= \mathbb{E}_t \left[m_{t+1}(\epsilon^{t+1}) I_{t+1}(\epsilon^{t+1}) \Pi^{-1}_{t+1}(\epsilon^{t+1}) \right], \\
P_t(\epsilon^t) &= \frac{M_{t+1}(\epsilon^t)}{w_t(\epsilon^t)},
\end{aligned}$$

as well as the government budget constraints, conditions (11.3) and (11.4). Walras' law implies that the household budget constraints then are satisfied as well.

We are interested in policies that implement the same allocation. Such policies include the same government consumption sequence but they might differ from each other with respect to taxes, debt instruments, money balances, or nominal interest rates. Our objective is to understand whether, and how changes in these policy instruments alter the equilibrium price level sequence.

Irrelevance of Debt Composition

Note first a neutrality result: A feasible change of the *composition* of *government debt* between real and nominal debt accompanied by no change of taxes or money supply does not alter inflation. Such a policy change neither affects total indebtedness in real terms nor total debt issuance. For example, a feasible policy φ with positive real and nominal debt implements the same equilibrium inflation as a modified policy $\bar{\varphi}$ with zero nominal debt and

$$\begin{aligned}
\bar{b}_t(\epsilon^{t-1}) \bar{R}_t(\epsilon^t) &= b_t(\epsilon^{t-1}) R_t(\epsilon^t) + \frac{B_t(\epsilon^{t-1}) I_t(\epsilon^t)}{P_t(\epsilon^t)}, \ t \geq 1, \\
1 &= \mathbb{E}_t \left[m_{t+1}(\epsilon^{t+1}) \bar{R}_{t+1}(\epsilon^{t+1}) \right].
\end{aligned}$$

Throughout the subsection, we therefore abstract from nominal debt, without loss of generality.

Policy Mixes

We consider feasible policy changes that affect taxes, seignorage and debt subject to (11.3), (11.4), the asset pricing condition, and the cash-in-advance constraint. To satisfy the equilibrium conditions, the policies before and after the change, φ and $\bar{\varphi}$ respectively, and the associated price level sequences, $\{P_t\}_{t\geq 0}$ and $\{\bar{P}_t\}_{t\geq 0}$, must be related as follows:

$$\begin{aligned}
& \bar{\tau}_t(\epsilon^t) + \frac{\bar{M}_{t+1}(\epsilon^t) - \bar{M}_t(\epsilon^{t-1})}{\bar{P}_t(\epsilon^t)} + \bar{b}_{t+1}(\epsilon^t) - \bar{b}_t(\epsilon^{t-1})\bar{R}_t(\epsilon^t) \\
&= \tau_t(\epsilon^t) + \frac{M_{t+1}(\epsilon^t) - M_t(\epsilon^{t-1})}{P_t(\epsilon^t)} + b_{t+1}(\epsilon^t) - b_t(\epsilon^{t-1})R_t(\epsilon^t), \\
\bar{P}_t(\epsilon^t) &= \bar{M}_{t+1}(\epsilon^t)/w_t(\epsilon^t), \\
0 &= \mathbb{E}_t\left[m_{t+1}(\epsilon^{t+1})(\bar{R}_{t+1}(\epsilon^{t+1}) - R_{t+1}(\epsilon^{t+1}))\right].
\end{aligned}$$

The new policy must also satisfy condition (11.4). We consider several special cases of such policy changes.

Current vs. Future Taxes

Delaying taxation and financing the temporary revenue shortfall by issuing government debt leaves the money supply unchanged. The equilibrium price level sequence then is unchanged as well. For example, altering taxes and debt issuance according to

$$\begin{aligned}
\bar{\tau}_0(\epsilon^0) &= \tau_0(\epsilon^0) - \Delta, \\
\bar{b}_1(\epsilon^0) &= b_1(\epsilon^0) + \Delta, \\
\bar{\tau}_1(\epsilon^1) &= \tau_1(\epsilon^1) + R_1(\epsilon^1)\Delta,
\end{aligned}$$

where $\Delta > 0$, has no effect on price levels.

Seignorage vs. Future Taxes

A one-time change of the composition of government liabilities between money and debt coupled with a subsequent change of taxes has a permanent effect on the price level. Formally, let

$$\begin{aligned}
\bar{\tau}_0 &= \tau_0, \\
\bar{b}_1(\epsilon^0) + \frac{\bar{M}_1(\epsilon^0) - M_0}{\bar{P}_0(\epsilon^0)} &= b_1(\epsilon^0) + \frac{M_1(\epsilon^0) - M_0}{P_0(\epsilon^0)}, \\
\bar{P}_0(\epsilon^0) &= \bar{M}_1(\epsilon^0)/w_0(\epsilon^0),
\end{aligned}$$

where the two seignorage terms differ. Since the policy $\bar{\varphi}$ does not involve further changes in seignorage the effect on the price level is permanent.[4] The change of debt issuance at date $t = 0$ implies $\bar{b}_1(\epsilon^0)\bar{R}_1(\epsilon^1) \neq b_1(\epsilon^0)R_1(\epsilon^1)$ for some history ϵ^1. Long-term budget balance therefore requires an appropriate adjustment of taxes subsequent to ϵ^1. With this adjustment, all equilibrium conditions are met.

[4] To see this, consider the deterministic case. The cash-in-advance constraint implies $\{\bar{M}_{t+1}/\bar{P}_t\}_{t\geq 0} = \{M_{t+1}/P_t\}_{t\geq 0}$. $(\bar{M}_1 - M_0)/\bar{P}_0 \neq (M_1 - M_0)/P_0$ implies $\bar{P}_0 \neq P_0$ and $\bar{M}_1 \neq M_1$. $(\bar{M}_2 - \bar{M}_1)/\bar{P}_1 = (M_2 - M_1)/P_1$ implies $\bar{M}_1/\bar{P}_1 = M_1/P_1$ and thus, $\bar{P}_1 \neq P_1$ and $\bar{M}_2 \neq M_2$. The argument extends to subsequent periods.

The example illustrates that monetary policy interventions that change the government's portfolio do not only affect the price level but also have fiscal consequences. An *open market operation* in which the government purchases government debt from households against cash constitutes an example of such an intervention.

Current vs. Future Seignorage

A debt financed reduction of seignorage that is accompanied by a subsequent increase of seignorage permanently alters the price level. In fact, such a monetary contraction coupled with a subsequent expansion implies a higher price level in the long run.

For a simple example, consider a deterministic environment with constant endowment, w, and gross interest rate, $R > 1$. Feasible policy φ involves no seignorage revenues, $M_t = M$, such that $P_t = P = M/w$. Under the modified policy, $\bar{\varphi}$, money balances are reduced at date $t = 0$ and kept constant until date $t = T - 1$ when they are increased again. That is,

$$\begin{aligned} \bar{M}_{t+1} &= M - \Delta_1,\ t = 0, \ldots, T-2, \\ \bar{M}_{t+1} &= M - \Delta_1 + \Delta_T,\ t = T-1, T, \ldots. \end{aligned}$$

The cash-in-advance constraints imply

$$\begin{aligned} \bar{P}_t &= (M - \Delta_1)/w,\ t = 0, \ldots, T-2, \\ \bar{P}_t &= (M - \Delta_1 + \Delta_T)/w,\ t = T-1, T, \ldots. \end{aligned}$$

Under $\bar{\varphi}$, seignorage revenues at date $t = 0$ and date $t = T - 1$, respectively, equal $-\Delta_1 w/(M - \Delta_1)$ and $\Delta_T w/(M - \Delta_1 + \Delta_T)$. To satisfy the budget constraint (11.4), the present value of these revenues must equal zero, implying

$$\frac{\Delta_T w}{M - \Delta_1 + \Delta_T} = R^{T-1} \frac{\Delta_1 w}{M - \Delta_1} \ \Rightarrow\ \frac{\Delta_T}{\Delta_1} = R^{T-1} \frac{M - \Delta_1 + \Delta_T}{M - \Delta_1}.$$

This implies that $\Delta_T > \Delta_1$, both since $R > 1$ and $\bar{P}_{T-1} > \bar{P}_0$. That is, following a monetary contraction at date $t = 0$, money balances and thus, the price level *increase* in the long run because the monetary contraction generates a revenue shortfall which requires higher future inflation to balance the budget. A postponement of the expansionary policy increases the long-run price level (P_T increases in T).

This so-called *unpleasant monetarist arithmetic* illustrates how the fiscal implications of a monetary policy intervention force an eventual policy reversal when fiscal policy does not accommodate the intervention; and that the inflationary effects of the reversal can dominate those of the intervention.

11.4.4 Game Of Chicken

When monetary and fiscal policy are controlled by separate authorities—a central bank on the one hand and a fiscal authority on the other—then the institutional structure governing the policy coordination can have important macroeconomic implications.

Suppose the fiscal authority moves first in the sense of committing to history-contingent tax and government consumption sequences before the monetary authority chooses the money supply. By moving first, the fiscal authority shifts responsibility for implementing an equilibrium to the monetary authority; the latter must generate sufficient seignorage to satisfy the intertemporal budget constraint. In this *game of chicken* the central bank's choice set is restricted by the actions of the fiscal authority. Although the central bank may wish to conduct a monetary policy aimed at stabilizing the price level say, its second mover status can frustrate this plan.

Threats to price stability of this kind can be countered by instituting an arrangement that guarantees *central bank independence* and assigns the first mover advantage to the monetary authority. An independent central bank is relieved of the responsibility for intertemporally balancing the budget; that responsibility lies with the fiscal authority.

11.4.5 Fiscal Theory of the Price Level

When nominal debt is outstanding at date $t = 0$, the government's intertemporal budget constraint (11.4) may not only be balanced by appropriate choices of government consumption, taxes, or seignorage revenues, but also by a revaluation of nominal debt through a change of the price level. The *fiscal theory of the price level* emphasizes this possibility. It views the intertemporal budget constraint (11.4) not as a constraint on government actions but as an equilibrium condition that determines the price level.

To motivate the theory, consider a static model. Suppose that nominal liabilities $B_0 I_0$ are outstanding at date $t = 0$ which is the last period; both money balances and seignorage are negligible; and there is no real debt. Suppose further that the choice of fiscal policy is *non-Ricardian*: Rather than balancing the government's budget by setting $\tau_0 - g_0 = B_0 I_0 / P_0$ for whatever equilibrium price level is realized (as would be the case in the *Ricardian* case), the government sets $\tau_0 - g_0$ independently of P_0. For an equilibrium to exist, P_0 must adjust to $B_0 I_0 / (\tau_0 - g_0)$; the price level is fiscally determined.[5]

For a more detailed analysis, consider the model introduced in subsection 11.4.3. We abstract from real debt and assume that the nominal interest rate is risk-free such that $B_{t+1}(\epsilon^t) I_{t+1}(\epsilon^t)$ is constant across all histories ϵ^{t+1} subsequent to history ϵ^t. Accordingly, the dynamic budget constraint of the government reads

$$\frac{B_t(\epsilon^{t-1}) I_t(\epsilon^{t-1})}{P_t(\epsilon^t)} = \tau_t(\epsilon^t) - g_t(\epsilon^t) + \frac{B_{t+1}(\epsilon^t)}{P_t(\epsilon^t)} + \frac{M_{t+1}(\epsilon^t) - M_t(\epsilon^{t-1})}{P_t(\epsilon^t)},$$

[5]There is an even simpler mechanism without debt to fiscally determine the price level. It relies on the government setting government consumption in real terms and tax revenue in nominal terms. The budget balance requirement pins down the price level.

and the intertemporal budget constraint at date $t=0$ is given by

$$\frac{B_0(\epsilon^{-1})I_0(\epsilon^{-1})+M_0(\epsilon^{-1})}{P_0(\epsilon^0)} = \sum_{j=0}^{\infty}\mathbb{E}_0\left[(m_1(\epsilon^1)\cdots m_j(\epsilon^j))\left(\tau_j(\epsilon^j)-g_j(\epsilon^j)+\frac{m_{j+1}(\epsilon^{j+1})\,M_{j+1}(\epsilon^j)\,i_{j+1}(\epsilon^j)}{P_{j+1}(\epsilon^{j+1})}\right)\right].$$

The remaining equilibrium conditions are

$$\begin{aligned} 1 &= \mathbb{E}_t\left[m_{t+1}(\epsilon^{t+1})I_{t+1}(\epsilon^t)\Pi_{t+1}^{-1}(\epsilon^{t+1})\right], \\ P_t(\epsilon^t) &= \frac{M_{t+1}(\epsilon^t)}{w_t(\epsilon^t)}, \end{aligned}$$

where $m_{t+1}(\epsilon^{t+1})$ is pinned down by the resource constraint.

A *policy regime* is a mapping. With each strictly positive price level sequence, it associates a policy (or set of policies); the price level sequence and policy satisfy the above equilibrium conditions, except possibly the intertemporal budget constraint. A policy regime is Ricardian if for each price level sequence, this sequence and the associated policy also satisfy the intertemporal budget constraint. Otherwise, the policy regime is non-Ricardian: there exist some price level sequences and associated policies that do not satisfy the intertemporal budget constraint. Since in equilibrium, the government must balance its budget, a non-Ricardian regime rules out certain price level sequences if they are associated with policies that do not satisfy the intertemporal budget constraint. That is, a non-Ricardian policy regime imposes restrictions on equilibrium price levels which a Ricardian regime does not impose.

Consider for simplicity a deterministic environment with constant endowments, $w_t = w$, and government consumption, $g_t = g$, implying $m_{t+1} = \beta$. A (Ricardian or non-Ricardian) policy regime imposes the following restrictions on policy:

$$\begin{aligned} M_{t+1} &= P_t w, \\ I_{t+1} &= \Pi_{t+1}/\beta, \\ B_{t+1} &= B_t I_t - P_t\left(\tau_t - g + w - \Pi_t^{-1}w\right). \end{aligned}$$

If the regime is Ricardian then policy also satisfies the intertemporal budget constraint for any strictly positive P_0 that is, policy satisfies

$$\frac{B_0 I_0 + M_0}{P_0} = \sum_{j=0}^{\infty}\beta^j\left(\tau_j - g + \frac{w\, i_{j+1}}{I_{j+1}}\right),$$

where $i_t \equiv I_t - 1$. If the regime is non-Ricardian, in contrast, then policy need not satisfy the latter restriction for arbitrary price level sequences. The non-Ricardian regime rules out price level sequences which in combination with the associated policy do not satisfy the restriction.

Suppose that $B_0 I_0 + M_0 \neq 0$ and monetary policy fixes the nominal interest rate at value I. Equilibrium inflation then is constant at value $\Pi = \beta I$ and money supply grows at the gross rate Π, implying $P_t = P_0 \Pi^t$ and $M_{t+1} = P_0 \Pi^t w$. We consider fiscal policy regimes that relate positive price level sequences that grow at a constant rate, to fiscal policies, $\{\tau_t, g, B_{t+1}\}_{t\geq 0}$. In any fiscal policy regime, fiscal policy satisfies

$$B_{t+1} = B_t I - P_0 \Pi^t \left(\tau_t - g + w - \Pi^{-1} w\right).$$

If the fiscal policy regime is Ricardian then, for any $P_0 > 0$, policy also satisfies

$$\frac{B_0 I_0 + M_0}{P_0} = \sum_{j=0}^{\infty} \beta^j \left(\tau_j - g + w \frac{I-1}{I}\right),$$

that is, P_0 imposes a constraint on fiscal policy. Since there is no other condition to determine the initial price level, P_0 is *indeterminate* (see section 11.5). Under a non-Ricardian fiscal policy regime, in contrast, fiscal policy is not constrained by the latter condition; as a consequence, it may determine the price level.

Suppose alternatively that monetary policy fixes the money supply at date t at value M_{t+1} such that the equilibrium price level equals $P_t = M_{t+1}/w$. Under a non-Ricardian fiscal policy regime the price level now is over determined; except for knife-edge cases, only a Ricardian fiscal policy regime is consistent with equilibrium.

That a non-Ricardian policy regime may determine the initial price level and thereby revalue initially outstanding nominal debt does not mean that the government can choose primary surpluses and seignorage revenues arbitrarily. Standard asset pricing and rational expectations imply that, when nominal debt is issued for the first time (before date $t = 0$) the government cannot raise more resources in present value terms than it repays in the future. Accordingly, the intertemporal budget constraint binds at the time of debt issuance. This can also be seen by noting that, at the "truly initial" date $t = -1$ say when $B_{-1} I_{-1} + M_{-1} = 0$ the price level P_{-1} cannot revalue outstanding liabilities. A non-Ricardian policy regime therefore does not allow the government to escape long-run budget balance. Similarly, a non-Ricardian policy regime does not provide a *nominal anchor*—it does not determine the price level—before nominal liabilities have been issued for the first time; it may only contribute, in a stochastic environment, to determining history-contingent inflation rates.

11.4.6 Stability under Policy Rules

Mechanically, a non-Ricardian policy regime determines the equilibrium price level conditional on outstanding nominal debt because only a specific price level prevents explosive debt dynamics: The equilibrium conditions without the intertemporal budget constraint determine the path of government debt in real terms, conditional on its starting value, and this path satisfies the government's no-Ponzi-game condition (or the household's transversality condition) only for a specific starting value and thus, a specific initial price level.

The same mechanism may be at work when a policy regime prescribes ad-hoc policy rules, for example rules specifying how the interest rate and taxes are set in response to inflation and the stock of outstanding debt. Consider a deterministic setting. Suppose the policy regime prescribes that the nominal interest rate responds to inflation, and taxes net of government consumption respond to the stock of real debt at the end of the previous period,

$$\begin{aligned} I_{t+1} &= \alpha \Pi_t, \\ \tau_t - g_t &= \gamma \frac{B_t}{P_{t-1}}, \end{aligned}$$

where α and γ are fixed parameters.[6] Suppose also, as before, that the equilibrium gross real interest rate equals β^{-1}. The Fisher equation, $I_{t+1} = \Pi_{t+1}/\beta$, and the interest rate rule then imply

$$\Pi_{t+1} = \alpha\beta\Pi_t.$$

We allow for a cash-in-advance constraint or a money demand function that depends on the interest rate; in either case, $M_{t+1}/P_t = w_t\zeta(I_{t+1})$ for some function ζ. The dynamic budget constraint thus can be expressed as

$$\begin{aligned} \frac{B_{t+1}}{P_t} &= \frac{B_t}{P_{t-1}}\left(\frac{I_t}{\Pi_t} - \gamma\right) - \frac{M_{t+1} - M_t}{P_t} \\ &= \frac{B_t}{P_{t-1}}(\beta^{-1} - \gamma) - \chi(w_t, w_{t-1}, \Pi_t) \end{aligned}$$

for some function χ, where the second equality uses the Fisher equation, the money demand function, the interest rate rule, and the equilibrium condition $\Pi_{t+1} = \alpha\beta\Pi_t$.

Linearizing the two dynamic equations yields a linear difference equation system in two endogenous variables, the deviation of Π_t from its steady-state value and the deviation of B_t/P_{t-1} from its steady-state value. The latter variable is predetermined, the former is not. The matrix determining the stability of the system is given by

$$\begin{bmatrix} \alpha\beta & 0 \\ \xi & \beta^{-1} - \gamma \end{bmatrix}$$

for some constant ξ; its eigenvalues equal $\alpha\beta$ and $\beta^{-1} - \gamma$.

Since we study a linear approximation around the system's steady state we restrict attention to bounded solutions.[7] Three cases may be distinguished, see appendix B.5. First, if both eigenvalues of the matrix are unstable then no bounded solution exists. Second, if both eigenvalues are stable then any initial inflation rate together with the predetermined real debt value gives rise to a bounded solution. As a consequence, sunspot shocks may buffet the system. Finally, if exactly one eigenvalue is stable then

[6]We disregard additive constant terms since they are irrelevant for the argument.

[7]This is a more stringent stability requirement than the no-Ponzi-game condition in the fiscal theory of the price level.

the system is saddle-path stable and the two policy rules pin down a unique inflation rate conditional on the predetermined real debt level.

Specifically, if $|\alpha\beta| > 1$ (*active monetary policy*), inflation in the initial period must equal a specific value to guarantee stable inflation dynamics (the difference equation for inflation is solved forward to yield a bounded solution). But in this case, debt dynamics only are bounded if $|\beta^{-1} - \gamma| < 1$ (*passive fiscal policy*). The situation parallels the one in the fiscal theory of the price level when the policy regime is Ricardian.

Alternatively, if $|\beta^{-1} - \gamma| > 1$ (*active fiscal policy*), inflation (and thus, the price level) in the initial period must adjust to guarantee stable debt dynamics. Stable inflation dynamics then require $|\alpha\beta < 1|$ (*passive monetary policy*). The situation is akin to a non-Ricardian policy regime in the fiscal theory of the price level when the government fixes the nominal interest rate.

11.5 Determinate Inflation and Output

In the model with flexible prices discussed in subsection 10.3.5, risk renders inflation indeterminate when the government sets the nominal interest rate. In the fiscal theory of the price level analyzed in subsection 11.4.5, a Ricardian policy regime may render the price level and the real value of government debt indeterminate. And in the model with ad-hoc policy rules considered in subsection 11.4.6, sufficiently passive rules also render inflation and real debt indeterminate.

We now study the source of price level indeterminacy in more detail and analyze the role of monetary policy in the determination of the price level. We first consider a flexible price environment before turning to rigid prices. Throughout the analysis we assume that fiscal policy is Ricardian. Since we analyze linearized equilibrium conditions we are looking for bounded solutions.

11.5.1 Flexible Prices

Consider the model with flexible prices analyzed in subsection 10.3.5, in which the classical dichotomy holds: The nominal interest rate or the money supply do not affect the real allocation; the real interest rate equals the natural interest rate, $r_t^n(\epsilon^t)$; and inflation is determined by the Fisher equation which reads, in linearized form,

$$\mathbb{E}_t[r_{t+1}^n(\epsilon^{t+1})] = i_{t+1}(\epsilon^t) - \mathbb{E}_t[\pi_{t+1}(\epsilon^{t+1})].$$

We assume that the natural interest rate follows a stationary process and thus, is bounded.

Suppose first that the government determines a history-contingent sequence of nominal interest rates which is independent of the values of other, endogenous variables. Such an *interest rate peg* only pins down expected inflation, not the actually realized inflation (see subsection 10.3.5). That is, inflation and thus, the price level (and, for a given money demand function, nominal balances) are indeterminate. The source of the indeterminacy is that the nominal and natural interest rate sequences

Following a positive monetary policy shock, $\zeta(\epsilon^t) > 0$, the nominal interest rate exceeds the level implied by the systematic part of the interest rate rule. This leads to lower inflation, a lower output gap, and (since the natural level of output is unaffected by the nominal interest rate) lower output. When $\rho > 0$, these effects are persistent, otherwise they are temporary.

Associated with the output contraction is an increase in the real interest rate. Paradoxically, however, the nominal interest rate need not rise; for sufficiently high values of ρ, the response of $i_{t+1}(\epsilon^t)$ to $\pi_t(\epsilon^t)$ and $\chi_t(\epsilon^t)$ through the systematic part of the policy rule may more than offset the direct positive effect of the policy shock. Intuitively, when ρ is sufficiently high, the policy shock generates expectations of negative inflation in subsequent periods and this lowers the nominal relative to the real interest rate.

Recall that κ, the coefficient on the output gap in the Phillips curve, depends on the frequency of price adjustments by firms, $1-\theta$. As $\theta \to 0$ (perfectly flexible prices), $\kappa \to \infty$ and the effect of the policy shock on the output gap is zero. The marginal effect on inflation equals $-(\phi_\pi - \rho)^{-1}$, and the marginal effect on the nominal interest rate as well as on expected inflation in the subsequent period equals $-\rho(\phi_\pi - \rho)^{-1}$; accordingly, the real interest rate is not affected. Consistent with the results in subsection 11.5.1, we thus find that expected inflation and the nominal interest rate move in tandem; and the impact effect of the monetary policy shock on inflation is determined by the restriction that the expected inflation sequence under the policy rule be bounded.

For $\theta \to 1$ (completely rigid prices), in contrast, $\kappa \to 0$ and inflation is unaffected by the shock. The marginal effect on the output gap equals $-(\phi_\chi + \sigma(1-\rho))^{-1}$ because, with zero inflation, it is determined by the difference equation

$$\chi_t(\epsilon^t) = \mathbb{E}_t \left[\chi_{t+1}(\epsilon^{t+1})\right] - \frac{1}{\sigma}\mathbb{E}_t \left[\phi_\chi \chi_t(\epsilon^t) + \zeta_t(\epsilon^t)\right]$$

and the requirement that expected future output gaps remain bounded. The marginal effect of the policy shock on both the nominal and the real interest rate equals $\sigma(1-\rho)(\phi_\chi + \sigma(1-\rho))^{-1}$. A higher persistence of the shock implies a stronger output effect but potentially a weaker interest rate response. When the persistence is very high ($\rho \to 1$) interest rates do not respond at all.

11.7 Bibliographic Notes

Baxter and King (1993) and Barro (1990) analyze tax financed government consumption or investment in the neoclassical growth model and the *Ak* model, respectively.

The modern formulation of the Ricardian equivalence proposition is due to Barro (1974). Diamond (1965) studies debt in the overlapping generations model. Auerbach et al. (1994) discuss generational accounting, Breyer (1989) and Rangel (1997) analyze equivalent social security reforms, and Ball and Mankiw (2007) analyze risk sharing properties of social security systems.

Gonzalez-Eiras and Niepelt (2015) state a general neutrality result. Classic statements are due to Wallace (1981) and Bryant (1983). Bassetto and Kocherlakota (2004) derive the neutrality result for policy changes that involve distorting taxes.

Cagan (1956) analyzes need-for-seignorage driven (hyper)inflation. The unpleasant monetarist arithmetic is due to Sargent and Wallace (1981). Subsection 11.4.3 follows Sargent (1987, 5.4). Leeper (1991) analyzes active and passive policy rules. The fiscal theory of the price level is due to Sims (1994) and Woodford (1995), see also Aiyagari and Gertler (1985) and Kocherlakota and Phelan (1999). For critiques, see Bassetto (2002), Buiter (2002), and Niepelt (2004a).

Sargent and Wallace (1975) establish price level indeterminacy under an interest rate rule. Interest rate rules often are referred to as *Taylor rules*, after Taylor (1993). Taylor (1999) and Woodford (2001) discuss the Taylor principle and Bullard and Mitra (2002) analyze the conditions for price level determinacy, see also Atkeson et al. (2010). Benhabib et al. (2002) analyze determinacy under interest rate rules in the presence of an effective lower bound. Brock (1974) and Obstfeld and Rogoff (1983) analyze determinacy when the government pegs the money supply.

Following Friedman (1968) and Phelps (1970), Lucas (1972) analyzes a model with imperfect information and real effects of monetary policy shocks. Grossman and Weiss (1983), Rotemberg (1984), and Lucas (1990) analyze models with segmented markets, see also Alvarez et al. (2002) on endogenous segmentation.

Related Topics and Additional References Farhi and Werning (2016a) analyze *fiscal multipliers*—the effects of increased government consumption on private consumption and output over time—in the New Keynesian model.

Woodford (1990) analyzes how public debt increases private sector liquidity and helps relax borrowing constraints, see also Holmström and Tirole (1998) and Aiyagari and McGrattan (1998). On bubbly debt, see Domeij and Ellingsen (2018) and section 9.5.

Wallace (1981) and Chamley and Polemarchakis (1984) derive neutrality results in economies with money as a store of value.

Del Negro and Sims (2015) and Hall and Reis (2015) study implications for monetary policy and *central bank insolvency* when the fiscal and monetary authorities satisfy separate budget constraints.

Poole (1970) assesses how interest and money supply targets stabilize output and prices.

Woodford (2003, 4), Galí (2008, 3), and Walsh (2017, 8) cover the stability properties of the New Keynesian model and its transmission mechanism. Walsh (2017, 5) covers models of monetary policy under imperfect information and with segmented markets.

Chapter 12

Optimal Policy

When policy affects the equilibrium allocation preferences over allocations induce preferences over policies. The *Ramsey program* consists of choosing the *optimal* or *Ramsey policy*, which implements the *Ramsey allocation*. A Ramsey policy must be feasible—it must implement an equilibrium—and it must be *admissible*—it may only use policy instruments at the government's disposal. Both requirements are costly; a social planner that directly chooses among feasible allocations does at least as good as a *Ramsey government* that chooses among feasible allocations which can be implemented as an equilibrium given the set of admissible instruments.

We analyze how a government optimally chooses deficits and taxes over time when it is attentive to the welfare costs of tax distortions and the wealth distribution. Thereafter, we study how social insurance undermines private incentives and how the resulting trade-off affects the optimal taxation of savings. Finally, we analyze the characteristics of optimal monetary policy, both under flexible and rigid prices. Many of our findings are instances of fundamental results from public finance, which we review in appendix B.6.

12.1 Tax Smoothing

The public finance problem of which goods to optimally tax, and at what rate, in order to minimize distortions or negative distributive implications (see appendix B.6) corresponds to the macroeconomic problem of *when* to tax. Since a decoupling of taxation from government spending relies on government debt (see section 11.2) the Ramsey policy determines the optimal sequence of government indebtedness.

12.1.1 Complete Markets

Consider a representative household economy without capital. The household is endowed with one unit of time per period which can be transformed into $w_t(\epsilon^t)$ units of the good. Household preferences over consumption, c, and leisure, x, are represented by the utility function $\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0[u(c_t(\epsilon^t), x_t(\epsilon^t))]$ where u is strictly concave and increasing and β denotes the discount factor.

To finance a given stream of government consumption, $\{g_t(\epsilon^t)\}_{t\geq 0}$, the government taxes labor income at rates $\{\tau_t(\epsilon^t)\}_{t\geq 0}$ and issues Arrow securities of arbitrary maturity; markets are complete. Without loss of generality, taxes on consumption are normalized to zero (see the discussion in subsection 11.3.2 and note that absent a technology to transform resources intertemporally, there are no intertemporal producer prices).

Variable ${}_tb_s(\epsilon^{t-1}, \epsilon^s), s \geq t$, denotes claims vis-à-vis the government held at date t, given that history ϵ^{t-1} occurred; one claim entitles to one unit of the consumption good at date s after history ϵ^s. The marginal distribution of ϵ^t is denoted by $H_t(\epsilon^t)$ and its density by $h_t(\epsilon^t)$; the conditional distribution of ϵ^s given $\epsilon^t, s \geq t$, is denoted $H_s(\epsilon^s|\epsilon^t)$.

The benevolent government maximizes household welfare subject to the resource constraint,

$$c_t(\epsilon^t) + g_t(\epsilon^t) = w_t(\epsilon^t)(1 - x_t(\epsilon^t)),$$

and the equilibrium conditions that characterize household choices, namely the household's (complete markets) intertemporal budget constraint,

$$\sum_{t=0}^{\infty} \int q_t(\epsilon^t)[c_t(\epsilon^t) - (1 - \tau_t(\epsilon^t))w_t(\epsilon^t)(1 - x_t(\epsilon^t)) - {}_0b_t(\epsilon^{-1}, \epsilon^t)]d\epsilon^t = 0,$$

and the household first-order conditions,

$$\begin{aligned} u_c(c_0, x_0)q_t(\epsilon^t) &= \beta^t h_t(\epsilon^t)u_c(c_t(\epsilon^t), x_t(\epsilon^t)), \\ u_c(c_t(\epsilon^t), x_t(\epsilon^t))w_t(\epsilon^t)(1 - \tau_t(\epsilon^t)) &= u_x(c_t(\epsilon^t), x_t(\epsilon^t)), \end{aligned}$$

which reflect the consumption-saving and labor-leisure trade-off, respectively. Variable $q_t(\epsilon^t)$ denotes the price at date $t = 0$ of the good at date t, and $\{{}_0b_t(\epsilon^{-1}, \epsilon^t)\}_{t=0}^{\infty}$ are the private sector claims vis-à-vis the government at the initial date. The resource and budget constraint imply the government's intertemporal budget constraint.

To represent the government's constraint set more compactly we adopt the primal approach (see appendix B.6). Substituting the first-order conditions into the budget constraint yields the *implementability constraint*,

$$\sum_{t=0}^{\infty} \int \beta^t [u_c(c_t(\epsilon^t), x_t(\epsilon^t))(c_t(\epsilon^t) - {}_0b_t(\epsilon^{-1}, \epsilon^t)) - u_x(c_t(\epsilon^t), x_t(\epsilon^t))(1 - x_t(\epsilon^t))]dH_t(\epsilon^t) = 0,$$

which comprises all equilibrium conditions in the household sector and expresses them solely in terms of the allocation, without direct reference to prices and after-tax wages. The government's problem is to maximize welfare subject to the resource constraint, the implementability constraint, and the first-order conditions. Conditional on an allocation that satisfies the resource and implementability constraint, the first-order conditions pin down prices and tax rates. The Ramsey allocation therefore solves the problem of maximizing welfare subject to the resource and implementability constraints.

Let ν and $-\beta^t \mu_t(\epsilon^t) h_t(\epsilon^t)$ denote the multipliers associated with the implementability and resource constraints, respectively. Suppressing histories to improve legibility, the first-order conditions for consumption and leisure are given by

$$\begin{aligned}(1+\nu)u_c(c_t, x_t) + \nu(u_{cc}(c_t, x_t)(c_t - {}_0b_t) - u_{xc}(c_t, x_t)(1-x_t)) &= \mu_t, \\ (1+\nu)u_x(c_t, x_t) + \nu(u_{cx}(c_t, x_t)(c_t - {}_0b_t) - u_{xx}(c_t, x_t)(1-x_t)) &= \mu_t w_t,\end{aligned}$$

respectively. The conditions state that the government accounts for three types of effects when increasing c_t or x_t. First, the direct effects on the objective function. Second, the resource costs, represented by the terms multiplying μ_t. And third, the marginal effects on the implementability constraint, represented by the terms multiplying ν. The latter effects reflect both higher outlays for consumption or leisure and changes of the marginal rates of substitution—corresponding to changed inter- and intratemporal prices.

Together with the implementability and resource constraints the first-order conditions of the government fully characterize the Ramsey allocation. Moreover, from the household's first-order conditions, the allocation implies the optimal tax rates,

$$\tau_t = 1 - \frac{u_x(c_t, x_t)}{u_c(c_t, x_t) w_t},$$

as well as prices. Finally, from the government's intertemporal budget constraint, the history-contingent sequence of taxes implies a unique sequence of optimal government indebtedness since the value of outstanding debt equals the market value of future primary surpluses (see subsection 11.4.1). Note that the level of indebtedness at date t,

$$\sum_{s=t}^{\infty} \int \frac{q_s \, {}_tb_s}{q_t} d\epsilon^s | \epsilon^t,$$

does not uniquely determine the *maturity structure* $\{{}_tb_s(\epsilon^{t-1}, \epsilon^s)\}_{s \geq t}$ that is, the composition of public debt by maturity.

We make four key observations. First, the multiplier associated with the implementability constraint, ν, represents the *shadow value of public funds*—the government's valuation of public relative to private sector wealth. To see this, recall that the implementability constraint incorporates all competitive equilibrium conditions beyond the resource constraint. The multiplier associated with the constraint represents the shadow cost of the competitive equilibrium requirement and specifically, of the government's need to levy distorting taxes. A marginal lump-sum transfer from the private to the public sector (or a reduction of the government's initial indebtedness) would relax the implementability constraint and increase the value of the program by ν.

Second, with complete markets, the shadow value is constant over time and across histories. This is just a restatement of the fact that the government faces a single implementability constraint with a single multiplier. Intuitively, with complete markets, households smooth the shadow value of income over time and across histories and the same holds true for the government, implying that the ratio of the shadow values, ν, is constant as well. If the government did not face complete markets but, to take an extreme example, had to balance the budget at each date and history then the single implementability constraint would be replaced by a history-contingent sequence of constraints with an associated sequence of multipliers. The government's inability to decouple tax collections and government spending would imply that the shadow cost of public funds varies over time and across histories. We return to this point in subsection 12.1.2.

Third, as established above, under the Ramsey policy government indebtedness generally differs from zero and in stochastic environments, it is stochastic as well. That is, the optimal return on the government's portfolio generally is not risk-free. This is an implication of the constancy of the shadow cost, ν. In parallel to households, the government uses financial claims to shift purchasing power across periods and histories. We return to this point below.

Finally, tax rates at date t only depend on $({}_0b_t, w_t, g_t)$. To see this, note that the two first-order conditions combine to an equation in $(\nu, {}_0b_t, w_t, c_t, x_t)$ while the variables (g_t, w_t, c_t, x_t) enter the resource constraint. Since the structure of either equation is not history dependent the equilibrium allocation at a date and history is an invariant function of the exogenous state, $({}_0b_t, w_t, g_t)$, as well as of the constant multiplier, ν. As a consequence, the tax rates in two histories with the same state are identical. In environments with additional state variables, e.g. capital, this complete markets result generalizes.

To characterize the Ramsey tax policy in more detail, we manipulate the optimality conditions to derive two auxiliary conditions,

$$\begin{aligned}(1+\nu)[u_c(c_t,x_t)(c_t-{}_0b_t)-u_x(c_t,x_t)(1-x_t)]+\nu Q_t+(g_t+{}_0b_t)\mu_t &= 0,\\ \nu Q+\sum_{t=0}^{\infty}\int\beta^t(g_t+{}_0b_t)\mu_t dH_t &= 0,\end{aligned}$$

where $Q_t<0, Q<0$, and $\mu_t>0$.[1] From the resource constraint, the first condition implies that, even with no government spending in a history ($g_t={}_0b_t=0$), the tax rate is strictly positive when public funds are scarce ($\nu>0$). According to the second condition, the shadow value of public funds equals zero if the market value of government consumption and initial government debt equals zero.

Suppose first that $\sum_{t=0}^{\infty}\int\beta^t u_c(c_t,x_t)(g_t+{}_0b_t)dH_t=0$, for example because $g_t+{}_0b_t=0$ in all histories. As we have just seen, $\nu=0$ in this case. The first-order conditions then imply that the allocation is not distorted, $u_c(c_t,x_t)w_t=u_x(c_t,x_t)$, and tax rates therefore equal zero. Intuitively, when the government's initial asset holdings (negative ${}_0b_t$) suffice to finance government consumption then there is no need to levy distorting taxes.

[1] The first equation results from multiplying the government's first-order conditions by $c_t-{}_0b_t$ and x_t-1, respectively, summing them, and using the resource constraint. The second equation follows from integrating the first condition, weighting by β^t, summing over time, and using the intertemporal budget constraint.

Suppose next that $\nu > 0$ such that the government needs to raise taxes, and that $(w_t, g_t, {}_0b_t)$ is constant across all histories. As shown above, (c_t, x_t) and thus, tax rates then are constant as well. Accordingly, the government budget is balanced at all times. From now on, we let ${}_0b_t = 0$ in all histories.

Third, consider a deterministic environment with constant productivity, w, and $g_t = 0$ at all dates except at date $t = T$ when $g_T > 0$. Our findings imply that tax rates at all dates $t \neq T$ are constant and strictly positive. Intuitively, since tax distortions are convex in the tax rate, optimal tax rates vary less than government consumption—the optimal *tax smoothing* policy spreads tax collections over time to reduce average tax distortions. Accordingly, the government accumulates assets before date $t = T$ and services debt thereafter.

Next, consider the same scenario except that at date $t = T$, government consumption is stochastic and can take two values: $g_T > 0$ or $g_T = 0$. Our findings imply that tax rates are constant and strictly positive except at date $t = T$ if $g_T > 0$. The Ramsey policy smoothes taxes both across time and histories. Since the tax revenue before and after date $t = T$ is constant the government's indebtedness at date $t = T + 1$ must be independent of the realization of g_T. Moreover, since the government budget at date $t = T$ is not balanced this requires that the government's indebtedness at date $t = T$ is history-contingent: With $g_T > 0$ government debt is lower than with $g_T = 0$. That is, between $t = T - 1$ and $t = T$, the rate of return on government debt is contingent on the realization of g_T—the private sector (partially) insures the government against the high government consumption shock.

Finally, if (w_t, g_t) follows a deterministic cycle then (c_t, x_t) and tax rates follow a deterministic cycle as well and the government's budget is balanced over the cycle. Similarly, if (w_t, g_t) follows a stationary Markov process then (c_t, x_t) and tax rates inherit the stochastic properties of the state.

We have seen that with complete markets and stochastic (w_t, g_t), the tax smoothing Ramsey policy relies on contingent government indebtedness. One mechanism to generate this contingency is to make the coupon payment contingent on the realization of the state. A more subtle mechanism, which works even when coupons are risk-free, relies on an appropriate choice of *maturity structure* and the fact that shocks that change the interest rate affect the market value of outstanding debt differently, depending on the debt's maturity. Suppose for example that a shock to productivity or government consumption alters equilibrium consumption and leads to a persistent increase in interest rates. This has no effect on the value of maturing liabilities but it devalues outstanding longer-term debt and this effect is stronger when the maturity is longer.

For a given level of indebtedness at date t, the indebtedness in the subsequent period thus depends on the choice of maturity structure at date t. Generically, the contingent government indebtedness under the complete markets Ramsey policy is spanned by the contingent term structure of interest rates associated with the Ramsey allocation. That is, the contingent indebtedness under the complete markets Ramsey policy can be generated even if the coupons on government debt are restricted to be risk-free, provided that a sufficiently rich maturity structure of government debt is admissible.

12.1.2 Incomplete Markets

Short-Term, Risk-Free Debt

Assume now that the government only issues one-period debt, with a risk-free coupon, implying that government indebtedness is non-contingent. The complete markets Ramsey allocation characterized in subsection 12.1.1 generally cannot be implemented in this case and the properties of the Ramsey policy change.

Let $b_t(\epsilon^{t-1})$ denote claims vis-à-vis the government that are due at date t in any history subsequent to history ϵ^{t-1} (the claims are measurable with respect to ϵ^{t-1} rather than ϵ^t as before). For convenience, we assume that productivity equals unity at all times. Using the resource constraint we adopt the shorthand notation $u_t(\epsilon^t) \equiv u(c_t(\epsilon^t), 1 - c_t(\epsilon^t) - g_t(\epsilon^t))$; $u_{c,t}(\epsilon^t) \equiv u_c(c_t(\epsilon^t), 1 - c_t(\epsilon^t) - g_t(\epsilon^t))$; and similarly for $u_{x,t}(\epsilon^t)$.

Since the government only issues debt with a risk-free return the household faces incomplete markets. In competitive equilibrium, the household satisfies its intratemporal first-order condition and stochastic Euler equation; the household or equivalently, the government satisfies its dynamic budget constraint; government debt or assets are bounded; and the resource constraint is met. From the intratemporal first-order condition and the resource constraint, we can express the government's primary surplus, $s_t(\epsilon^t) \equiv \tau_t(\epsilon^t)(1 - x_t(\epsilon^t)) - g_t(\epsilon^t)$, as

$$s_t(\epsilon^t) = \left(1 - \frac{u_{x,t}(\epsilon^t)}{u_{c,t}(\epsilon^t)}\right)(c_t(\epsilon^t) + g_t(\epsilon^t)) - g_t(\epsilon^t).$$

Accordingly, the government's dynamic budget constraint incorporating the household optimality conditions and the resource constraint reads

$$b_t(\epsilon^{t-1}) \leq s_t(\epsilon^t) + \beta \mathbb{E}_t \left[\frac{u_{c,t+1}(\epsilon^{t+1})}{u_{c,t}(\epsilon^t)} b_{t+1}(\epsilon^t) \right],$$

where we assume that the government may pay lump-sum transfers, thus the inequality constraint.

Iterating this equation (with equality) forward, applying the law of iterated expectations, and assuming $\lim_{T\to\infty} \beta^T u_{c,T}(\epsilon^T) = 0$ almost surely, yields the implementability constraint

$$u_{c,0} b_0 = \sum_{t=0}^{\infty} \int \beta^t u_{c,t} s_t dH_t(\epsilon^t),$$

where we suppress histories to improve legibility. There are two differences between this implementability constraint and the one in subsection 12.1.1. First, the constraint here incorporates the resource constraint. Second, it is derived from the government's rather than the private sector's intertemporal budget constraint. Both differences affect the exposition but not the underlying economic structure.

However, the implementability constraint does not yet reflect the restriction that indebtedness be non-contingent (see the discussion of equation (4.1) on page 48). To incorporate this restriction, we need to impose the intertemporal budget constraint along each history. Equivalently, we require that in addition to the implementability constraint, the indebtedness at date t and thus, the present value of primary surpluses from that node onwards be the same for all ϵ^t conditional on ϵ^{t-1}. This *measurability constraint* can be stated as

$$u_{c,t}b_t = \sum_{j=t}^{\infty} \int \beta^{j-t} u_{c,j} s_j dH_j(\epsilon^j|\epsilon^t) \ \forall \epsilon^t|\epsilon^{t-1}, \ t \geq 1,$$

where b_t on the left-hand side of the equation is measurable with respect to ϵ^{t-1}. Note that the measurability constraint at date $t \geq 1$ has the same form as the implementability constraint that holds at date $t = 0$. We also impose boundedness conditions, requiring that b_t and thus, the right-hand side of the measurability constraint normalized by $u_{c,t}$, lies between some bounds $\underline{M}$ and $\bar{M}$.

Let $\beta^t h_t(\epsilon^t)\gamma_t(\epsilon^t)$ denote the multiplier associated with the implementability constraint (for $t = 0$) and the measurability constraints (for $t \geq 1$) with $\gamma_0 \leq 0$; and let $\beta^t h_t(\epsilon^t)\xi_{1,t}(\epsilon^t)$ and $\beta^t h_t(\epsilon^t)\xi_{2,t}(\epsilon^t)$ denote the multipliers associated with the upper and lower bounds, respectively. The Lagrangian of the government's program reads

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty} \int \beta^t \{ & u_t + u_{c,t}(\gamma_t b_t + \xi_{1,t}\bar{M} - \xi_{2,t}\underline{M}) \\ & -(\gamma_t + \xi_{1,t} - \xi_{2,t}) \left(\sum_{j=t}^{\infty} \int \beta^{j-t} u_{c,j} s_j dH_j(\epsilon^j|\epsilon^t) \right) \Bigg\} dH_t(\epsilon^t).\end{aligned}$$

This can be rewritten as

$$\mathcal{L} = \sum_{t=0}^{\infty} \int \beta^t \left\{ u_t + u_{c,t}(\gamma_t b_t + \xi_{1,t}\bar{M} - \xi_{2,t}\underline{M}) - u_{c,t}s_t \sum_{j=0}^{t} (\gamma_j + \xi_{1,j} - \xi_{2,j}) \right\} dH_t(\epsilon^t),$$

where the multipliers $\gamma_j, \xi_{1,j}$, and $\xi_{2,j}$ in the sum $\sum_{j=0}^{t}(\gamma_j + \xi_{1,j} - \xi_{2,j})$ denote multipliers along the branch of the event tree whose nodes precede the node ϵ^t. We thus have

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{\infty} \int \beta^t \{ u_t + u_{c,t}(\gamma_t b_t + \xi_{1,t}\bar{M} - \xi_{2,t}\underline{M}) - u_{c,t}s_t \nu_t \} dH_t(\epsilon^t) \\ \text{s.t.} \quad & \nu_t = \nu_{t-1} + \gamma_t + \xi_{1,t} - \xi_{2,t}, \ \nu_{-1} = 0.\end{aligned}$$

Differentiating with respect to $c_t(\epsilon^t)$ and $b_{t+1}(\epsilon^t)$ yields the first-order conditions

$$\begin{aligned}u_{c,t} - u_{x,t} - \nu_t((u_{cc,t} - u_{cx,t})s_t + u_{c,t}s_{c,t}) + (u_{cc,t} - u_{cx,t})(\gamma_t b_t + \xi_{1,t}\bar{M} - \xi_{2,t}\underline{M}) &= 0, \\ \int \gamma_{t+1} u_{c,t+1} dH_{t+1}(\epsilon^{t+1}|\epsilon^t) &= 0,\end{aligned}$$

respectively. The first optimality condition relates the allocation to the level of debt as well as to time-varying multipliers ($\gamma_t, \xi_{1,t}, \xi_{2,t}$) and their cumulative sum (ν_t). The second condition states that the shadow cost of the measurability constraint in utility terms, $\gamma_{t+1}u_{c,t+1}$, should equal zero on average.

To build intuition for these conditions, consider first the hypothetical complete-markets case where debt service at date t is measurable with respect to ϵ^t rather than ϵ^{t-1}, and $\xi_{1,t} = \xi_{2,t} = 0$. The second optimality condition then changes to $\gamma_{t+1} = 0$ and ν_t is constant across histories; the optimality conditions thus reduce to the equivalent of the conditions in subsection 12.1.1. Intuitively, with complete markets, there is no cost associated with having to satisfy the intertemporal budget constraint after the initial period, conditional on satisfying it in the initial period. Stated differently, the optimal choice of contingent indebtedness equalizes the shadow cost of the government budget constraint across histories.

With incomplete markets, in contrast, the government cannot equalize the shadow cost across histories. With risk-free indebtedness it can only equalize this cost on average, over time. After a negative shock to the budget the intertemporal budget constraint tightens ($\gamma_t < 0$ and ν_t decreases) and going forward, the Ramsey allocation is more distorted than it would have been after a positive shock which leads to an increase of ν_t. The tightening or relaxation of the budget constraint is permanently reflected in the multiplier ν and thus, in the Ramsey allocation and tax policy. In contrast to the complete markets case, tax rates therefore do not only inherit the stochastic properties of the government consumption shock but also reflect its history.

It is instructive to consider the special case of quasilinear utility, $u(c, x) = c + G(x)$ where G is increasing and concave. From the household's first-order conditions, the stochastic discount factor then equals β, and $\tau_t = 1 - G'(x)$. Tax revenue thus is a function of x_t,

$$\rho(x_t) = (1 - G'(x_t))(1 - x_t)$$

say. Under standard assumptions, it is a strictly concave function over the domain $[\underline{x}, \bar{x}]$ where $\underline{x}$ denotes the undistorted level of leisure ($\rho(\underline{x}) = 0$) and $\bar{x} < 1$ denotes the level where $\rho(\bar{x})$ attains the maximum of the Laffer curve. Inverting ρ yields leisure as a strictly convex function of tax revenue, $\chi(\rho_t)$ say. Function χ is defined over the domain $[0, \rho(\bar{x})]$.

Since utility at date t equals $1 - \chi(\rho_t) - g_t + G(\chi(\rho_t))$ we may formulate the Ramsey program with tax revenue and debt as the choice variables. This program reads

$$\begin{aligned}
\max_{\{\rho_t(\epsilon^t), b_{t+1}(\epsilon^t)\}_{t \geq 0}} \quad & -\sum_{t=0}^{\infty} \int \beta^t \mathcal{D}(\rho_t) dH_t(\epsilon^t) \\
\text{s.t.} \quad & b_t \leq \rho_t - g_t + \beta b_{t+1}, \\
& \underline{M} \leq b_{t+1} \leq \bar{M},
\end{aligned}$$

where we define the deadweight loss $\mathcal{D}(\rho_t) \equiv \chi(\rho_t) - G(\chi(\rho_t))$. Note that $\mathcal{D}$ is strictly convex over the domain $[0, \rho(\bar{x})]$ and reaches a minimum at $\rho_t = 0$.

With quasilinear utility the optimality conditions simplify to

$$\begin{aligned}
1 - G'(x_t) - \nu_t[1 - G'(x_t) + (1 - x_t)G''(x_t)] &= 0, \\
\nu_t = \nu_{t-1} + \gamma_t + \xi_{1,t} - \xi_{2,t},\ \nu_{-1} &= 0, \\
\mathbb{E}_t[\gamma_{t+1}] &= 0,
\end{aligned}$$

where the first condition can be expressed as $-\mathcal{D}'(\rho_t) = \nu_t$. Whenever the debt limits do not bind ν_t and thus, the marginal deadweight loss follows a martingale. That is, the Ramsey policy keeps the expected marginal tax distortion constant over time. Compare this to the complete markets environment where the Ramsey policy stabilizes the actual marginal tax distortion across all histories.

The Ramsey program with quasilinear utility is isomorphic to a consumption-saving problem with the utility function $-\mathcal{D}(\rho_t)$; an interest rate equal to the inverse of the time discount factor; negative income shocks (government consumption); an asset with a risk-free return; and the natural borrowing limit.[2] The difference between this saving problem and the problem analyzed in subsection 4.3.2 is that the utility function $-\mathcal{D}(\rho_t)$ has a bliss point (at $\rho_t = 0$, corresponding to no deadweight loss).

Since ν_t is nonpositive and $\mathbb{E}_t[\nu_{t+1}] = \nu_t + \mathbb{E}_t[\xi_{1,t+1}] \geq \nu_t$, ν_t is a nonpositive submartingale. Due to the bliss point of $-\mathcal{D}$, convergence of the submartingale does not require an infinite asset level, unlike in the problem analyzed in subsection 4.3.2. If the Markov process for government consumption has a nontrivial invariant distribution then government utility converges to the bliss point; the Ramsey tax rate and ν_t converge to zero; and the government accumulates a sufficiently large stock of assets to finance an infinite sequence of maximal government consumption. Whenever the realization of government consumption is lower than its maximal value the government pays lump-sum transfers to the households.[3]

Broader Portfolio

Returning to the case with general preferences, assume next that the government holds a broader portfolio of liabilities and assets, including physical capital. Markets are incomplete.

We assume that a Markov process governs government consumption and productivity, and we formulate the Ramsey program recursively. Output $f(k_\circ, 1 - x_\circ(\epsilon_\circ), \epsilon_\circ)$ depends on the predetermined capital stock, $k_\circ$; labor input, $1 - x_\circ(\epsilon_\circ)$; and a productivity shock, reflected by $\epsilon_\circ$. To simplify the notation we let $u_c(\epsilon_\circ) \equiv u_c(c_\circ(\epsilon_\circ), x_\circ(\epsilon_\circ))$, $f_K(\epsilon_\circ) \equiv f_K(k_\circ, 1 - x_\circ(\epsilon_\circ), \epsilon_\circ)$, etc.

The state at the beginning of a period, before the realization of the shock, includes the economy's capital stock, $k_\circ$; the government's net liabilities, $b_\circ$; the shock in the previous period, ϵ_-; and marginal utility in the previous period, $u_c(\epsilon_-)$. The choice variables in the government's program include the gross real risk-free interest rate on government debt, $R_\circ$; government holdings of capital, $k_\circ^g$; exposures to arbitrary securities (in zero net supply), $\{e_\circ^i\}_i$, with exogenous gross returns $\{R^i(\epsilon_\circ)\}_i$; as well as variables which vary with the shock realization, namely consumption and leisure, $c_\circ(\epsilon_\circ)$ and $x_\circ(\epsilon_\circ)$; the capital stock at the beginning of the subsequent period, $k_+(\epsilon_\circ)$; government net liabilities at the beginning of the subsequent period, $b_+(\epsilon_\circ)$; and the

[2] The lower bound $\underline{M}$ does not bind, $\xi_{2,t} = 0$, because the government can pay lump-sum transfers.

[3] If the process for government consumption has an absorbing state then ν_t and taxes converge to a strictly negative and positive value, respectively. Ad-hoc restrictions on asset accumulation would imply that $\xi_{2,t}$ differs from zero, undermining the convergence result.

labor income tax rate, $\tau_\circ(\epsilon_\circ)$. In the initial period, the risk-free interest rate is given.

The constraints of the government's program are given by

$$
\begin{aligned}
&u_c(\epsilon_-) = \beta\mathbb{E}[u_c(\epsilon_\circ)R_\circ|\epsilon_-],\\
&u_c(\epsilon_-) = \beta\mathbb{E}[u_c(\epsilon_\circ)(1+f_K(\epsilon_\circ)-\delta)|\epsilon_-],\\
&u_c(\epsilon_-) = \beta\mathbb{E}[u_c(\epsilon_\circ)R^i(\epsilon_\circ)|\epsilon_-],\\
&\tau_\circ(\epsilon_\circ) = 1 - \frac{u_x(\epsilon_\circ)}{u_c(\epsilon_\circ)f_L(\epsilon_\circ)},\\
&R_\circ b_\circ - \omega_\circ(\epsilon_\circ) + g_\circ(\epsilon_\circ) \leq \tau_\circ(\epsilon_\circ)(1-x_\circ(\epsilon_\circ))f_L(\epsilon_\circ) + b_+(\epsilon_\circ),\\
&c_\circ(\epsilon_\circ) + g_\circ(\epsilon_\circ) + k_+(\epsilon_\circ) = (1-\delta)k_\circ + f(\epsilon_\circ),\\
&\underline{M}(\cdot) \leq u_c(\epsilon_\circ)b_+(\epsilon_\circ) \leq \bar{M}(\cdot),
\end{aligned}
$$

where

$$
\omega_\circ(\epsilon_\circ) \equiv \sum_i e^i_\circ(R^i(\epsilon_\circ) - R_\circ) + k^g_\circ(1+f_K(\epsilon_\circ)-\delta-R_\circ)
$$

denotes the return on the government's portfolio $(\{e^i_\circ\}_i, k^g_\circ)$. The first three constraints represent the household's Euler equations for risk-free government debt, capital, and the other assets. The fourth constraint relates the labor income tax rate to the household's marginal rate of substitution. The remaining constraints represent the (government) budget constraint, the resource constraint, and the debt limits.

Using the household's first-order conditions to substitute out $\tau_\circ(\epsilon_\circ)$ and $R_\circ$ and letting $\tilde{b}_\circ \equiv b_\circ u_c(\epsilon_-)$, we can express the budget constraint as

$$
\left(\frac{\tilde{b}_\circ}{\beta\mathbb{E}[u_c(\epsilon_\circ)|\epsilon_-]} - \tilde{\omega}_\circ(\epsilon_\circ) + g_\circ(\epsilon_\circ)\right)u_c(\epsilon_\circ) \leq (u_c(\epsilon_\circ)f_L(\epsilon_\circ) - u_x(\epsilon_\circ))(1-x_\circ(\epsilon_\circ)) + \tilde{b}_+(\epsilon_\circ),
$$

where $\tilde{\omega}_\circ(\epsilon_\circ)$ differs from $\omega_\circ(\epsilon_\circ)$ in that $R_\circ$ is replaced by $u_c(\epsilon_-)/(\beta\mathbb{E}[u_c(\epsilon_\circ)|\epsilon_-])$. The constraint set of the government is characterized by this modified budget constraint as well as the Euler equations, the resource constraint, and the debt limits. The Bellman equation reads

$$
\begin{aligned}
V(k_\circ, \tilde{b}_\circ, u_c(\epsilon_-), \epsilon_-) \quad &= \quad \max \mathbb{E}[u(\epsilon_\circ) + \beta V(k_+(\epsilon_\circ), \tilde{b}_+(\epsilon_\circ), u_c(\epsilon_\circ), \epsilon_\circ)|\epsilon_-]\\
\text{s.t.} \quad & \quad \text{constraint set},
\end{aligned}
$$

and the choice variables are $k^g_\circ, \{e^i_\circ\}_i, \{c_\circ(\epsilon_\circ), x_\circ(\epsilon_\circ), k_+(\epsilon_\circ), \tilde{b}_+(\epsilon_\circ)\}_{\epsilon_\circ}$. Note that in accordance with our definition of the state, the value function represents the unconditional value, prior to the realization of $\epsilon_\circ$.

Let $\nu_\circ(\epsilon_\circ)\cdot\text{prob}(\epsilon_\circ|\epsilon_-)$ denote the multiplier associated with the budget constraint (the shadow value of public funds) when $\epsilon_\circ$ is realized. The government's first-order conditions with respect to $\tilde{b}_+(\epsilon_\circ)$ (assuming debt limits do not bind), $e^i_\circ$, and $k^g_\circ$, respectively, are given by

$$
\begin{aligned}
&\nu_\circ(\epsilon_\circ) + \beta V_b(k_+(\epsilon_\circ), \tilde{b}_+(\epsilon_\circ), u_c(\epsilon_\circ), \epsilon_\circ) = 0,\\
&\mathbb{E}[\nu_\circ(\epsilon_\circ)u_c(\epsilon_\circ)(R^i(\epsilon_\circ) - R_\circ)|\epsilon_-] = 0,\\
&\mathbb{E}[\nu_\circ(\epsilon_\circ)u_c(\epsilon_\circ)(1+f_K(\epsilon_\circ)-\delta-R_\circ)|\epsilon_-] = 0,
\end{aligned}
$$

and the envelope condition implies

$$V_b(k_\circ, \tilde{b}_\circ, u_c(\epsilon_-), \epsilon_-) = -\sum_{\epsilon_\circ} \nu_\circ(\epsilon_\circ)\text{prob}(\epsilon_\circ|\epsilon_-)\frac{u_c(\epsilon_\circ)}{\beta\mathbb{E}[u_c(\epsilon_\circ)|\epsilon_-]} = -\frac{R_\circ}{u_c(\epsilon_-)}\mathbb{E}[\nu_\circ(\epsilon_\circ)u_c(\epsilon_\circ)|\epsilon_-].$$

Combined, these equations yield the optimality conditions

$$\begin{aligned}
\nu_-(\epsilon_-)u_c(\epsilon_-) &= \beta\mathbb{E}[\nu_\circ(\epsilon_\circ)u_c(\epsilon_\circ)R_\circ|\epsilon_-], \\
\nu_-(\epsilon_-)u_c(\epsilon_-) &= \beta\mathbb{E}[\nu_\circ(\epsilon_\circ)u_c(\epsilon_\circ)R^i(\epsilon_\circ)|\epsilon_-], \\
\nu_-(\epsilon_-)u_c(\epsilon_-) &= \beta\mathbb{E}[\nu_\circ(\epsilon_\circ)u_c(\epsilon_\circ)(1 + f_K(\epsilon_\circ) - \delta)|\epsilon_-].
\end{aligned}$$

These conditions resemble the stochastic Euler equations characterizing a household's portfolio choice (see section 5.1). They differ insofar as marginal utility is replaced by the product of marginal utility and the shadow value of public funds. Intuitively, the Ramsey policy equalizes the return weighted average valuation of public funds over time exactly as a household equalizes the return weighted average marginal utility.

Note that the first condition coincides with the result derived earlier, namely that the change of the government budget multiplier, weighted by marginal utility, equals zero on average. This follows from

$$\mathbb{E}\left[\beta R_\circ \nu_\circ(\epsilon_\circ)u_c(\epsilon_\circ) - \nu_-(\epsilon_-)u_c(\epsilon_-)|\epsilon_-\right] = \beta R_\circ \mathbb{E}\left[(\nu_\circ(\epsilon_\circ) - \nu_-(\epsilon_-))u_c(\epsilon_\circ)|\epsilon_-\right] = 0.$$

The second and third optimality condition generalize this result. For any asset or liability in the government's portfolio, the Ramsey policy satisfies

$$\beta\mathbb{E}\left[\nu_\circ(\epsilon_\circ)u_c(\epsilon_\circ)\left(R^i(\epsilon_\circ) - R_\circ\right)|\epsilon_-\right] = 0.$$

That is, a more diversified portfolio results in a smoother multiplier and thus, better insurance for the government. If the portfolio were sufficiently diversified for the government (and the household) to face complete markets then the multiplier would be constant across histories.

The optimality conditions can be used to derive an *stochastic discount factor* for *government projects*. Letting $\mu_\circ(\epsilon_\circ) \cdot \text{prob}(\epsilon_\circ|\epsilon_-)$ denote the multiplier associated with the resource constraint when $\epsilon_\circ$ is realized, this discount factor is given by

$$\beta\frac{\nu_\circ(\epsilon_\circ)u_c(\epsilon_\circ) + \mu_\circ(\epsilon_\circ)}{\nu_-(\epsilon_-)u_c(\epsilon_-) + \mu_-(\epsilon_-)}.$$

12.1.3 Capital Income Taxation

Neutrality Result

Consider the model with capital of subsection 12.1.2 and assume that the government may impose state-contingent taxes on the return on capital, in addition to labor income taxes. Capital income tax rates, $\tau^k_\circ(\epsilon_\circ)$, only enter the household's Euler equation for capital and the government's budget constraint:

$$u_c(\epsilon_-) = \beta\mathbb{E}[u_c(\epsilon_\circ)(1 + (1 - \tau^k_\circ(\epsilon_\circ))(f_K(\epsilon_\circ) - \delta))|\epsilon_-],$$
$$R_\circ b_\circ - \omega_\circ(\epsilon_\circ) + g_\circ(\epsilon_\circ) \leq \tau_\circ(\epsilon_\circ)(1 - x_\circ(\epsilon_\circ))f_L(\epsilon_\circ) + \tau^k_\circ(\epsilon_\circ)k_\circ(f_K(\epsilon_\circ) - \delta) + b_+(\epsilon_\circ).$$

In the household's Euler equation, only the average tax wedge (suitably weighted) matters. In the budget constraint, effects from changes in tax rates can be neutralized by appropriately adjusting the government's portfolio when markets are complete. With complete markets, an equilibrium allocation thus can be implemented with different combinations of instruments, for instance, with state-contingent capital income taxes and risk-free-coupon bonds, or with non-contingent capital income taxes and state-contingent-coupon bonds.

We have thus derived a neutrality result: Optimal state-contingent capital income tax rates are indeterminate if the government faces complete markets.

Zero Capital Income Taxation

Consider a deterministic setting and suppose that the economy is inhabited by an infinitely lived, representative agent whose date-t capital and labor incomes are taxed at rates τ^k_t and τ_t, respectively. The household's intertemporal budget constraint and first-order conditions are given by

$$\begin{aligned} 0 &= k_0 R_0(1 - \tau^k_0) + \sum_{t=0}^{\infty} \tilde{q}_t \left(w_t(1 - x_t)(1 - \tau_t) - c_t\right), \\ \tilde{q}_t &= \beta^t u_c(c_t, x_t)/u_c(c_0, x_0), \\ w_t(1 - \tau_t) &= u_x(c_t, x_t)/u_c(c_t, x_t), \end{aligned}$$

where we define $\tilde{q}_t \equiv (\tilde{R}_1 \cdots \tilde{R}_t)^{-1}$ and $\tilde{R}_t$ denotes the after-tax gross interest rate. In equilibrium, the latter equals $\tilde{R}_t = 1 + (1 - \tau^k_t)(f_K(k_t, 1 - x_t) - \delta)$. Substituting the first-order conditions into the intertemporal budget constraint we arrive at the implementability constraint which is a function of $\{c_t, x_t\}_{t\geq 0}$, $k_0 R_0$, and the tax rate on the initial capital stock, τ^k_0, which we assume to be capped by an exogenous upper bound.[4] Write this constraint as

$$\iota_0(c_0, x_0, k_0 R_0, \tau^k_0) + \sum_{t=0}^{\infty} \beta^t \iota(c_t, x_t) = 0$$

for some functions ι_0 and ι. The Lagrangian associated with the Ramsey program reads

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{v(c_t, x_t, \nu) - \mu_t(c_t + g_t + k_{t+1} - (1 - \delta)k_t - f(k_t, 1 - x_t))\} + \nu\iota_0(c_0, x_0, k_0 R_0, \tau^k_0),$$

[4]Without an upper bound the Ramsey problem would be trivial: The Ramsey policy would only tax the inelastically supplied initial capital stock.

where we define $v(c_t, x_t, \nu) \equiv u(c_t, x_t) + \nu\, \iota(c_t, x_t)$; ν and $\beta^t \mu_t$ denote the multipliers associated with the implementability and resource constraints, respectively.

The first-order conditions with respect to $c_t, t \geq 1$, and k_{t+1}, respectively, are given by

$$\begin{aligned} v_c(c_t, x_t, \nu) &= \mu_t,\ t \geq 1, \\ \mu_t &= \beta \mu_{t+1}(1 - \delta + f_K(k_{t+1}, 1 - x_{t+1})). \end{aligned}$$

Combining these conditions yields the key equation of interest which we report together with the household's Euler equation:

$$\begin{aligned} v_c(c_t, x_t, \nu) &= \beta(1 - \delta + f_K(k_{t+1}, 1 - x_{t+1}))v_c(c_{t+1}, x_{t+1}, \nu),\ t \geq 1, \\ u_c(c_t, x_t) &= \beta(1 + (1 - \tau^k_{t+1})(f_K(k_{t+1}, 1 - x_{t+1}) - \delta))u_c(c_{t+1}, x_{t+1}). \end{aligned}$$

The two equations imply that under the Ramsey policy capital income is not taxed at dates $t \geq 2$ whenever $v_c(c_t, x_t, \nu)$ and $u_c(c_t, x_t)$ grow at the same rate. Two alternative conditions guarantee such equal growth and thus, optimality of *zero capital income taxation*. The first condition relates to preferences. If preferences are separable between consumption and leisure, and homothetic, then $v_c(c_t, x_t, \nu)$ is proportional to $u_c(c_t, x_t)$. In this case, the zero capital taxation implication is an instance of the uniform commodity taxation result discussed in appendix B.6.1. Recall from subsection 11.3.2 that capital income taxation is equivalent to time varying taxation of consumption. When consumption taxes are normalized to zero and the structure of preferences calls for uniform taxation of consumption, capital income must not be taxed. Standard CIES preferences satisfy the separability and homotheticity condition.

The second, alternative condition relates to the economy's dynamics. If the Ramsey allocation converges to a path along which both $v_c(c_t, x_t, \nu)$ and $u_c(c_t, x_t)$ are constant over time (i.e., a steady state) or growing at equal rates then the optimality of zero capital income taxation follows. In fact, it also follows in richer environments, for instance when in steady state the derivative of the implementability constraint(s) with respect to capital equal(s) zero and the multiplier(s) of the constraint(s) are constant.[5]

An upper bound on capital income tax rates, $\tau^k_{t+1} \leq \bar{\tau}^k$, which can be expressed as

$$1 - \left(\frac{u_c(c_t, x_t)}{\beta u_c(c_{t+1}, x_{t+1})} - 1 \right) / (f_K(k_{t+1}, 1 - x_{t+1}) - \delta) \leq \bar{\tau}^k,$$

introduces additional terms in the government's optimality conditions. As long as the constraint binds (forcing taxes to be spread over a longer horizon) the key equation discussed above contains additional terms, and this undermines the zero-tax-rate implication. The constraint may bind forever.

[5]This holds true, for example, in the steady state of an economy with heterogenous households whose capital income—but not labor income—is taxed at a uniform rate.

12.1.4 Heterogeneous Households

When households are homogeneous the assumption that the government levies distorting taxes rests on weak foundations. After all, if everybody is the same, non-distorting lump-sum taxes can be implemented and clearly are preferable to distorting taxes. With heterogeneous agents, in contrast, non-distorting taxes may not be implementable if the government does not observe an agent's type, for example whether a household is productive or not. When the government is motivated by distributive concerns, in addition to efficiency considerations, an *equity-efficiency trade-off* arises and this can rationalize distorting taxation of an endogenous tax base.

Consider a variant of the economy analyzed in subsection 12.1.1 with two rather than one group of households, groups a and b with population shares η^a and $\eta^b = 1 - \eta^a$, respectively. Both groups have the same preferences but their labor productivities differ, $w_t^a \neq w_t^b$. For simplicity, we abstract from risk. The resource constraint is given by

$$\eta^a c_t^a + \eta^b c_t^b + g_t = \eta^a w_t^a (1 - x_t^a) + \eta^b w_t^b (1 - x_t^b).$$

In each period, the government has two tax instruments at its disposal: A proportional labor income tax levied at rate τ_t, and a lump-sum tax, θ_t. The government's objective function is given by the *social welfare function* $\omega \eta^a U^a + \eta^b U^b$ where U^i denotes welfare of a member of group i and ω denotes some positive weight. The intertemporal budget constraint of a household in group i reads

$$\sum_{t=0}^{\infty} q_t [c_t^i - (1 - \tau_t) w_t^i (1 - x_t^i) + \theta_t] = 0.$$

Substituting the household first-order conditions,

$$\begin{aligned} u_c(c_0^i, x_0^i) q_t &= \beta^t u_c(c_t^i, x_t^i), \\ u_c(c_t^i, x_t^i) w_t^i (1 - \tau_t) &= u_x(c_t^i, x_t^i), \end{aligned}$$

into the budget constraints yields the implementability constraints,

$$\sum_{t=0}^{\infty} \beta^t [u_c(c_t^i, x_t^i)(c_t^i + \theta_t) - u_x(c_t^i, x_t^i)(1 - x_t^i)] = 0, \quad (12.1)$$

$$u_c(c_t^a, x_t^a) / u_c(c_0^a, x_0^a) = u_c(c_t^b, x_t^b) / u_c(c_0^b, x_0^b), \quad (12.2)$$

$$u_c(c_t^a, x_t^a) w_t^a / u_x(c_t^a, x_t^a) = u_c(c_t^b, x_t^b) w_t^b / u_x(c_t^b, x_t^b). \quad (12.3)$$

Constraints (12.2) and (12.3) capture the restriction that households in both groups face the same prices and tax rates.

Let ν^a and ν^b denote the multipliers associated with the implementability constraints (12.1) for groups a and b. The optimal choice of lump-sum tax at date t satisfies

$$\nu^a u_c(c_t^a, x_t^a) + \nu^b u_c(c_t^b, x_t^b) = 0,$$

that is, the Ramsey policy sets the average multiplier equal to zero. To understand this result suppose first that group b did not exist such that only the implementability constraint (12.1) for group a were present. The optimality condition for θ_t then would collapse to $\nu^a = 0$, indicating that the competitive equilibrium constraint were not binding for the Ramsey government. Intuitively, the Ramsey policy could implement the first best in this case because the government could costlessly transfer resources from the private to the public sector.

With heterogeneous households, the lump-sum tax still allows the government to extract resources without distorting household choices. But since the lump-sum tax cannot be differentiated across groups the government cannot in general extract resources and attain the preferred wealth distribution without distorting household choices. Against this background, the optimal choice of θ_t "at least" equalizes the average value of the multiplier with zero. If, by chance, the optimal lump-sum tax happens to implement the preferred wealth distribution, then ν^a and ν^b individually equal zero as well. Otherwise, the Ramsey policy also employs labor income taxes, at the cost of generating tax distortions.

From the implementability constraint (12.2), marginal utility grows at identical rates across groups. This implies that all lump-sum taxes but one are redundant instruments (their first-order conditions are multiples of each other) and a Ricardian equivalence result holds: A change of timing of lump-sum taxes accompanied by suitable debt operations does not alter the equilibrium allocation.

To see how the timing of labor income taxes can affect the wealth distribution let $u(c, x) \equiv \ln(c) + \gamma \ln(x)$ and disregard lump-sum taxes. The implementability constraints then read

$$\sum_{t=0}^{\infty} \beta^t [1 - \gamma(1 - x_t^i)/x_t^i] = 0,$$

$$\frac{c_0^a}{c_0^b} = \frac{c_t^a}{c_t^b} = \frac{w_t^a}{w_t^b} \frac{x_t^a}{x_t^b}$$

or, more compactly,

$$\sum_{t=0}^{\infty} \beta^t \left[1 - \gamma \left(\frac{1}{x_t^a} - 1\right)\right] = 0,$$

$$\sum_{t=0}^{\infty} \beta^t \left[1 - \gamma \left(\frac{c_0^a}{c_0^b} \frac{w_t^b}{w_t^a} \frac{1}{x_t^a} - 1\right)\right] = 0.$$

From the first equation, raising x_0^a requires lowering x_t^a at some other date. From the second equation, this translates into a change of relative wealth and consumption, c_0^a/c_0^b, if relative productivity varies over time, $w_0^a/w_0^b \neq w_t^a/w_t^b$. Specifically, an increase in x_0^a (corresponding to a tax hike at date $t = 0$) and corresponding decrease of x_t^a raises c_0^a/c_0^b if $w_0^a/w_0^b \leq w_t^a/w_t^b$. Wealth is redistributed for two reasons. First, collecting taxes in periods where one group is relatively more productive shifts the tax burden to that group. Second, tax induced changes in consumption and thus, interest rates affect debtors and creditors asymmetrically.

respectively. At date $t = 0$, the first-order condition with respect to inflation does not contain the $\lambda_{t-1}(\epsilon^{t-1})$ term. Combining these equations yields the optimality condition

$$\chi_t(\epsilon^t) = -\frac{\kappa}{\omega}\left(\pi_0(\epsilon^0) + \cdots + \pi_t(\epsilon^t)\right) = -\frac{\kappa}{\omega}\left(p_t(\epsilon^t) - p_{-1}(\epsilon^{-1})\right). \tag{12.4}$$

Condition (12.4) confirms the intuition developed above. It indicates that the Ramsey policy *targets* the *price level* that is, the negative output gap evolves proportionally to the cumulative inflation rate and it disappears only when the price level has reverted to its starting value, $p_{-1}(\epsilon^{-1})$. A characterization of optimal policy in terms of relations between policy targets is referred to as a *targeting rule*. The targeting rule (12.4) implies an associated interest rate rule; when the latter satisfies the Taylor principle then it uniquely implements the Ramsey allocation.

12.4 Bibliographic Notes

Lucas and Stokey (1983) analyze the model presented in subsection 12.1.1. Angeletos (2002) shows that markets are complete even with non-contingent coupons as long as the maturity structure is sufficiently rich (see also Gale; 1990). Aiyagari et al. (2002), Werning (2003), and Farhi (2010) analyze the model presented in subsection 12.1.2. The special case with quasilinear utility provides micro foundations for the tax-smoothing results in Barro (1979) and Bohn (1990). The indeterminacy result for optimal capital income taxes is due to Zhu (1992) and Chari et al. (1994). Chamley (1986) and Chari et al. (1994) derive the optimality of zero capital income taxes with CIES preferences. Chamley (1986) and Judd (1985) derive steady-state results and Straub and Werning (2014) clarify the conditions under which these results apply. Subsection 12.1.4 follows Werning (2007) and Niepelt (2004b).

Diamond and Mirrlees (1978), Rogerson (1985), and Golosov et al. (2003) establish that moral hazard renders the optimal intertemporal wedge non-zero.

Bailey (1956) and Friedman (1969) analyze the welfare costs of inflation. Phelps (1973) questions the applicability of the Friedman rule in environments with distorting taxes, see also Chari et al. (1996), Mulligan and Sala-i-Martin (1997), and Correia and Teles (1999). Clarida et al. (1999) analyze optimal policy in the New Keynesian model, and Rotemberg and Woodford (1997) and Woodford (2003, 6) derive the loss function in that model.

Related Topics and Additional References Bhandari et al. (2017) characterize the stationary distribution of Ramsey taxes and debt when markets are incomplete.

Aiyagari (1995) shows that the optimal long-run capital income tax is strictly positive when uninsurable idiosyncratic labor income risk leads households to excessively accumulate capital. Farhi (2010) derives optimal capital income taxes in the model discussed in subsection 12.1.2. Atkinson and Sandmo (1980) and King (1980) derive conditions for optimal capital income taxes in the overlapping generations model.

Atkeson and Lucas (1992), Phelan (2006), and Farhi and Werning (2007) study the implications of optimal social insurance in dynastic economies with private information; constrained efficient allocations display *immiseration*—inheritable and increasing welfare inequality—unless the Ramsey government values the welfare of future generations directly, not only indirectly through the altruism of their ancestors.

Siu (2004) analyzes the Ramsey policy in an environment with rigid prices when inflation serves as fiscal shock absorber, due to non-contingent, nominal debt; he finds that even a small degree of price rigidity renders optimal inflation non-volatile and tax rates volatile. Adam and Weber (2019) study the optimal steady-state inflation rate when the productivity of firms displays experience and cohort effects.

Correia et al. (2008) establish that with sufficiently flexible tax instruments, the Ramsey policy can neutralize the effects of price rigidity; see also Correia et al. (2013).

Chari and Kehoe (1999) cover optimal taxation and Ramsey policies; see also the references in appendix B.6. Ljungqvist and Sargent (2018, 20) cover models with a conflict between insurance and incentives, and Golosov et al. (2016) cover dynamic incentive problems. Woodford (2003, 6, 7), Galí (2008, 4, 5), and Walsh (2017, 8) cover optimal policy in the New Keynesian model.

Chapter 13

Time Consistent Policy

Ramsey policies of the type analyzed in chapter 12 specify history-contingent values for all policy instruments. These values are optimal ex ante, from the perspective of the initial date, and they are chosen under *commitment*: Ex post, as time proceeds, the government implements the chosen path and does not re-optimize. An alternative specification of government behavior is that governments *sequentially* determine the values of the policy instruments, under *discretion*, at the time when the instruments actually are implemented. Governments acting under discretion perceive private sector choices in earlier periods as bygones, unlike a Ramsey government that determines all instruments before the private sector makes its choices. This alters the trade-offs that governments face and introduces a layer of dynamic interaction between the private sector and successive governments (or selves of one and the same government) which is absent when the government acts under commitment.

We illustrate this interaction in the context of a simple two-period model. Thereafter, we study the taxation of labor and capital income when the government chooses tax rates under discretion, and we analyze public debt policy when the government cannot commit to honoring its liabilities. Finally, we study redistribution in politico-economic equilibrium and analyze how discretionary monetary policy shapes inflation and output differently than a committed or rules-based policy.

13.1 Time Consistency and the Role of State Variables

Sequential policy choice introduces the possibility that the government at a date $t > 0$ chooses values for the policy instruments that differ from the preferred values of the government at date $t = 0$. This may occur for two reasons. Either, because the objectives of the two governments (or of the same government at different points in time) are not dynamically consistent, similarly to the preferences of a household with non-geometrically declining discount factors (see subsection 2.2.2). Or, because the constraints faced by the government change over time.

To see how the passing of time can alter a government's constraint set consider a setting with two periods. At date $t = 0$, the private sector makes a choice, s say (for instance, it saves), and at date $t = 1$, the government takes an action, τ (for example, it imposes a tax). The private sector's choice depends on the expected government action, τ^e. When the government moves first, choosing τ before the household determines s, then the choice of τ also determines τ^e and thus, affects s. When the government moves second, in contrast, then its actual choice, τ, is irrelevant for s; all that matters for s is the action the private sector anticipates the government to take, τ^e. Ex post (conditional on $s(\tau^e)$), the government's choice of τ may differ from τ^e.

In equilibrium, the government's seeming degree of freedom to choose $\tau \neq \tau^e$ vanishes because the private sector rationally anticipates the government's ex-post optimal action: τ^e thus equals τ in equilibrium. In fact, the government's choice set when moving first—under commitment—is larger than when it moves second—under discretion—because only in the former case can the government steer private sector expectations away from anticipating the ex-post optimal government action.

These findings generalize. When the government cannot commit, an *equilibrium policy* satisfies two requirements: It implements an economic equilibrium and it is *time consistent* that is, the continuation of the policy is ex-post optimal. Compared to a Ramsey policy, an equilibrium policy thus satisfies additional *incentive compatibility constraints*. The optimal policy under commitment (the Ramsey policy) is optimal in the set of feasible and admissible policies while the optimal policy under discretion is optimal in the set of policies that are both feasible and admissible and ex-post optimal. Since the former set always is weakly larger than the latter the ability to commit always has a non-negative value. When the Ramsey policy is *time inconsistent* then the ability to commit has strictly positive value.

When a government lacks commitment, (other) state variables may partly or even fully compensate for this deficiency. Consider again the two-period example and assume for simplicity that the government can make an announcement at date $t = 0$, $\hat{\tau}$, whose only effect is to render it very costly for the government at date $t = 1$ to implement a τ that differs from $\hat{\tau}$ by more than ε. The announcement and the cost of deviating from it constitute a state variable. When $\varepsilon = 0$, then this state variable is a perfect substitute for commitment. But even when $\varepsilon > 0$ the announcement is useful when it enhances the date-$t = 0$ government's *credibility* that is, when it induces the government at date $t = 1$ to choose a policy closer to $\hat{\tau}$ than it would choose without the announcement. In a stochastic environment stronger credibility may come at the cost of reduced flexibility: When $\hat{\tau}$ cannot be contingent then the announcement may limit the ability of the government at date $t = 1$ to appropriately respond to shocks.

The ideal state variable that helps implement the Ramsey allocation when the government cannot commit has two properties. Ex ante, it is neutral in the sense that it does not impose additional constraints on the equilibrium allocation. Ex post, its presence provides incentives for the government not to deviate from the ex-ante optimal policy choice. In the model discussed in section 13.2 the maturity structure of public debt constitutes such an ideal state variable.

13.2 Credible Tax Policy

Consider the complete markets tax smoothing problem analyzed in subsection 12.1.1 and suppose that the government cannot commit to the ex-ante optimal tax policy while it can commit to repaying debt. Recall that the Ramsey allocation uniquely determines the value of government indebtedness at date t,

$$\sum_{s=t}^{\infty} \int \frac{q_s(\epsilon^s)}{q_t(\epsilon^t)} {}_t b_s(\epsilon^{t-1}, \epsilon^s) d\epsilon^s | \epsilon^t,$$

while it does not determine the *maturity structure*. This ex-ante indeterminacy can be exploited to render the Ramsey policy time consistent. There exists a unique maturity structure of debt that both supports the Ramsey policy and renders it time consistent.[1]

To see this, consider date $t = 1$ (parallel arguments apply for subsequent periods) when the government is bound to honor the outstanding promises $\{{}_1 b_t(\epsilon^0, \epsilon^t)\}_{t \geq 1}$ but free to choose any tax sequence $\{\tau_t(\epsilon^t)\}_{t \geq 1}$ satisfying the date-$t = 1$ implementability constraint as well as the resource constraints. The government's program at date $t = 1$ resembles the program at date $t = 0$ except that the debt maturities outstanding are given by $\{{}_1 b_t(\epsilon^0, \epsilon^t)\}_{t \geq 1}$ rather than $\{{}_0 b_t(\epsilon^{-1}, \epsilon^t)\}_{t \geq 0}$. Accordingly, the first-order conditions of the date-$t = 1$ program are given by

$$\begin{aligned}
(1 + \nu_1) u_c(c_t, x_t) + \nu_1 (u_{cc}(c_t, x_t)(c_t - {}_1 b_t) - u_{xc}(c_t, x_t)(1 - x_t)) &= \mu_{1t}, \\
(1 + \nu_1) u_x(c_t, x_t) + \nu_1 (u_{cx}(c_t, x_t)(c_t - {}_1 b_t) - u_{xx}(c_t, x_t)(1 - x_t)) &= \mu_{1t} w_t
\end{aligned}$$

for $t \geq 1$ where we suppress histories for legibility and use the notation from subsection 12.1.1. Note that the multipliers of the date-$t = 1$ program are indexed by "1" to distinguish them from the multipliers of the date-$t = 0$ program.

If these conditions together with the implementability constraint as of date $t = 1$ and the resource constraints are to prescribe the same allocation as the conditions resulting from the program at date $t = 0$, then the first-order conditions of the two programs must be satisfied for the same sequences $\{c_t, x_t\}_{t \geq 1}$. From the first-order conditions of the date-$t = 0$ and date-$t = 1$ programs, this implies the restrictions

$$\nu_1 \, {}_1 b_t = \nu \, {}_0 b_t + (\nu_1 - \nu) A_t, \; t \geq 1 \;\; \forall \epsilon^t | \epsilon^1,$$

where A_t is a function of the Ramsey allocation.[2] For every history ϵ^1, the multiplier ν and the sequences $\{A_t\}_{t \geq 1}$ and $\{{}_0 b_t\}_{t \geq 1}$ are given; the above restrictions and the implementability constraint as of date $t = 1$ thus pin down ν_1 and $\{{}_1 b_t\}_{t \geq 1}$. We conclude that the Ramsey allocation, the multiplier ν, and the initial maturity structure $\{{}_0 b_t\}_{t \geq 1}$ uniquely determine a maturity structure of debt issued at date $t = 0$ and outstanding at date $t = 1$ that renders the Ramsey policy time consistent.

[1] We assume that the Ramsey policy operates on the increasing segment of the Laffer curve.

[2] To derive the restriction, multiply the first-order condition with respect to c_t by w_t; subtract the first-order condition with respect to x_t from the resulting equality; follow the same steps with the first-order conditions from the date-$t = 0$ program; and collect terms. We have $A_t \equiv (w_t u_c(c_t, x_t) - u_x(c_t, x_t) + c_t(w_t u_{cc}(c_t, x_t) - u_{cx}(c_t, x_t)) + (1 - x_t)(u_{xx}(c_t, x_t) - w_t u_{cx}(c_t, x_t)))/(w_t u_{cc}(c_t, x_t) - u_{cx}(c_t, x_t))$.

Intuitively, the choice of maturity structure at date $t = 0$ affects the trade-offs faced by the government at date $t = 1$ because the extent to which changes in the allocation and thus, the stochastic discount factor affect the intertemporal budget constraint depend on the debt positions, $\{{}_1b_t\}_{t\geq 1}$. By appropriately structuring these positions the government at date $t = 0$ can assure that the preferred tax sequences as of date $t = 0$ and date $t = 1$ coincide from date $t = 1$ onwards. The choice of maturity structure at date $t = 0$ thus allows the government to effectively tie its hands, as if it had commitment.

13.3 Capital Income Taxation

Capital and capital income constitute elastic tax bases ex ante when households adjust their saving and investment in response to anticipated capital income tax rates, but a completely inelastic tax base ex post when no such adjustment is possible any more. Ex ante, capital income taxes thus are distorting while they are non-distorting ex post. This renders capital income taxation under the Ramsey policy particularly exposed to problems of time inconsistency.

Consider an infinite horizon economy with two subperiods at each date t, a "morning" and an "evening." In the morning, the representative household is endowed with w units of the good which it may consume, c_{1t}, or invest in capital, k_t,

$$w = c_{1t} + k_t.$$

In the evening, capital yields the exogenous return $R > 1$ and depreciates thereafter. Households supply labor, $1 - x_t$, whose productivity equals unity; consume leisure, x_t; and consume their after-tax capital and labor income. Capital income is taxed at rate $\tau_t^k \leq 1$ and labor income, $1 - x_t$, is taxed at rate $\tau_t \leq 1$. Consumption in the evening, c_{2t}, thus equals

$$c_{2t} = k_t R(1 - \tau_t^k) + (1 - x_t)(1 - \tau_t).$$

Households discount the future at the factor $\beta \in (0, 1)$. Their period utility function, u, is strictly increasing and concave in consumption and leisure. For simplicity, we assume that c_{1t} and c_{2t} are perfect substitutes such that utility from consumption and leisure at date t equals $u(c_{1t} + c_{2t}, x_t)$. The government is benevolent, faces an exogenous revenue requirement, g, in the evening, and does not issue debt. The government budget constraint thus reads

$$g = k_t R\tau_t^k + (1 - x_t)\tau_t.$$

We start by characterizing the Ramsey policy before studying time consistent policies in the absence of commitment.

13.3.1 Commitment Benchmark

Since capital immediately depreciates and the government issues no debt the Ramsey problem consists of a sequence of subproblems, one for each date. Focusing on the problem at date t consider the savings choice of a household. When the after-tax return on a unit of capital exceeds unity that is, when $\tau_t^k \leq (R-1)/R$, then the household saves its first-period endowment in full and the capital income tax is non-distorting. The tax rate affects the tax revenue in this case but not the tax base. When the after-tax return is strictly smaller than unity, in contrast, then the savings rate equals zero and the government collects no capital income taxes. The maximal capital income tax revenue thus is attained for $\tau_t^k = (R-1)/R$ and equals $w(R-1)$.

We assume that $g > w(R-1)$ such that the Ramsey tax rate on capital income equals $(R-1)/R$ and the Ramsey policy also taxes labor income. The tax rate τ_t and equilibrium labor supply, $1-x_t$, then solve

$$\begin{aligned} \frac{u_x(w+(1-x_t)(1-\tau_t),x_t)}{u_c(w+(1-x_t)(1-\tau_t),x_t)} &= 1-\tau_t, \\ w(R-1)+(1-x_t)\tau_t &= g. \end{aligned}$$

The first condition equates the household's marginal rate of substitution between consumption and leisure with the marginal rate of transformation, and the second condition represents the government budget constraint. Both conditions are evaluated at the equilibrium household choices $c_{1t}=0$, $k_t=w$, and $c_{2t}=w+(1-x_t)(1-\tau_t)$. Let u^R denote the period utility under the Ramsey policy.

13.3.2 No Commitment

Suppose next that the government cannot commit. Households choose their saving in the morning before the government fixes the tax rates that are imposed in the evening.

Static Equilibrium

For now, we also assume that there are no state variables other than capital. Since the capital income tax is non-distorting ex post while the labor income tax does distort labor supply the government's optimal policy choice in the evening amounts to taxing capital income as much as possible (and needed). That is, for any level of k_t that the household saves the government sets $\tau_t^k = \min[1, g/(k_t R)]$.

Anticipating this ex-post optimal policy choice households do not save at all and the government is forced to collect all revenue from labor income taxes.[3] Labor supply and the labor income tax rate in this *static time consistent equilibrium* satisfy

$$\begin{aligned} \frac{u_x(w+(1-x_t)(1-\tau_t),x_t)}{u_c(w+(1-x_t)(1-\tau_t),x_t)} &= 1-\tau_t, \\ (1-x_t)\tau_t &= g. \end{aligned}$$

The allocation in the static time consistent equilibrium gives strictly lower utility than the Ramsey allocation. Intuitively, both the Ramsey policy and the static time consistent policy implement a competitive equilibrium but only the latter satisfies the incentive compatibility constraint that the policy choice be ex-post optimal. This additional

[3]Note that $g/(k_t R) \geq g/(wR) > w(R-1)/(wR) = (R-1)/R$.

constraint is costly. Let u^s denote the period utility in the static equilibrium and let $V^s \equiv u^s/(1-\beta)$ denote the continuation value in a static equilibrium.[4]

If the government announced any tax rate $\tau_t^k \leq (R-1)/R$ (e.g., the Ramsey tax rate) and households believed the announcement and accumulated capital the government could always do better than following the announcement by imposing $\tau_t^k = \min[1, g/(k_t R)]$ instead and levying low labor income taxes. The period utility for households (and the government) of inducing capital accumulation k_t and deviating from the announcement ex post, $u^d(k_t)$ say, would exceed the utility from not deviating—a policy of taxing capital at a low rate therefore is not time consistent.

Note that the time inconsistency of policies with $\tau_t^k \leq (R-1)/R$ reflects a *lack of instruments*. If the government had access to a non-distorting tax then it would use this instrument rather than the labor income tax and the Ramsey policy would be time consistent.

History as a State Variable

Consider now the effect of introducing a new state variable—the *history* of preceding policy choices. We let $\pi_t \equiv (\tau_t^k, \tau_t)$ denote the choice of tax rates at date t and π^t the history of such choices up to and including date t. When households and the government condition their decisions on π^t this introduces a dynamic link across periods which has not been present so far. Moreover, when this link provides incentives for the government not to overburden capital ex post then this opens the possibility for time consistent equilibria that Pareto dominate the static equilibrium.

To explore this possibility, we study an *equilibrium* of *household* and *government plans*, ϕ and ψ respectively. The timing of events at date t is as follows. First, each household chooses the morning allocation according to the date-t morning component of the household plan, ϕ_{1t}, which maps the history π^{t-1} into (c_{1t}, k_t); moreover, the household determines a continuation plan for future choices contingent on future histories. Second, the government chooses feasible tax rates according to the date-t component of the government plan, ψ_t, which maps the history π^{t-1} into the policy choice π_t, and it determines a continuation plan. And finally, each household chooses the evening allocation according to the evening component of the household plan, ϕ_{2t}, which maps the history $\pi^t = (\pi^{t-1}, \pi_t)$ into (c_{2t}, x_t), and it determines a continuation plan.

Note that conditional on history π^{t-1}, a government plan of feasible policies induces the history $\pi^t = (\pi^{t-1}, \psi_t(\pi^{t-1}))$, $\pi^{t+1} = (\pi^t, \psi_{t+1}(\pi^t))$, etc. Jointly, the household and government plans therefore induce a continuation utility for the household (and the government) from date t onward which we denote by $V_t(\pi^{t-1}, \phi, \psi)$. We are interested in plans that implement an equilibrium with higher continuation value than in the static equilibrium, V^s.

Household and government plans perform two functions. On the one hand, they steer household expectations. On the other hand, they render it advantageous for

[4] The conditions characterizing (x_t, τ_t) in the static time consistent equilibrium coincide with the equilibrium conditions under the Ramsey policy when the government faces a larger revenue requirement, namely $g + w(R-1)$ rather than g. This implies that $u^R > u^s$.

the government to act in accordance with these expectations ex post by letting non-compliance trigger a change of expectations that implies worse outcomes in the future. An optimal plan leads households to expect sufficiently low capital income tax rates and thus, to accumulate capital while at the same time assuring ex-post incentive compatibility on the part of the government. Since the incentive to comply derives from the desire to avert worse outcomes in the future the mechanism can only work if the economy has an infinite horizon.

A *sustainable equilibrium* is a pair of incentive compatible, *sustainable plans*, ϕ and ψ, that induce a policy which implements a competitive equilibrium: Given ψ, the continuation of ϕ in the morning and the evening of date t is optimal for the household for any history π^{t-1} and π^t, respectively; and given ϕ, the continuation of ψ at date t is feasible and optimal for the government for any history π^{t-1}. To find the *best sustainable equilibrium* we first determine the *worst* one because the threat of a reversal to the latter incentivizes the government to implement the former.

The worst sustainable equilibrium is given by the static equilibrium characterized above. The static equilibrium is sustainable because given the expectation of a high capital income tax rate households optimally do not save; and given zero savings in the present and the future, the government is indifferent between setting a high or low capital income tax rate. It is the worst sustainable equilibrium because any feasible policy balances the government budget and in the static equilibrium the government levies taxes in the least efficient manner, relying exclusively on labor income taxes.

We have established that plans which implement the static equilibrium are incentive compatible and that they implement the worst sustainable equilibrium. Plans which trigger a reversion to the static equilibrium in response to non-compliance by the government thus provide the strongest possible incentive for the government to remain compliant. This implies that a pair of plans constitutes a sustainable equilibrium as long as the value of complying with the plans exceeds the value from deviating and implementing the static equilibrium forever after.

Formally, consider plans $\phi^\star$ and $\psi^\star$ that prescribe the policy $\pi_t = \pi^\star$ as long as the government only chose $\pi^\star$ in the past; and the policy implementing the static equilibrium in all future periods once the government has deviated from $\pi^\star$. The plans form a sustainable equilibrium if

$$V_t(\pi^{t-1}, \phi^\star, \psi^\star) \geq u^d(k_t) + \beta V^s.$$

In this case, infinite repetition of the policy choice $\pi^\star$ constitutes a *credible policy*.

Note that the static equilibrium is a sustainable equilibrium because $V^s \geq u^d(0) + \beta V^s$. But due to the dynamic link introduced by the state variable π^t, equilibria with strictly positive capital accumulation are sustainable as well as long as the long-term loss from reverting to the static equilibrium outweighs the short-term gain from imposing a high tax rate on the capital income tax base ex post. For a sufficiently high discount factor, even the Ramsey allocation can be sustained: As $\beta \to 1$, the sustainability condition

$$\frac{u^R}{1-\beta} \geq u^d(w) + \beta \frac{u^s}{1-\beta}$$

necessarily is satisfied.

13.4 Sovereign Debt and Default

When a debtor-creditor pair engages in borrowing and lending both parties benefit from gains of trade (see section 7.3). Once the debt comes due, however, their interests are no longer aligned since debt repayment constitutes a transfer from the debtor to the creditor. If the prospective debtor lacks commitment this ex-post conflict of interest can undermine the viability of the arrangement ex ante. For if the lender foresees that the borrower would not repay then the lender does not enter into the credit relationship in the first place (see also section 8.3).

We study a government that issues *sovereign debt* to a representative, competitive, risk neutral international investor. The government levies lump-sum taxes or pays lump-sum transfers to the representative domestic household who does not have access to international financial markets and takes no decisions. Unlike the investor, the government cannot commit. Rather than making contractually agreed payments to the investor it might choose to *default*.

13.4.1 Insurance

Consider first a static model. The country's endowment is risky. It takes the value $w_1(\epsilon^1)$ with probability $\eta(\epsilon^1)$. Since the household's period utility function, u, is strictly concave but the investor is risk neutral, the investor and the benevolent government agree on an insurance contract that stipulates (positive or negative) contingent payments, $T(\epsilon^1)$, from the country to the investor. In equilibrium, the investor must break even on average that is, the contract must satisfy the investor's participation constraint (or the country's budget constraint), $\sum_{\epsilon^1} \eta(\epsilon^1) T(\epsilon^1) \geq 0$.

Violating the contract ex post triggers exogenous default costs, $L(\epsilon^1) \geq 0$, which are born by the domestic household whose consumption equals $w(\epsilon^1) - T(\epsilon^1) - L(\epsilon^1)$. Since the government lacks commitment it cannot credibly promise to make a positive payment to the investor unless the amount is smaller than the costs the country would have to bear in case of default. Incentive compatibility therefore requires that, in addition to the participation constraint, the contract also satisfies the incentive constraints $L(\epsilon^1) \geq T(\epsilon^1)$. Note that the commitment outcome can be implemented in equilibrium when the default costs are sufficiently large such that the incentive constraints never bind. When $L(\epsilon^1) = 0$ in all histories, in contrast, then insurance is not viable because the government cannot credibly promise to make any payments to the investor.

In equilibrium, the country never defaults. If it did, the insurance contract could be improved by lowering the contractual payment to the investor in the history with default to the value of the default costs; this would lower the expected cost for the domestic household and weakly increase the expected payments to the investor. Letting λ and $\eta(\epsilon^1)\mu(\epsilon^1)$ denote the multipliers attached to the participation and incentive compatibility constraints, respectively, the optimal contracting program thus can be represented by the Lagrangian

$$\mathcal{L} = \sum_{\epsilon^1} \eta(\epsilon^1) u(w(\epsilon^1) - T(\epsilon^1)) + \lambda \left(\sum_{\epsilon^1} \eta(\epsilon^1) T(\epsilon^1) \right) + \sum_{\epsilon^1} \eta(\epsilon^1) \mu(\epsilon^1) (L(\epsilon^1) - T(\epsilon^1)),$$

and the first-order condition with respect to $T(\epsilon^1)$ is given by

$$u'(w(\epsilon^1) - T(\epsilon^1)) + \mu(\epsilon^1) = \lambda.$$

The condition states that the equilibrium contract provides insurance across all histories in which the incentive constraint does not bind ($\mu(\epsilon^1) = 0$). In histories where the constraint does bind ($\mu(\epsilon^1) > 0$) marginal utility is lower and thus, consumption higher than in the insured histories. If the constraint never binds, as would be the case with commitment, then consumption is perfectly insured (see subsection 4.2.1).

Intuitively, in histories with a binding incentive constraint, the government ex ante would like to make higher payments to the investor than is incentive compatible ex post. The cap that incentive compatibility imposes on payments from the country to the investor implies that in other histories, the payments to the country are smaller than under commitment as well. Relative to the Ramsey allocation, the country therefore consumes more in histories with high endowments and a binding incentive constraint, and less in histories with low endowments.

13.4.2 Borrowing with Contingent Debt

Suppose next that the country borrows an amount b at date $t = 0$ and repays the contingent amount $T(\epsilon^1)$ at date $t = 1$. The household's discount factor is β and the investor requires an expected gross interest rate R. We assume that either R or the endowment at date $t = 0$, w_0, is sufficiently low such that the government has a borrowing motive. The Lagrangian associated with this modified program reads

$$\begin{aligned} \mathcal{L} &= \beta^{-1} u(w_0 + b) + \sum_{\epsilon^1} \eta(\epsilon^1) u(w(\epsilon^1) - T(\epsilon^1)) + \lambda \left(\sum_{\epsilon^1} \eta(\epsilon^1) T(\epsilon^1) - Rb \right) \\ &\quad + \sum_{\epsilon^1} \eta(\epsilon^1) \mu(\epsilon^1) (L(\epsilon^1) - T(\epsilon^1)). \end{aligned}$$

The first-order conditions derived in the insurance case continue to apply; they are augmented by the first-order condition with respect to b, namely $u'(w_0 + b) = \lambda \beta R$. Combined, the conditions yield the Euler equation

$$u'(w_0 + b) = \beta R (u'(w(\epsilon^1) - T(\epsilon^1)) + \mu(\epsilon^1)),$$

which states that marginal utility declines by more in histories where the incentive constraint binds ($\mu(\epsilon^1) > 0$). The intuition parallels the one from the insurance case.

The explicitly history-contingent payment, $T(\epsilon^1)$, can be interpreted as the payment on a bond with notionally risk-free return that is *renegotiated* ex post. To see this, focus on the set of histories in which the incentive constraint binds and disregard all other histories. According to the interpretation, the bond has *face value* $b\rho = \max[T(\epsilon^1)]$ in the second period where ρ denotes the contractual interest rate. When $L(\epsilon^1)$ is revealed the government threatens to default and the parties agree to lower the payment to the amount the country would forfeit in case of default, $T(\epsilon^1) = L(\epsilon^1)$. Note that the default costs serve as a threat point during the renegotiation but do not materialize in equilibrium.

13.4.3 Borrowing with Non-Contingent Debt

When both explicitly history-contingent payments and renegotiation are ruled out, for example because the investor cannot verify the realization of the default costs, then the government only has a choice between repaying the non-contingent face value in full or defaulting; in contrast to the case with renegotiation, default costs therefore generally materialize in equilibrium.

Let $b\rho$ denote the face value of debt at date $t = 1$. Since the government chooses to default when $b\rho \geq L(\epsilon^1)$ the contracting program reads

$$\begin{aligned}\mathcal{L} \;=\; & \beta^{-1}u(w_0+b) + \sum_{\epsilon^1\in\mathcal{E}^r}\eta(\epsilon^1)u(w(\epsilon^1)-b\rho) + \sum_{\epsilon^1\notin\mathcal{E}^r}\eta(\epsilon^1)u(w(\epsilon^1)-L(\epsilon^1)) \\ & +\lambda\left(\sum_{\epsilon^1\in\mathcal{E}^r}\eta(\epsilon^1)b\rho - Rb\right),\end{aligned}$$

where $\mathcal{E}^r$ denotes the (endogenous) set of repayment histories in which $L(\epsilon^1) \geq b\rho$. The first-order condition with respect to b implies a stochastic Euler equation,[5]

$$u'(w_0+b) = \beta\rho\sum_{\epsilon^1\in\mathcal{E}^r}\eta(\epsilon^1)u'(w(\epsilon^1)-b\rho),$$

which relates the marginal utility gain from debt issuance in the first period to marginal utility losses in histories in which the country services the debt. From the participation constraint, which holds with equality, $\rho = R/\sum_{\epsilon^1\in\mathcal{E}^r}\eta(\epsilon^1)$.

Since the government repays in full if it repays, the country receives no insurance across histories $\epsilon^1 \in \mathcal{E}^r$. In the other histories the country bears default costs in equilibrium, reflected in the $-L(\epsilon^1)$ terms in the sum $\sum_{\epsilon^1\notin\mathcal{E}^r}\eta(\epsilon^1)u(w(\epsilon^1)-L(\epsilon^1))$. These costs constitute a *social loss* as they are a cost for the borrower that does not correspond with a benefit for the investor. Without renegotiation, the contractual interest rate thus is higher and the default costs generate an *external finance premium* (see subsection 8.3.1).

13.4.4 Loan Size Determinants

To study the role of social losses and the determinants of equilibrium loan size in more detail we focus on the interaction between b, ρ, and L. We assume that default costs

[5] We assume that at the margin, changing b does not affect $\mathcal{E}^r$. More on this below.

rium condition

$$\chi_t(\epsilon^t) = -\frac{\kappa}{\omega}\pi_t(\epsilon^t).$$

Without commitment the government is unable to smooth welfare losses over time because it cannot control expectations of future policy. It may only smooth welfare losses within a period, by trading off output gaps and deviations of inflation from target. In contrast to the Ramsey policy, output and inflation deviate from their target values only as long as a cost push shock is present. That is, without commitment, the reversion of output and inflation to their long-run target values occurs faster than under the Ramsey policy—the discretionary policy exhibits a *stabilization bias*. Related, and also in contrast to the Ramsey policy, the price level does not revert to its starting value.

In both the program with and without commitment, inflation and the output gap eventually return to their long-run values of zero. This is a consequence of the assumption that at $\chi_t(\epsilon^t) = \pi_t(\epsilon^t) = 0$, the government does not have an incentive to drive inflation up or down. We turn next to a model where such an incentive is present.

13.6.2 Inflation Bias

Consider an infinite-horizon economy with no exogenous shocks. The government at date t chooses contemporaneous inflation, π_t, to minimize the reduced-form loss function

$$L_t = \sum_{j=0}^{\infty} \beta^j \ell_{t+j} \text{ with } \ell_{t+j} = \frac{\alpha}{2}\pi_{t+j}^2 - \gamma(\pi_{t+j} - \pi_{t+j}^e),$$

where $\beta \in (0,1)$ denotes the discount factor; α and γ are strictly positive parameters; and π_{t+j}^e denotes private sector inflation expectations. The loss components ℓ_{t+j} reflect the assumption that deviations of inflation from zero generate costs while unexpectedly high inflation rates generate benefits. The costs arise, for example, because inflation distorts relative prices (see section 10.2). The benefits may derive from the fact that an inflation surprise stimulates output (due to a Phillips curve relationship in the background, see subsection 10.3.5) or reduces the real value of public debt and thus, the need to levy distorting taxes (see subsection 11.4.1).

Absent commitment, the government at date t chooses π_t after the private sector has formed inflation expectations. The equilibrium inflation rate under discretion therefore solves $\partial L_t/\partial \pi_t = 0$ for given inflation expectations and it equals $\bar{\pi} \equiv \gamma/\alpha$. Anticipating this ex-post optimal policy choice, the private sector forms expectations $\pi_t^e = \bar{\pi}$ and the loss at date t (and in all future periods) equals $\ell_t = \gamma^2/(2\alpha)$.

The discretionary outcome is suboptimal because strictly positive inflation generates costs but no benefits when it is anticipated. If the government was able to commit it could control both inflation and inflation expectations and improve outcomes; specifically, the Ramsey policy amounts to $\pi_{t+j} = \pi_{t+j}^e = 0$ in all periods and generates losses $\ell_{t+j} = 0$. Absent commitment, however, inflation expectations are beyond the government's control and since $\gamma > 0$, the ex-post optimal choice is the discretionary one. As a consequence, the economy suffers from an *inflation bias*.

If the government announced a policy rule to implement the Ramsey policy, then this announcement would not be credible. For if the private sector believed the announcement and formed expectations accordingly, $\pi^e_{t+j} = 0$, then the ex-post optimal policy choice would still be given by $\bar{\pi}$ and would generate a negative loss, $\ell_t = -\gamma^2/(2\alpha)$. That is, after announcing to follow the rule the government would be tempted to renege on the announcement, in particular if the private sector believed the announcement.

Trigger Strategy

The situation changes and certain announced rules $\pi^\star$ become credible in spite of the government's lack of commitment when we introduce an additional state variable. Suppose that the government and the private sector play a *trigger strategy*, conditioning expectations on the history of equilibrium outcomes (see subsection 13.3.2). Specifically, the private sector expects the announced policy, $\pi^\star$, if expected and actual inflation in the previous period coincided; and the discretionary policy, $\bar{\pi}$, if this was not the case:

$$\pi^e_t = \begin{cases} \pi^\star & \text{if } \pi_{t-1} = \pi^e_{t-1} \\ \bar{\pi} & \text{if } \pi_{t-1} \neq \pi^e_{t-1} \end{cases} .$$

With these expectations (which are validated in equilibrium), a deviation of the government's choice from $\pi^\star$ triggers a one period punishment phase where the private sector expects the discretionary outcome and the government optimally responds accordingly. The expectation formation mechanism thus links the contemporaneous policy choice to the constraints (private sector expectations) the government faces in the subsequent period. As a consequence, some rules that dominate discretion become credible.

Conditional on the announced rule $\pi^\star$ the government's *temptation* to deviate from the rule and implement the discretionary outcome is given by

$$-\left(\frac{\alpha}{2}\bar{\pi}^2 - \gamma(\bar{\pi} - \pi^\star)\right) + \left(\frac{\alpha}{2}\pi^{\star 2} - \gamma(0)\right),$$

which equals the difference between $-\ell_t$ when deviating from the rule and when following it. Note that as long as $\pi^\star < \bar{\pi}$, the government is tempted to deviate—this is the source of the inflation bias discussed earlier. At the same time, the fact that a deviation affects expectation formation provides incentives not to deviate. The strength of this incentive—the *enforcement*—equals

$$\beta\left\{-\left(\frac{\alpha}{2}\pi^{\star 2} - \gamma(0)\right) + \left(\frac{\alpha}{2}\bar{\pi}^2 - \gamma(0)\right)\right\},$$

namely the discounted difference between $-\ell_t$ when following the rule (and this is expected) and implementing the discretionary policy (and this is expected).

An announced rule is credible if enforcement exceeds temptation that is, if

$$\beta\frac{\alpha}{2}\left(\bar{\pi}^2 - \pi^{\star 2}\right) \geq \frac{\alpha}{2}\left(\pi^{\star 2} - \bar{\pi}^2\right) + \gamma(\bar{\pi} - \pi^\star) = \frac{\alpha}{2}\left(\bar{\pi} - \pi^\star\right)^2.$$

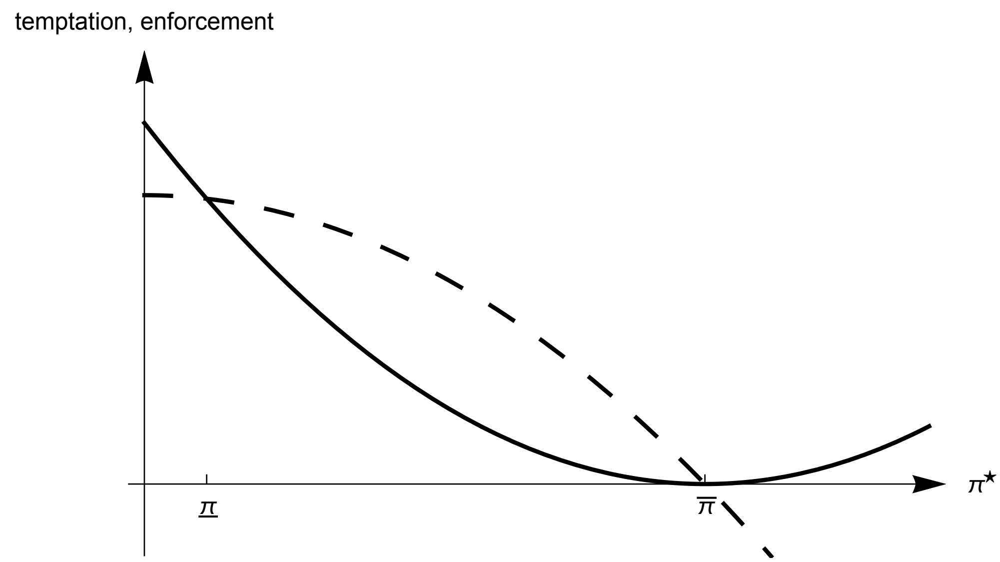

Figure 13.1: Time consistent monetary policy: Temptation and enforcement.

The best enforceable rule is given by the smallest $\pi^\star$ that satisfies this condition, namely

$$\underline{\pi} = \frac{\gamma(1-\beta)}{\alpha(1+\beta)}.$$

Note that for $\beta \to 0$, the best credible rule reduces to the discretionary outcome because enforcement equals zero in this case. For $\beta \to 1$, the best credible rule approaches the Ramsey policy.

Figure 13.1 plots temptation and enforcement, represented by the solid and dashed lines respectively, against the announced rule, $\pi^\star$. The two schedules intersect at the discretionary inflation rate, $\bar{\pi}$; and the lower inflation rate $\underline{\pi}$ (because the figure is plotted for $\beta > 0$) which represents the optimal rule.

Delegation

Irrevocable *delegation* of decision power introduces an alternative state variable that can help reduce the inflation bias. If it is possible to delegate monetary policy to a bureaucrat whose appointment cannot be overturned ex post, because of *central bank independence*, then the bureaucrat's preferences constitute a state variable under the control of the appointing government. When the government appoints a *conservative* that is, inflation averse *central bank governor* then future policy choices and thus, the private sector's inflation expectations reflect this aversion.

Suppose, for example, that preferences of the government and of society at large are represented by the parameters α and γ in the government's loss function while preferences of the central bank governor are represented by $\hat{\alpha}$ and $\hat{\gamma}$. When $\hat{\gamma}/\hat{\alpha} < \gamma/\alpha$ that is, when the governor attaches more weight to the cost of inflation than society then the non-revocable appointment reduces the inflation bias. In the extreme case where $\hat{\gamma} = 0$ the discretionary policy choice of the governor supports the Ramsey policy.

When all agents in society have identical preferences, delegation per se cannot solve the time inconsistency problem. But a non-revocable contract which makes the central bank governor's salary depend negatively on realized inflation effectively changes the governor's preferences and therefore achieves the same goal.

Reputation

Endogenous beliefs constitute yet another state variable that can affect government behavior. Suppose that the government may have two types, a committed one that always implements $\pi^{\star}$, and a non-committed or opportunistic one that always behaves ex-post optimally. The private sector does not observe the government's type but rationally infers it based on Bayes' rule from the observed inflation choices.

The government's *reputation* is the probability that the private sector assigns to the event that the government is committed. In equilibrium, high reputation leads the private sector to expect low inflation, and vice versa. When the remaining horizon is sufficiently long—not necessarily infinitely long—the opportunistic type therefore *mimics* the committed type and implements low inflation in order to build or maintain good (high) reputation and thus, support low inflation expectations. As the final period approaches, however, the opportunistic type eventually surprises the private sector with higher than expected inflation and destroys its reputation.

13.7 Bibliographic Notes

Kydland and Prescott (1977) emphasize that private sector decisions depend on expectations about ex-post optimal policy choices; they discuss examples of Ramsey policies that are not time consistent. Calvo (1978) analyzes time inconsistency in an environment where the government may use inflation to devalue money ex post.

Section 13.2 follows Lucas and Stokey (1983). Debortoli et al. (2018) show that access to a rich maturity structure may not suffice to render the Ramsey policy time consistent when this policy operates on the declining segment of the Laffer curve.

Fischer (1980) analyzes time consistent capital income taxation. Section 13.3 follows Chari and Kehoe (1990) who also relate sustainable plans to game theoretic equilibrium notions; see also Stokey's (1989; 1991) credible policies. Abreu, Pearce and Stacchetti (1986; 1990) use recursive methods to identify the worst and best sustainable equilibrium in infinite horizon economies, see Ljungqvist and Sargent (2018, 24) for a textbook treatment.

Eaton and Gersovitz (1981) study the sovereign debt model with incomplete markets. Subsection 13.4.4 follows Eaton and Fernandez (1995, 3.1). Calvo (1988) analyzes multiplicity of equilibrium in a model where the government chooses loan size rather than face value of maturing debt. Bulow and Rogoff (1989) and Grossman and

Han (1999) analyze the conditions under which financial autarky constitutes a credible threat.

Krusell and Ríos-Rull (1996) and Krusell et al. (1997) define dynamic politico-economic equilibrium. The analysis in section 13.5 follows Gonzalez-Eiras and Niepelt (2008).

Clarida et al. (1999) analyze equilibrium policy in the New Keynesian model. The model in subsection 13.6.2 is due to Barro and Gordon (1983). Rogoff (1985) and Walsh (1995) analyze delegation to a conservative central banker or a central banker that is incentivized. Backus and Driffill (1985) analyze the sequential equilibrium with reputation in the Barro and Gordon (1983) setup.

Related Topics and Additional References Chang (1998) and Phelan and Stacchetti (2001) extend the approach proposed by Abreu, Pearce and Stacchetti (1986; 1990) to economies with fundamental state variables, see Ljungqvist and Sargent (2018, 25) for a textbook treatment. For the theory of repeated games, see Fudenberg and Maskin (1986) and Abreu (1988); for sequential equilibrium, see Kreps and Wilson (1982).

Myers (1977) and Krugman (1989) analyze debt overhang when the debtor must be incentivized.

Gonzalez-Eiras and Niepelt (2015) derive conditions for *politico-economic equivalence* in environments where policy is endogenous and chosen sequentially; a set of political institutions and a state are equivalent to another such pair if both pairs give rise to the same allocation in politico-economic equilibrium.

Woodford (2003, 6) derives the government's loss function in the New Keynesian model in the presence of small steady-state distortions; Benigno and Woodford (2005) propose a solution method for models with large steady-state distortions. Dixit and Lambertini (2003) analyze games between fiscal and monetary authorities under commitment and discretion.

Ljungqvist and Sargent (2018, 21) cover models with two-sided lack of commitment. Mueller (1989) and Persson and Tabellini (2000) cover models of public choice and politico-economic equilibrium. Woodford (2003, 7), Galí (2008, 5), and Walsh (2017, 6,8) cover discretionary policy in the New Keynesian model.

Bibliography

Abreu, D. (1988). On the theory of infinitely repeated games with discounting, *Econometrica* **56**(2): 383–396. (Cited on p. 235.)

Abreu, D., Pearce, D. and Stacchetti, E. (1986). Optimal cartel equilibria with imperfect monitoring, *Journal of Economic Theory* **39**(1): 251–269. (Cited on pp. 234 and 235.)

Abreu, D., Pearce, D. and Stacchetti, E. (1990). Toward a theory of discounted repeated games with imperfect monitoring, *Econometrica* **58**(5): 1041–1063. (Cited on pp. 234 and 235.)

Acemoglu, D. (2009). *Introduction to Modern Economic Growth*, Princeton University Press, Princeton. (Cited on pp. 86, 240, and 258.)

Adam, K. and Weber, H. (2019). Optimal trend inflation, *American Economic Review* **109**(2): 702–737. (Cited on p. 209.)

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving, *Quarterly Journal of Economics* **109**(3): 659–684. (Cited on pp. 53 and 86.)

Aiyagari, S. R. (1995). Optimal capital income taxation with incomplete markets, borrowing constraints, and constant discounting, *Journal of Political Economy* **103**(6): 1158–1175. (Cited on p. 208.)

Aiyagari, S. R. and Gertler, M. (1985). The backing of government bonds and Monetarism, *Journal of Monetary Economics* **16**(1): 19–44. (Cited on p. 185.)

Aiyagari, S. R., Marcet, A., Sargent, T. J. and Seppälä, J. (2002). Optimal taxation without state-contingent debt, *Journal of Political Economy* **110**(6): 1220–1254. (Cited on p. 208.)

Aiyagari, S. R. and McGrattan, E. R. (1998). The optimal quantity of debt, *Journal of Monetary Economics* **42**(3): 447–469. (Cited on p. 185.)

Akerlof, G. A. (1970). The market for lemons: Qualitative uncertainty and the market mechanism, *Quarterly Journal of Economics* **84**(3): 488–500. (Cited on p. 125.)

Akerlof, G. A. and Yellen, J. L. (1985). A near-rational model of the business cycle, with wage and price inertia, *Quarterly Journal of Economics* **100**(Supplement): 823–838. (Cited on p. 155.)

Allais, M. (1947). *Economie et Interet*, Imprimerie Nationale, Paris. (Cited on p. 37.)

Alvarez, F., Atkeson, A. and Kehoe, P. J. (2002). Money, interest rates, and exchange rates with endogenously segmented markets, *Journal of Political Economy* **110**(1): 73–112. (Cited on p. 185.)

Alvarez, F. and Jermann, U. J. (2000). Efficiency, equilibrium, and asset pricing with risk of default, *Econometrica* **68**(4): 775–797. (Cited on p. 125.)

Alvarez, F. and Jermann, U. J. (2004). Using asset prices to measure the cost of business cycles, *Journal of Political Economy* **112**(6): 1223–1256. (Cited on p. 86.)

Angeletos, G.-M. (2002). Fiscal policy with noncontingent debt and the optimal maturity structure, *Quarterly Journal of Economics* **117**(3): 1105–1131. (Cited on p. 208.)

Angeletos, G.-M. (2007). Uninsured idiosyncratic investment risk and aggregate saving, *Review of Economic Dynamics* **10**(1): 1–30. (Cited on p. 53.)

Angeletos, G.-M. and Lian, C. (2016). Incomplete information in macroeconomics: Accommodating frictions in coordination, *in* J. B. Taylor and H. Uhlig (eds), *Handbook of Macroeconomics*, Vol. 2A, North Holland, Amsterdam, chapter 14, pp. 1065–1240. (Cited on p. xiv.)

Aristotle (350 B.C.E.). *Politics, Book I*, Aristotle, Athens. (Cited on p. 142.)

Arrow, K. J. (1953). Le rôle des valeurs boursières pour la répartition la meilleure des risques, *Économétrie, Colloques Internationaux du Centre National de la Recherche Scientifique* **11**: 41–47. (Cited on pp. 8 and 53.)

Arrow, K. J. (1964). The role of securities in the optimal allocation of risk-bearing, *Review of Economic Studies* **31**(2): 91–96. (Cited on pp. 8 and 53.)

Arrow, K. J. and Debreu, G. (1954). Existence of an equilibrium for a competitive economy, *Econometrica* **22**(3): 265–290. (Cited on p. 8.)

Atkeson, A., Chari, V. V. and Kehoe, P. J. (2010). Sophisticated monetary policies, *Quarterly Journal of Economics* **125**(1): 47–89. (Cited on p. 185.)

Atkeson, A. and Lucas, R. E. (1992). On efficient distribution with private information, *Review of Economic Studies* **59**: 427–453. (Cited on p. 208.)

Atkinson, A. B. and Sandmo, A. (1980). Welfare implications of the taxation of savings, *Economic Journal* **90**(359): 529–549. (Cited on p. 208.)

Atkinson, A. B. and Stiglitz, J. E. (1972). The structure of indirect taxation and economic efficiency, *Journal of Public Economics* **1**(1): 97–119. (Cited on p. 258.)

Atkinson, A. B. and Stiglitz, J. E. (1976). The design of tax structure: Direct versus indirect taxation, *Journal of Public Economics* **6**(1–2): 55–75. (Cited on p. 258.)

Atkinson, A. B. and Stiglitz, J. E. (1980). *Lectures on Public Economics*, McGraw-Hill, London. (Cited on p. 258.)

Auclert, A. (2017). Monetary policy and the redistribution channel, *Working Paper 23451*, NBER, Cambridge, Massachusetts. (Cited on p. 155.)

Auerbach, A. J., Gokhale, J. and Kotlikoff, L. J. (1994). Generational accounting: A meaningful way to evaluate fiscal policy, *Journal of Economic Perspectives* **8**(1): 73–94. (Cited on p. 184.)

Azariadis, C. (1981). Self-fulfilling prophecies, *Journal of Economic Theory* **25**(3): 380–396. (Cited on p. 86.)

Backus, D. and Driffill, J. (1985). Inflation and reputation, *American Economic Review* **75**(3): 530–538. (Cited on p. 235.)

Backus, D. K. and Smith, G. W. (1993). Consumption and real exchange rates in dynamic economies with non-traded goods, *Journal of International Economics* **35**(3–4): 297–316. (Cited on p. 96.)

Bailey, M. J. (1956). The welfare cost of inflationary finance, *Journal of Political Economy* **64**(2): 93–110. (Cited on p. 208.)

Balasko, Y. and Shell, K. (1980). The overlapping-generations model, I: The case of pure exchange without money, *Journal of Economic Theory* **23**(3): 281–306. (Cited on p. 37.)

Balassa, B. (1964). The purchasing-power parity doctrine: A reappraisal, *Journal of Political Economy* **72**(6): 584–596. (Cited on p. 96.)

Baldwin, C. Y. and Meyer, R. F. (1979). Liquidity preference under uncertainty: A model of dynamic investment in illiquid opportunities, *Journal of Financial Economics* **7**(4): 347–374. (Cited on p. 124.)

Ball, L. and Mankiw, N. G. (2007). Intergenerational risk sharing in the spirit of Arrow, Debreu, and Rawls, with applications to social security design, *Journal of Political Economy* **115**(4): 523–547. (Cited on p. 184.)

Bansal, R. and Yaron, A. (2004). Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* **59**(4): 1481–1509. (Cited on p. 258.)

Baqaee, D. R. and Farhi, E. (2018). Unbundling macroeconomics via heterogeneous agents and input-output networks, *Working Paper 24684*, NBER, Cambridge, Massachusetts. (Cited on p. 258.)

Barbie, M., Hagedorn, M. and Kaul, A. (2007). On the interaction between risk sharing and capital accumulation in a stochastic OLG model with production, *Journal of Economic Theory* **137**(1): 568–579. (Cited on p. 38.)

Barro, R. J. (1974). Are government bonds net wealth?, *Journal of Political Economy* **82**(6): 1095–1117. (Cited on p. 184.)

Barro, R. J. (1979). On the determination of the public debt, *Journal of Political Economy* **87**(5): 940–971. (Cited on p. 208.)

Barro, R. J. (1990). Government spending in a simple model of endogenous growth, *Journal of Political Economy* **98**(5): S103–S125. (Cited on p. 184.)

Barro, R. J. and Gordon, D. B. (1983). Rules, discretion, and reputation in a model of monetary policy, *Journal of Monetary Economics* **12**: 101–121. (Cited on p. 235.)

Barro, R. J. and Grossman, H. I. (1971). A general disequilibrium model of income and employment, *American Economic Review* **61**(1): 82–93. (Cited on p. 8.)

Bassetto, M. (2002). A game-theoretic view of the fiscal theory of the price level, *Econometrica* **70**(6): 2167–2196. (Cited on p. 185.)

Bassetto, M. and Kocherlakota, N. (2004). On the irrelevance of government debt when taxes are distortionary, *Journal of Monetary Economics* **51**(2): 299–304. (Cited on p. 185.)

Baumol, W. J. (1952). The transactions demand for cash, *Quarterly Journal of Economics* **67**(4): 545–556. (Cited on p. 142.)

Baumol, W. J. and Bowen, W. G. (1966). *Performing Arts—The Economic Dilemma: A Study of Problems common to Theater, Opera, Music and Dance*, MIT Press, Cambridge, Massachusetts. (Cited on p. 96.)

Baxter, M. and King, R. G. (1993). Fiscal policy in general equilibrium, *American Economic Review* **83**: 315–334. (Cited on p. 184.)

Becker, G. S. (1965). A theory of the allocation of time, *Economic Journal* **75**(299): 493–517. (Cited on p. 85.)

Benhabib, J. and Farmer, R. E. A. (1994). Indeterminacy and increasing returns, *Journal of Economic Theory* **63**(1): 19–41. (Cited on p. 86.)

Benhabib, J. and Farmer, R. E. A. (1999). Indeterminacy and sunspots in macroeconomics, *in* J. B. Taylor and M. Woodford (eds), *Handbook of Macroeconomics*, Vol. 1A, North-Holland, Amsterdam, chapter 6, pp. 387–448. (Cited on p. 87.)

Benhabib, J., Schmitt-Grohé, S. and Uribe, M. (2002). Avoiding liquidity traps, *Journal of Political Economy* **110**(3): 535–563. (Cited on p. 185.)

Benigno, P. and Woodford, M. (2005). Inflation stabilization and welfare: The case of a distorted steady state, *Journal of the European Economic Association* **3**(6): 1185–1236. (Cited on p. 235.)

Bernanke, B. S. and Gertler, M. (1989). Agency costs, net worth, and business fluctuations, *American Economic Review* **79**(1): 14–31. (Cited on p. 125.)

Bernanke, B. S., Gertler, M. and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework, *in* J. B. Taylor and M. Woodford (eds), *Handbook of Macroeconomics*, Vol. 1C, North-Holland, Amsterdam, chapter 21, pp. 1341–1393. (Cited on p. 125.)

Bewley, T. F. (1972). Existence of equilibria in economies with infinitely many commodities, *Journal of Economic Theory* **4**(3): 514–540. (Cited on p. 8.)

Bewley, T. F. (1977). The permanent income hypothesis: A theoretical formulation, *Journal of Economic Theory* **16**(2): 252–292. (Cited on p. 53.)

Bewley, T. F. (1980). The optimum quantity of money, *in* J. H. Kareken and N. Wallace (eds), *Models of Monetary Economies*, Federal Reserve Bank of Minneapolis, Minneapolis, pp. 169–210. (Cited on pp. 53, 63, and 142.)

Bewley, T. F. (1986). Stationary monetary equilibrium with a continuum of independently fluctuating consumers, *in* W. Hildenbrand and A. Mas-Colell (eds), *Contributions to Mathematical Economics in Honor of Gerard Debreu*, North Holland, Amsterdam, pp. 79–102. (Cited on p. 53.)

Bhandari, A., Evans, D., Golosov, M. and Sargent, T. J. (2017). Fiscal policy and debt management with incomplete markets, *Quarterly Journal of Economics* **132**(2): 617–663. (Cited on p. 208.)

Blanchard, O. J. (1985). Debt, deficits, and finite horizons, *Journal of Political Economy* **93**(2): 223–247. (Cited on p. 38.)

Blanchard, O. J. and Kahn, C. M. (1980). The solution of linear difference models under rational expectations, *Econometrica* **48**(5): 1305–1311. (Cited on p. 258.)

Blanchard, O. J. and Kiyotaki, N. (1987). Monopolistic competition and the effects of aggregate demand, *American Economic Review* **77**(4): 647–666. (Cited on p. 155.)

Bohn, H. (1990). Tax smoothing with financial instruments, *American Economic Review* **80**(5): 1217–1230. (Cited on p. 208.)

Borch, K. (1962). Equilibrium in a reinsurance market, *Econometrica* **30**(3): 424–444. (Cited on p. 53.)

Breeden, D. T. (1979). An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* **7**(3): 265–296. (Cited on p. 63.)

Breyer, F. (1989). On the intergenerational Pareto efficiency of pay-as-you-go financed pension systems, *Journal of Institutional and Theoretical Economics* **145**(4): 643–658. (Cited on p. 184.)

Brock, W. A. (1974). Money and growth: The case of long run perfect foresight, *International Economic Review* **15**(3): 750–777. (Cited on p. 185.)

Brock, W. A. and Mirman, L. J. (1972). Optimal economic growth and uncertainty: The discounted case, *Journal of Economic Theory* **4**(3): 479–513. (Cited on p. 86.)

Broer, T., Harbo Hansen, N.-J., Krusell, P. and Öberg, E. (2019). The New Keynesian transmission mechanism: A heterogeneous-agent perspective, *Review of Economic Studies* . (Cited on p. 155.)

Brunnermeier, M. K., Eisenbach, T. M. and Sannikov, Y. (2012). Macroeconomics with financial frictions: A survey, *Working Paper 18102*, NBER, Cambridge, Massachusetts. (Cited on p. 126.)

Brunnermeier, M. K. and Niepelt, D. (2019). On the equivalence of private and public money, *Mimeo* . (Cited on p. 142.)

Brunnermeier, M. K. and Oehmke, M. (2013). Bubbles, financial crises, and systemic risk, *in* G. M. Constantinides, M. Harris and R. M. Stulz (eds), *Handbook of the Economics of Finance*, Vol. 2B, North Holland, Amsterdam, chapter 18, pp. 1221–1288. (Cited on p. 63.)

Brunnermeier, M. K. and Sannikov, Y. (2014). A macroeconomic model with a financial sector, *American Economic Review* **104**(2): 379–421. (Cited on p. 126.)

Brunnermeier, M. K. and Sannikov, Y. (2016). The I theory of money, *Working Paper 22533*, NBER, Cambridge, Massachusetts. (Cited on p. 142.)

Bryant, J. (1983). Government irrelevance results: A simple exposition, *American Economic Review* **73**(4): 758–761. (Cited on p. 185.)

Buiter, W. H. (1981). Time preference and international lending and borrowing in an overlapping-generations model, *Journal of Political Economy* **89**(4): 769–797. (Cited on p. 96.)

Buiter, W. H. (2002). The fiscal theory of the price level: A critique, *Economic Journal* **112**(481): 459–480. (Cited on p. 185.)

Bullard, J. and Mitra, K. (2002). Learning about monetary policy rules, *Journal of Monetary Economics* **49**(6): 1105–1129. (Cited on p. 185.)

Bulow, J. and Rogoff, K. (1989). Sovereign debt: Is to forgive to forget?, *American Economic Review* **79**(1): 43–50. (Cited on p. 234.)

Caballero, R. J. and Krishnamurthy, A. (2003). Excessive dollar debt: Financial development and underinsurance, *Journal of Finance* **58**(2): 867–893. (Cited on p. 126.)

Cagan, P. (1956). The monetary dynamics of hyperinflation, *in* M. Friedman (ed.), *Studies in the Quantity Theory of Money*, University of Chicago Press, Chicago, pp. 25–117. (Cited on p. 185.)

Calvo, G. A. (1978). On the time consistency of optimal policy in a monetary economy, *Econometrica* **46**: 1411–1428. (Cited on p. 234.)

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics* **12**(3): 383–398. (Cited on p. 155.)

Calvo, G. A. (1988). Servicing the public debt: The role of expectations, *American Economic Review* **78**: 647–661. (Cited on p. 234.)

Campbell, J. Y. (1986). Bond and stock returns in a simple exchange model, *Quarterly Journal of Economics* **101**(4): 785–804. (Cited on p. 63.)

Cao, D. and Werning, I. (2018). Saving and dissaving with hyperbolic discounting, *Econometrica* **86**(3): 805–857. (Cited on p. 24.)

Carlstrom, C. T. and Fuerst, T. S. (1997). Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis, *American Economic Review* **87**(5): 893–910. (Cited on p. 125.)

Carroll, C. D. (1997). Buffer-stock saving and the life cycle/permanent income hypothesis, *Quarterly Journal of Economics* **112**(1): 1–55. (Cited on p. 53.)

Caselli, F. and Ventura, J. (2000). A representative consumer theory of distribution, *American Economic Review* **90**(4): 909–926. (Cited on p. 258.)

Cass, D. (1965). Optimum growth in an aggregative model of capital accumulation, *Review of Economic Studies* **32**: 233–240. (Cited on p. 37.)

Cass, D. (1972). On capital overaccumulation in the aggregative, neoclassical model of economic growth: A complete characterization, *Journal of Economic Theory* **4**: 200–223. (Cited on p. 37.)

Cass, D. and Shell, K. (1983). Do sunspots matter?, *Journal of Political Economy* **91**(2): 193–227. (Cited on p. 53.)

Cassel, G. (1918). Abnormal deviations in international exchanges, *Economic Journal* **28**(112): 413–415. (Cited on p. 142.)

Chamberlain, G. and Wilson, C. A. (2000). Optimal intertemporal consumption under uncertainty, *Review of Economic Dynamics* **3**(3): 365–395. (Cited on p. 53.)

Chamley, C. (1986). Optimal taxation of capital income in general equilibrium with infinite lives, *Econometrica* **54**(3): 607–622. (Cited on p. 208.)

Chamley, C. and Polemarchakis, H. (1984). Assets, general equilibrium and the neutrality of money, *Review of Economic Studies* **51**(1): 129–138. (Cited on p. 185.)

Chang, R. (1998). Credible monetary policy in an infinite horizon model: Recursive approaches, *Journal of Economic Theory* **81**(2): 431–461. (Cited on p. 235.)

Chari, V. V., Christiano, L. J. and Kehoe, P. J. (1994). Optimal fiscal policy in a business cycle model, *Journal of Political Economy* **102**(4): 617–652. (Cited on p. 208.)

Chari, V. V., Christiano, L. J. and Kehoe, P. J. (1996). Optimality of the Friedman rule in economies with distorting taxes, *Journal of Monetary Economics* **37**(2): 203–223. (Cited on p. 208.)

Chari, V. V. and Kehoe, P. J. (1990). Sustainable plans, *Journal of Political Economy* **98**(4): 783–802. (Cited on p. 234.)

Chari, V. V. and Kehoe, P. J. (1999). Optimal fiscal and monetary policy, *in* J. B. Taylor and M. Woodford (eds), *Handbook of Macroeconomics*, Vol. 1C, North-Holland, Amsterdam, chapter 26, pp. 1671–1745. (Cited on p. 209.)

Chari, V. V., Kehoe, P. J. and McGrattan, E. R. (2000). Sticky price models of the business cycle: Can the contract multiplier solve the persistence problem?, *Econometrica* **68**(5): 1151–1180. (Cited on p. 155.)

Chattopadhyay, S. and Gottardi, P. (1999). Stochastic OLG models, market structure, and optimality, *Journal of Economic Theory* **89**(1): 21–67. (Cited on p. 38.)

Clarida, R., Galí, J. and Gertler, M. (1999). The science of monetary policy: A New Keynesian perspective, *Journal of Economic Literature* **37**(4): 1661–1707. (Cited on pp. 208 and 235.)

Clower, R. W. (1967). A reconsideration of the microfoundations of monetary theory, *Western Economic Journal* **6**(1): 1–8. (Cited on p. 142.)

Cobb, C. W. and Douglas, P. H. (1928). A theory of production, *American Economic Review, Papers and Proceedings* **18**(1): 139–165. (Cited on p. 8.)

Cochrane, J. H. (2001). *Asset Pricing*, Princeton University Press, Princeton. (Cited on p. 63.)

Constantinides, G. M. and Duffie, D. (1996). Asset pricing with heterogeneous consumers, *Journal of Political Economy* **104**(2): 219–240. (Cited on p. 85.)

Cooley, T. F. (ed.) (1995). *Frontiers of Business Cycle Research*, Princeton University Press, Princeton. (Cited on p. 86.)

Cooper, R. and John, A. (1988). Coordinating coordination failures in Keynesian models, *Quarterly Journal of Economics* **103**(3): 441–463. (Cited on p. 155.)

Correia, I., Farhi, E., Nicolini, J. P. and Teles, P. (2013). Unconventional fiscal policy at the zero bound, *American Economic Review* **103**(4): 1172–1211. (Cited on p. 209.)

Correia, I., Nicolini, J. P. and Teles, P. (2008). Optimal fiscal and monetary policy: Equivalence results, *Journal of Political Economy* **116**(1): 141–170. (Cited on p. 209.)

Correia, I. and Teles, P. (1999). The optimal inflation tax, *Review of Economic Dynamics* **2**(2): 325–346. (Cited on p. 208.)

Cox, J. C., Ingersoll, J. E. and Ross, S. A. (1985). A theory of the term structure of interest rates, *Econometrica* **53**(2): 385–407. (Cited on p. 63.)

Croushore, D. (1993). Money in the utility function: Functional equivalence to a shoppingtime model, *Journal of Macroeconomics* **15**(1): 175–182. (Cited on p. 142.)

Cvitanić, J. and Zapatero, F. (2004). *Introduction to the Economics and Mathematics of Financial Markets*, MIT Press, Cambridge, Massachusetts. (Cited on p. 63.)

Dávila, E. and Korinek, A. (2018). Pecuniary externalities in economies with financial frictions, *Review of Economic Studies* **85**(1): 352–395. (Cited on p. 125.)

Dávila, J., Hong, J. H., Krusell, P. and Ríos-Rull, J.-V. (2012). Constrained efficiency in the neoclassical growth model with uninsurable idiosyncratic shocks, *Econometrica* **80**(6): 2431–2467. (Cited on p. 126.)

Deaton, A. (1991). Saving and liquidity constraints, *Econometrica* **59**(5): 1221–1248. (Cited on p. 53.)

Deaton, A. (1992). *Understanding Consumption*, Oxford University Press, Oxford. (Cited on pp. 24 and 53.)

Deaton, A. and Muellbauer, J. (1980). *Economics and Consumer Behavior*, Cambridge University Press, Cambridge. (Cited on p. 258.)

Debortoli, D., Nunes, R. and Yared, P. (2018). Optimal taxation and debt management without commitment, *Working Paper 24522*, NBER, Cambridge, Massachusetts. (Cited on p. 234.)

Debreu, G. (1959). *Theory of Value*, John Wiley, New York. (Cited on p. 8.)

Del Negro, M. and Sims, C. A. (2015). When does a central bank's balance sheet require fiscal support?, *Journal of Monetary Economics* **73**(C): 1–19. (Cited on p. 185.)

Diamond, P. A. (1965). National debt in a neoclassical growth model, *American Economic Review* **55**(5): 1126–1150. (Cited on pp. 37 and 184.)

Diamond, P. A. (1975). A many-person Ramsey tax rule, *Journal of Public Economics* **4**(4): 335–342. (Cited on p. 258.)

Diamond, P. A. (1982). Aggregate demand management in search equilibrium, *Journal of Political Economy* **90**: 881–894. (Cited on p. 124.)

Diamond, P. A. and Mirrlees, J. A. (1971a). Optimal taxation and public production I: Production efficiency, *American Economic Review* **61**(1): 8–27. (Cited on p. 258.)

Diamond, P. A. and Mirrlees, J. A. (1971b). Optimal taxation and public production II: Tax rules, *American Economic Review* **61**(3): 261–278. (Cited on p. 258.)

Diamond, P. A. and Mirrlees, J. A. (1978). A model of social insurance with variable retirement, *Journal of Public Economics* **10**(3): 295–336. (Cited on p. 208.)

Dixit, A. K. (1989). Entry and exit decisions under uncertainty, *Journal of Political Economy* **97**(3): 620–638. (Cited on p. 124.)

Dixit, A. K. and Lambertini, L. (2003). Interactions of commitment and discretion in monetary and fiscal policies, *American Economic Review* **93**(5): 1522–1542. (Cited on p. 235.)

Dixit, A. K. and Pindyck, R. S. (1994). *Investment under Uncertainty*, Princeton University Press, Princeton. (Cited on p. 125.)

Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity, *American Economic Review* **67**(3): 297–308. (Cited on pp. 24 and 155.)

Domeij, D. and Ellingsen, T. (2018). Rational bubbles and public debt policy: A quantitative analysis, *Journal of Monetary Economics* **96**(C): 109–123. (Cited on p. 185.)

Dornbusch, R. (1976). Expectations and exchange rate dynamics, *Journal of Political Economy* **84**(6): 1161–1176. (Cited on p. 142.)

Dornbusch, R., Fischer, S. and Samuelson, P. A. (1977). Comparative advantage, trade, and payments in a Ricardian model with a continuum of goods, *American Economic Review* **67**(5): 823–839. (Cited on p. 96.)

Dotsey, M., King, R. G. and Wolman, A. L. (1999). State-dependent pricing and the general equilibrium dynamics of money and output, *Quarterly Journal of Economics* **114**(2): 655–690. (Cited on p. 155.)

Duffie, D. (2001). *Dynamic Asset Pricing Theory*, 3rd edn, Princeton University Press, Princeton. (Cited on p. 63.)

Eaton, J. and Fernandez, R. (1995). Sovereign debt, *in* G. M. Grossman and K. Rogoff (eds), *Handbook of International Economics*, Vol. 3, North-Holland, Amsterdam, chapter 39, pp. 2031–2077. (Cited on p. 234.)

Eaton, J. and Gersovitz, M. (1981). Debt with potential repudiation: Theoretical and empirical analysis, *Review of Economic Studies* **48**(2): 289–309. (Cited on p. 234.)

Eatwell, J., Milgate, M. and Newman, P. (eds) (1989). *The New Palgrave: General Equilibrium*, Norton, New York. (Cited on p. 9.)

Epstein, L. G. and Zin, S. E. (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* **57**(4): 937–969. (Cited on p. 258.)

Erceg, C. J., Henderson, D. W. and Levin, A. T. (2000). Optimal monetary policy with staggered wage and price contracts, *Journal of Monetary Economics* **46**(2): 281–313. (Cited on p. 155.)

Evans, G. W. and Honkapohja, S. (1999). Learning dynamics, *in* J. B. Taylor and M. Woodford (eds), *Handbook of Macroeconomics*, Vol. 1A, North-Holland, Amsterdam, chapter 7, pp. 449–452. (Cited on p. xiv.)

Evans, G. W. and Honkapohja, S. (2001). *Learning and Expectations in Macroeconomics*, Princeton University Press, Princeton. (Cited on p. xiv.)

Farhi, E. (2010). Capital taxation and ownership when markets are incomplete, *Journal of Political Economy* **118**(5): 908–948. (Cited on p. 208.)

Farhi, E. and Werning, I. (2007). Inequality and social discounting, *Journal of Political Economy* **115**(3): 365–402. (Cited on p. 209.)

Farhi, E. and Werning, I. (2016a). Fiscal multipliers: Liquidity traps and currency unions, *in* J. B. Taylor and H. Uhlig (eds), *Handbook of Macroeconomics*, Vol. 2B, North Holland, Amsterdam, chapter 31, pp. 2417–2492. (Cited on p. 185.)

Farhi, E. and Werning, I. (2016b). A theory of macroprudential policies in the presence of nominal rigidities, *Econometrica* **84**(5): 1645–1704. (Cited on p. 155.)

Feenstra, R. C. (1986). Functional equivalence between liquidity costs and the utility of money, *Journal of Monetary Economics* **17**(2): 271–291. (Cited on p. 142.)

Fischer, S. (1977). Long-term contracts, rational expectations, and the optimal money supply rule, *Journal of Political Economy* **85**(1): 191–205. (Cited on p. 155.)

Fischer, S. (1980). Dynamic inconsistency, cooperation, and the benevolent dissembling government, *Journal of Economic Dynamics and Control* **2**: 93–107. (Cited on p. 234.)

Fisher, I. (1896). Appreciation and interest, *Publications of the American Economic Association* **11**(4): 1–98. (Cited on p. 142.)

Fisher, I. (1930). *The Theory of Interest, as determined by Impatience to Spend Income and Opportunity to Invest it*, Macmillan, New York. (Cited on pp. 37, 125, and 142.)

Fried, J. (1980). The intergenerational distribution of the gains from technical change and from international trade, *The Canadian Journal of Economics* **13**(1): 65–81. (Cited on p. 96.)

Friedman, M. (1957). *A Theory of the Consumption Function*, Princeton University Press, Princeton. (Cited on pp. 24 and 52.)

Friedman, M. (1968). The role of monetary policy, *American Economic Review* **58**(1): 1–17. (Cited on pp. 8 and 185.)

Friedman, M. (1969). The optimum quantity of money, *in* M. Friedman (ed.), *The Optimum Quantity of Money and Other Essays*, Aldine, Chicago, chapter 1, pp. 1–50. (Cited on p. 208.)

Fudenberg, D. and Maskin, E. (1986). The folk theorem in repeated games with discounting or with incomplete information, *Econometrica* **54**(3): 533–554. (Cited on p. 235.)

Gale, D. (1973). Pure exchange equilibrium of dynamic economic models, *Journal of Economic Theory* **6**(1): 12–36. (Cited on p. 38.)

Gale, D. (1990). The efficient design of public debt, *in* R. Dornbusch and M. Draghi (eds), *Public Debt Management: Theory and History*, Cambridge University Press, Cambridge, England, chapter 2, pp. 14–47. (Cited on p. 208.)

Galí, J. (2008). *Monetary Policy, Inflation, and the Business Cycle*, Princeton University Press, Princeton. (Cited on pp. 155, 185, 209, and 235.)

Geanakoplos, J. D. and Polemarchakis, H. M. (1986). Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete, *in* W. P. Heller, R. M. Starr and D. A. Starrett (eds), *Uncertainty, Information, and Communication: Essays in Honor of Kenneth J. Arrow*, Vol. 3, Cambridge University Press, Cambridge, England, chapter 3, pp. 65–95. (Cited on p. 125.)

Gertler, M. and Leahy, J. (2008). A Phillips curve with an Ss foundation, *Journal of Political Economy* **116**(3): 533–572. (Cited on p. 155.)

Gollier, C. (2001). *The Economics of Risk and Time*, MIT Press, Cambridge, Massachusetts. (Cited on p. 53.)

Golosov, M., Kocherlakota, N. and Tsyvinski, A. (2003). Optimal indirect and capital taxation, *Review of Economic Studies* **70**: 569–587. (Cited on p. 208.)

Golosov, M. and Lucas, R. E. (2007). Menu costs and Phillips curves, *Journal of Political Economy* **115**(2): 171–199. (Cited on p. 155.)

Golosov, M., Tsyvinski, A. and Werquin, N. (2016). Recursive contracts and endogenously incomplete markets, *in* J. B. Taylor and H. Uhlig (eds), *Handbook of Macroeconomics*, Vol. 2A, North Holland, Amsterdam, chapter 10, pp. 725–841. (Cited on pp. 126 and 209.)

Gonzalez-Eiras, M. and Niepelt, D. (2008). The future of social security, *Journal of Monetary Economics* **55**(2): 197–218. (Cited on p. 235.)

Gonzalez-Eiras, M. and Niepelt, D. (2015). Politico-economic equivalence, *Review of Economic Dynamics* **18**(4): 843–862. (Cited on pp. 184 and 235.)

Goodfriend, M. and King, R. G. (1997). The new neoclassical synthesis and the role of monetary policy, *in* B. S. Bernanke and J. J. Rotemberg (eds), *NBER Macroeconomics Annual 1997*, Vol. 12, MIT Press, Cambridge, Massachusetts, pp. 231–283. (Cited on p. 155.)

Gorman, W. M. (1953). Community preference fields, *Econometrica* **21**(1): 63–80. (Cited on p. 258.)

Grandmont, J.-M. (1977). Temporary general equilibrium theory, *Econometrica* **45**(3): 535–572. (Cited on p. 8.)

Grandmont, J.-M. (1985). On endogenous competitive business cycles, *Econometrica* **53**(5): 995–1045. (Cited on p. 38.)

Grandmont, J.-M. and Younes, Y. (1972). On the role of money and the existence of a monetary equilibrium, *Review of Economic Studies* **39**(3): 355–372. (Cited on p. 142.)

Greenwald, B. C. and Stiglitz, J. E. (1986). Externalities in economies with imperfect information and incomplete markets, *Quarterly Journal of Economics* **101**(2): 229–264. (Cited on p. 125.)

Grossman, H. I. and Han, T. (1999). Sovereign debt and consumption smoothing, *Journal of Monetary Economics* **44**(1): 149–158. (Cited on p. 234.)

Grossman, S. and Weiss, L. (1983). A transactions-based model of the monetary transmission mechanism, *American Economic Review* **73**(5): 871–880. (Cited on p. 185.)

Hahn, F. H. (1971). Equilibrium with transaction costs, *Econometrica* **39**(3): 417–439. (Cited on p. 8.)

Hall, R. E. (1978). Stochastic implications of the life cycle-permanent income hypothesis: Theory and evidence, *Journal of Political Economy* **86**(6): 971–987. (Cited on p. 53.)

Hall, R. E. (2005). Employment fluctuations with equilibrium wage stickiness, *American Economic Review* **95**(1): 50–65. (Cited on p. 125.)

Hall, R. E. and Reis, R. (2015). Maintaining central-bank financial stability under new-style central banking, *Working Paper 21173*, NBER, Cambridge, Massachusetts. (Cited on p. 185.)

Hansen, G. D. (1985). Indivisible labor and the business cycle, *Journal of Monetary Economics* **16**(3): 309–327. (Cited on p. 85.)

Harrod, R. F. (1933). *International Economics*, Cambridge University Press, Cambridge. (Cited on p. 96.)

Hart, O. D. (1975). On the optimality of equilibrium when the market structure is incomplete, *Journal of Economic Theory* **11**: 418–443. (Cited on p. 125.)

Hart, O. and Moore, J. (1994). A theory of debt based on the inalienability of human capital, *Quarterly Journal of Economics* **109**(4): 841–879. (Cited on p. 125.)

Hart, O. and Moore, J. (1998). Default and renegotiation: A dynamic model of debt, *Quarterly Journal of Economics* **113**(1): 1–41. (Cited on p. 125.)

Hayashi, F. (1982). Tobin's marginal q and average q: A neoclassical interpretation, *Econometrica* **50**(1): 213–224. (Cited on p. 124.)

Heathcote, J., Storesletten, K. and Violante, G. L. (2014). Consumption and labor supply with partial insurance: An analytical framework, *American Economic Review* **104**(7): 2075–2126. (Cited on p. 85.)

Heckman, J. J. (1974). Life cycle consumption and labor supply: An explanation of the relationship between income and consumption over the life cycle, *American Economic Review* **64**(1): 188–194. (Cited on p. 85.)

Hicks, J. R. (1939). *Value and Capital: An Inquiry into Some Fundamental Principles of Economic Theory*, Clarendon Press, Oxford. (Cited on p. 8.)

Holmström, B. (1983). Equilibrium long-term labor contracts, *Quarterly Journal of Economics* **98**(Supplement): 23–54. (Cited on p. 125.)

Holmström, B. and Tirole, J. (1998). Private and public supply of liquidity, *Journal of Political Economy* **106**(1): 1–40. (Cited on p. 185.)

Homer (800 B.C.E.). *The Odyssey*, Homer. (Cited on p. 24.)

Hosios, A. J. (1990). On the efficiency of matching and related models of search and unemployment, *Review of Economic Studies* **57**(2): 279–298. (Cited on p. 125.)

Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies, *Journal of Economic Dynamics and Control* **17**(5–6): 953–969. (Cited on p. 53.)

Jones, L. E. and Manuelli, R. (1990). A convex model of equilibrium growth: Theory and policy implications, *Journal of Political Economy* **98**(5): 1008–1038. (Cited on p. 85.)

Jorgensen, D. W. (1963). Capital theory and investment behavior, *American Economic Review, Papers and Proceedings* **53**(2): 247–259. (Cited on p. 124.)

Judd, K. L. (1985). Redistributive taxation in a simple perfect foresight model, *Journal of Public Economics* **28**: 59–83. (Cited on p. 208.)

Kaldor, N. (1961). Capital accumulation and economic growth, *in* F. A. Lutz and D. C. Hague (eds), *The Theory of Capital*, St. Martins Press, New York, pp. 177–222. (Cited on p. 85.)

Kaplan, G., Moll, B. and Violante, G. L. (2018). Monetary policy according to HANK, *American Economic Review* **108**(3): 697–743. (Cited on p. 155.)

Kareken, J. and Wallace, N. (1981). On the indeterminacy of equilibrium exchange rates, *Quarterly Journal of Economics* **96**(2): 207–222. (Cited on p. 142.)

Kehoe, T. J. and Levine, D. K. (1985). Comparative statics and perfect foresight in infinite horizon economies, *Econometrica* **53**(2): 433–453. (Cited on p. 38.)

Kehoe, T. J. and Levine, D. K. (1993). Debt-constrained asset markets, *Review of Economic Studies* **60**(4): 865–888. (Cited on p. 125.)

Keynes, J. M. (1923). *A Tract on Monetary Reform*, Macmillan, London. (Cited on p. 142.)

Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*, Macmillan, London. (Cited on p. 8.)

Kimball, M. S. (1990). Precautionary saving in the small and in the large, *Econometrica* **58**(1): 53–73. (Cited on p. 53.)

King, M. A. (1980). Savings and taxation, *in* G. Hughes and G. Heal (eds), *Public Policy and the Tax System*, George Allen & Unwin, London, chapter 1, pp. 1–35. (Cited on p. 208.)

King, R. G., Plosser, C. I. and Rebelo, S. T. (1988). Production, growth, and business cycles I: The basic neoclassical model, *Journal of Monetary Economics* **21**: 195–232. (Cited on pp. 85 and 86.)

King, R. G., Plosser, C. I. and Rebelo, S. T. (2002). Production, growth, and business cycles: Technical appendix, *Computational Economics* **20**(1): 87–116. (Cited on pp. 85 and 86.)

King, R. G. and Wolman, A. L. (1996). Inflation targeting in a St. Louis model of the 21st century, *Federal Reserve Bank of St. Louis Review* **78**(3): 83–107. (Cited on p. 155.)

Kirman, A. P. (1992). Whom or what does the representative individual represent?, *Journal of Economic Perspectives* **6**(2): 117–136. (Cited on p. 258.)

Kiyotaki, N. and Moore, J. (1997). Credit cycles, *Journal of Political Economy* **105**(2): 211–248. (Cited on p. 125.)

Kiyotaki, N. and Wright, R. (1989). On money as a medium of exchange, *Journal of Political Economy* **97**(4): 927–954. (Cited on p. 142.)

Kiyotaki, N. and Wright, R. (1993). A search-theoretic approach to monetary economics, *American Economic Review* **83**(1): 63–77. (Cited on p. 142.)

Kocherlakota, N. and Phelan, C. (1999). Explaining the fiscal theory of the price level, *Federal Reserve Bank of Minneapolis Quarterly Review* **23**(4): 14–23. (Cited on p. 185.)

Kocherlakota, N. R. (1996). Implications of efficient risk sharing without commitment, *Review of Economic Studies* **63**(4): 595–609. (Cited on p. 125.)

Kollmann, R. (2001). The exchange rate in a dynamic-optimizing business cycle model with nominal rigidities: A quantitative investigation, *Journal of International Economics* **55**(2): 243–262. (Cited on p. 155.)

Koopmans, T. C. (1965). On the concept of optimal economic growth, *The Econometric Approach to Development Planning*, North-Holland / Rand McNally, Amsterdam, chapter 4, pp. 225–300. Reissue of Pontificiae Academiae Scientiarum Scripta Varia 28. (Cited on p. 37.)

Korinek, A. and Simsek, A. (2016). Liquidity trap and excessive leverage, *American Economic Review* **106**(3): 699–738. (Cited on p. 155.)

Kraay, A. and Ventura, J. (2000). Current accounts in debtor and creditor countries, *Quarterly Journal of Economics* **115**(4): 1137–1166. (Cited on p. 96.)

Kreps, D. M. and Porteus, E. L. (1978). Temporal resolution of uncertainty and dynamic choice theory, *Econometrica* **46**(1): 185–200. (Cited on p. 258.)

Kreps, D. M. and Wilson, R. (1982). Reputation and imperfect information, *Journal of Economic Theory* **27**(2): 253–279. (Cited on p. 235.)

Krugman, P. R. (1989). Market-based debt-reduction schemes, *in* J. A. Frenkel, M. P. Dooley and P. Wickham (eds), *Analytical Issues in Debt*, International Monetary Fund, Washington, pp. 258–278. (Cited on p. 235.)

Krusell, P., Quadrini, V. and Ríos-Rull, J.-V. (1997). Politico-economic equilibrium and economic growth, *Journal of Economic Dynamics and Control* **21**(1): 243–272. (Cited on p. 235.)

Krusell, P. and Ríos-Rull, J.-V. (1996). Vested interests in a positive theory of stagnation and growth, *Review of Economic Studies* **63**(2): 301–329. (Cited on p. 235.)

Krusell, P. and Smith, A. A. (1998). Income and wealth heterogeneity in the macroeconomy, *Journal of Political Economy* **106**(5): 867–896. (Cited on p. 86.)

Krusell, P. and Smith, A. A. (2003). Consumption-savings decisions with quasi-geometric discounting, *Econometrica* **71**(1): 365–375. (Cited on p. 24.)

Kydland, F. E. and Prescott, E. C. (1977). Rules rather than discretion: The inconsistency of optimal plans, *Journal of Political Economy* **85**(3): 473–491. (Cited on p. 234.)

Kydland, F. E. and Prescott, E. C. (1982). Time to build and aggregate fluctuations, *Econometrica* **50**(6): 1345–1370. (Cited on p. 86.)

Laffont, J.-J. (1989). *The Economics of Uncertainty and Information*, MIT Press, Cambridge, Massachusetts. (Cited on p. 53.)

Lagos, R. and Wright, R. (2005). A unified framework for monetary theory and policy analysis, *Journal of Political Economy* **113**(3): 463–484. (Cited on p. 142.)

Laibson, D. (1997). Golden eggs and hyperbolic discounting, *Quarterly Journal of Economics* **62**: 443–477. (Cited on p. 24.)

Leeper, E. M. (1991). Equilibria under 'active' and 'passive' monetary and fiscal policies, *Journal of Monetary Economics* **27**(1): 129–147. (Cited on p. 185.)

Leland, H. E. (1968). Saving and uncertainty: The precautionary demand for saving, *Quarterly Journal of Economics* **82**(3): 465–473. (Cited on p. 53.)

LeRoy, S. F. and Werner, J. (2014). *Principles of Financial Economics*, Cambridge University Press, Cambridge. (Cited on p. 63.)

Levhari, J. D. and Srinivasan, T. N. (1969). Optimal savings under uncertainty, *Review of Economic Studies* **36**(2): 153–163. (Cited on p. 52.)

Lewbel, A. (1989). Exact aggregation and a representative consumer, *Quarterly Journal of Economics* **104**(3): 621–633. (Cited on p. 258.)

Lindbeck, A. and Weibull, J. W. (1987). Balanced-budget redistribution as the outcome of political competition, *Public Choice* **52**: 273–297. (Cited on p. 258.)

Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics* **47**(1): 13–37. (Cited on p. 63.)

Ljungqvist, L. and Sargent, T. J. (2018). *Recursive Macroeconomic Theory*, 4. edn, MIT Press, Cambridge, Massachusetts. (Cited on pp. 125, 209, 234, and 235.)

Long, J. B. and Plosser, C. I. (1983). Real business cycles, *Journal of Political Economy* **91**(11): 39–69. (Cited on p. 86.)

Lorenzoni, G. (2008). Inefficient credit booms, *Review of Economic Studies* **75**(3): 809–833. (Cited on p. 126.)

Lucas, R. E. (1972). Expectations and the neutrality of money, *Journal of Economic Theory* **4**(2): 103–124. (Cited on pp. 8 and 185.)

Lucas, R. E. (1976). Econometric policy evaluation: A critique, *Carnegie-Rochester Conference Series on Public Policy* pp. 19–46. (Cited on p. 9.)

Lucas, R. E. (1978). Asset prices in an exchange economy, *Econometrica* **46**(6): 1429–1445. (Cited on p. 63.)

Lucas, R. E. (1980). Equilibrium in a pure currency economy, *in* J. H. Kareken and N. Wallace (eds), *Models of Monetary Economies*, Federal Reserve Bank of Minneapolis, Minneapolis, pp. 131–145. (Cited on p. 142.)

Lucas, R. E. (1982). Interest rates and currency prices in a two-country world, *Journal of Monetary Economics* **10**(3): 335–359. (Cited on p. 142.)

Lucas, R. E. (1987). *Models of Business Cycles*, Basil Blackwell, New York. (Cited on p. 86.)

Lucas, R. E. (1990). Liquidity and interest rates, *Journal of Economic Theory* **50**(2): 237–264. (Cited on p. 185.)

Lucas, R. E. and Prescott, E. C. (1971). Investment under uncertainty, *Econometrica* **39**(5): 659–681. (Cited on pp. 86 and 124.)

Lucas, R. E. and Prescott, E. C. (1974). Equilibrium search and unemployment, *Journal of Economic Theory* **7**(2): 188–209. (Cited on p. 125.)

Lucas, R. E. and Rapping, L. A. (1969). Real wages, employment, and inflation, *Journal of Political Economy* **77**(5): 721–754. (Cited on p. 85.)

Lucas, R. E. and Stokey, N. L. (1983). Optimal fiscal and monetary policy in an economy without capital, *Journal of Monetary Economics* **12**(1): 55–93. (Cited on pp. 208 and 234.)

Lucas, R. E. and Stokey, N. L. (1987). Money and interest rates in a cash-in-advance economy, *Econometrica* **55**(3): 491–513. (Cited on p. 142.)

Magill, M. and Quinzii, M. (1996). *Theory of Incomplete Markets*, Vol. 1, MIT Press, Cambridge, Massachusetts. (Cited on pp. 63 and 126.)

Malinvaud, E. (1953). Capital accumulation and efficient allocation of resources, *Econometrica* **21**(2): 233–268. (Cited on p. 37.)

Mankiw, N. G. (1985). Small menu costs and large business cycles: A macroeconomic model of monopoly, *Quarterly Journal of Economics* **100**(2): 529–537. (Cited on p. 155.)

Mankiw, N. G. and Reis, R. (2002). Sticky information versus sticky prices: A proposal to replace the New Keynesian Phillips curve, *Quarterly Journal of Economics* **117**(4): 1295–1328. (Cited on p. 155.)

Marcet, A. and Marimon, R. (1992). Communication, commitment, and growth, *Journal of Economic Theory* **58**(2): 219–249. (Cited on p. 125.)

Marcet, A. and Marimon, R. (1999). Recursive contracts. Mimeo, Universitat Pompeu Fabra, Barcelona. (Cited on p. 125.)

Martin, A. and Ventura, J. (2012). Economic growth with bubbles, *American Economic Review* **102**(6): 3033–3058. (Cited on p. 63.)

Martin, A. and Ventura, J. (2018). The macroeconomics of rational bubbles: A user's guide, *Annual Review of Economics* **10**: 505–539. (Cited on p. 63.)

Mas-Colell, A., Whinston, M. D. and Green, J. R. (1995). *Microeconomic Theory*, Oxford University Press, New York. (Cited on pp. 9, 240, and 258.)

McCall, J. (1970). Economics of information and job search, *Quarterly Journal of Economics* **84**(1): 113–126. (Cited on p. 125.)

McDonald, R. and Siegel, D. (1986). The value of waiting to invest, *Quarterly Journal of Economics* **101**(4): 707–727. (Cited on p. 124.)

McKenzie, L. (1954). On equilibrium in Graham's model of world trade and other competitive systems, *Econometrica* **22**(2): 147–161. (Cited on p. 8.)

Merz, M. (1995). Search in the labor market and the real business cycle, *Journal of Monetary Economics* **36**(2): 269–300. (Cited on p. 125.)

Miao, J. (2014). *Economic Dynamics in Discrete Time*, MIT Press, Cambridge, Massachusetts. (Cited on p. 258.)

Mirrlees, J. A. (1965). Optimum accumulation under uncertainty. Mimeo. (Cited on p. 85.)

Mirrlees, J. A. (1971). An exploration in the theory of optimum income taxation, *Review of Economic Studies* **38**(114): 175–208. (Cited on p. 258.)

Modigliani, F. and Brumberg, R. (1954). Utility analysis and the consumption function: An interpretation of cross-section data, *in* K. K. Kurihara (ed.), *Post Keynesian Economics*, Rutgers University Press, New Brunswick. (Cited on pp. 24, 37, and 52.)

Modigliani, F. and Miller, M. H. (1958). The cost of capital, corporation finance and the theory of investment, *American Economic Review* **48**(3): 261–297. (Cited on p. 125.)

Mortensen, D. T. (1982). Property rights and efficiency in mating, racing, and related games, *American Economic Review* **72**(5): 968–979. (Cited on p. 125.)

Mossin, J. (1966). Equilibrium in a capital asset market, *Econometrica* **34**(4): 768–783. (Cited on p. 63.)

Mueller, D. C. (1989). *Public Choice II*, Cambridge University Press, Cambridge. (Cited on p. 235.)

Mulligan, C. B. and Sala-i-Martin, X. (1997). The optimum quantity of money: Theory and evidence, *Journal of Money, Credit, and Banking* **29**(4): 687–715. (Cited on p. 208.)

Muth, J. F. (1961). Rational expectations and the theory of price movements, *Econometrica* **29**(3): 315–335. (Cited on p. 8.)

Myers, S. C. (1977). Determinants of corporate borrowing, *Journal of Financial Economics* **5**(2): 147–175. (Cited on p. 235.)

Negishi, T. (1960). Welfare economics and existence of an equilibrium for a competitive economy, *Metroeconomica* **12**(2–3): 92–97. (Cited on p. 37.)

Niehans, J. (1994). *A History of Economic Theory: Classic Contributions, 1720–1980*, Johns Hopkins University Press, Baltimore. (Cited on p. 9.)

Niepelt, D. (2004a). The fiscal myth of the price level, *Quarterly Journal of Economics* **119**(1): 277–300. (Cited on p. 185.)

Niepelt, D. (2004b). Tax smoothing versus tax shifting, *Review of Economic Dynamics* **7**(1): 27–51. (Cited on p. 208.)

Obstfeld, M. (1982). Aggregate spending and the terms of trade: Is there a Laursen-Metzler effect?, *Quarterly Journal of Economics* **97**(2): 251–270. (Cited on p. 96.)

Obstfeld, M. and Rogoff, K. (1983). Speculative hyperinflations in maximizing models: Can we rule them out?, *Journal of Political Economy* **91**(4): 675–687. (Cited on p. 185.)

Obstfeld, M. and Rogoff, K. (1995). Exchange rate dynamics redux, *Journal of Political Economy* **103**(3): 624–660. (Cited on p. 155.)

Obstfeld, M. and Rogoff, K. (1996). *Foundations of International Macroeconomics*, MIT Press, Cambridge, Massachusetts. (Cited on p. 96.)

Persson, T. and Tabellini, G. (2000). *Political Economics*, MIT Press, Cambridge, Massachusetts. (Cited on pp. 235 and 258.)

Phelan, C. (2006). Opportunity and social mobility, *Review of Economic Studies* **73**(2): 487–504. (Cited on p. 209.)

Phelan, C. and Stacchetti, E. (2001). Sequential equilibria in a Ramsey tax model, *Econometrica* **69**(6): 1491–1518. (Cited on p. 235.)

Phelps, E. S. (1962). The accumulation of risky capital: A sequential utility analysis, *Econometrica* **30**(4): 729–743. (Cited on p. 52.)

Phelps, E. S. (1965). Second essay on the golden rule of accumulation, *American Economic Review* **55**(4): 793–814. (Cited on p. 37.)

Phelps, E. S. (1973). Inflation in the theory of public finance, *Swedish Journal of Economics* **75**: 67–82. (Cited on p. 208.)

Phelps, E. S. (ed.) (1970). *Microeconomic Foundations of Employment and Inflation Theory*, Norton, New York. (Cited on pp. 8 and 185.)

Phelps, E. S. and Pollak, R. A. (1968). On second-best national saving and game-equilibrium growth, *Review of Economic Studies* **35**(2): 185–199. (Cited on p. 24.)

Pissarides, C. A. (1985). Short-run equilibrium dynamics of unemployment, vacancies, and real wages, *American Economic Review* **75**(4): 676–690. (Cited on p. 125.)

Pissarides, C. A. (1990). *Equilibrium Unemployment Theory*, Basil Blackwell, Oxford. (Cited on p. 125.)

Pissarides, C. A. (2000). *Equilibrium Unemployment Theory*, MIT Press, Cambridge, Massachusetts. (Cited on p. 125.)

Pollak, R. A. (1968). Consistent planning, *Review of Economic Studies* **35**(2): 201–208. (Cited on p. 24.)

Poole, W. (1970). Optimal choice of monetary policy instruments in a simple stochastic macro model, *Quarterly Journal of Economics* **84**(2): 197–216. (Cited on p. 185.)

Prescott, E. C. and Mehra, R. (1980). Recursive competitive equilibrium: The case of homogeneous households, *Econometrica* **48**(6): 1365–1379. (Cited on p. 86.)

Radner, R. (1972). Existence of equilibrium of plans, prices, and price expectations in a sequence of markets, *Econometrica* **40**(2): 289–303. (Cited on pp. 8 and 53.)

Ramsey, F. P. (1927). A contribution to the theory of taxation, *Economic Journal* **37**(145): 47–61. (Cited on p. 258.)

Ramsey, F. P. (1928). A mathematical theory of saving, *Economic Journal* **38**(152): 543–559. (Cited on p. 37.)

Rangel, A. (1997). Social security reform: Efficiency gains or intergenerational redistribution. Mimeo, Harvard University. (Cited on p. 184.)

Rebelo, S. (1991). Long-run policy analysis and long-run growth, *Journal of Political Economy* **99**(3): 500–521. (Cited on p. 85.)

Rocheteau, G. and Nosal, E. (2017). *Money, Payments, and Liquidity*, 2. edn, MIT Press, Cambridge, Massachusetts. (Cited on p. 142.)

Rogerson, R. (1988). Indivisible labor, lotteries and equilibrium, *Journal of Monetary Economics* **21**(1): 3–16. (Cited on p. 85.)

Rogerson, W. P. (1985). Repeated moral hazard, *Econometrica* **53**(1): 69–76. (Cited on p. 208.)

Rogoff, K. (1985). The optimal degree of commitment to an intermediate monetary target, *Quarterly Journal of Economics* **100**: 1169–1190. (Cited on p. 235.)

Romer, P. M. (1986). Increasing returns and long-run growth, *Journal of Political Economy* **94**(5): 1002–1037. (Cited on p. 85.)

Ross, S. A. (1976). The arbitrage theory of capital asset pricing, *Journal of Economic Theory* **13**(3): 341–360. (Cited on p. 63.)

Rotemberg, J. J. (1982). Monopolistic price adjustment and aggregate output, *Review of Economic Studies* **49**(4): 517–531. (Cited on p. 155.)

Rotemberg, J. J. (1984). A monetary equilibrium model with transactions costs, *Journal of Political Economy* **92**(1): 40–58. (Cited on p. 185.)

Rotemberg, J. J. and Woodford, M. (1997). An optimization-based econometric framework for the evaluation of monetary policy, *in* B. S. Bernanke and J. Rotemberg (eds), *NBER Macroeconomics Annual 1997*, MIT Press, Cambridge, Massachusetts, pp. 297–346. (Cited on p. 208.)

Sachs, J. D. (1981). The current account and macroeconomic adjustment in the 1970s, *Brookings Papers on Economic Activity*, Vol. 12, Brookings Institution, Washington, pp. 201–282. (Cited on p. 96.)

Salanié, B. (2003). *The Economics of Taxation*, MIT Press, Cambridge, Massachusetts. (Cited on p. 258.)

Samuelson, P. A. (1939). The gains from international trade, *Canadian Journal of Economics and Political Science* **5**(2): 195–205. (Cited on p. 96.)

Samuelson, P. A. (1955). *Economics*, 3. edn, McGraw Hill, New York. (Cited on p. 8.)

Samuelson, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money, *Journal of Political Economy* **66**(6): 467–482. (Cited on pp. 37 and 142.)

Samuelson, P. A. (1964). Theoretical notes on trade problems, *Review of Economics and Statistics* **46**(2): 145–154. (Cited on p. 96.)

Sandmo, A. (1970). The effect of uncertainty on saving decisions, *Review of Economic Studies* **37**(3): 353–360. (Cited on p. 53.)

Santos, M. S. and Woodford, M. (1997). Rational asset pricing bubbles, *Econometrica* **65**(1): 19–57. (Cited on p. 63.)

Sargent, T. J. (1971). A note on the 'accelerationist' hypothesis, *Journal of Money, Credit, and Banking* **3**(3): 721–725. (Cited on p. 8.)

Sargent, T. J. (1987). *Dynamic Macroeconomic Theory*, Harvard University Press, Cambridge, Massachusetts. (Cited on p. 185.)

Sargent, T. J. and Wallace, N. (1973). The stability of models of money and growth with perfect foresight, *Econometrica* **41**(6): 1043–1048. (Cited on p. 258.)

Sargent, T. J. and Wallace, N. (1975). 'Rational' expectations, the optimal monetary instrument, and the optimal money supply rule, *Journal of Political Economy* **83**(2): 241–254. (Cited on p. 185.)

Sargent, T. J. and Wallace, N. (1981). Some unpleasant monetarist arithmetic, *Federal Reserve Bank of Minneapolis Quarterly Review* **5**(3): 1–17. (Cited on p. 185.)

Saving, T. R. (1971). Transactions costs and the demand for money, *American Economic Review* **61**(3): 407–420. (Cited on p. 142.)

Schlicht, E. (2006). A variant of Uzawa's theorem, *Economics Bulletin* **5**(6): 1–5. (Cited on p. 85.)

Schmitt-Grohé, S. and Uribe, M. (2004). Solving dynamic general equilibrium models using a second-order approximation to the policy function, *Journal of Economic Dynamics and Control* **28**(4): 755–775. (Cited on p. 86.)

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* **19**(3): 425–442. (Cited on p. 63.)

Shell, K. (1971). Notes on the economics of infinity, *Journal of Political Economy* **79**(5): 1002–1011. (Cited on pp. 37 and 142.)

Shimer, R. (2010). *Labor Markets and Business Cycles*, Princeton University Press, Princeton. (Cited on p. 125.)

Shleifer, A. and Vishny, R. W. (1992). Liquidation values and debt capacity: A market equilibrium approach, *Journal of Finance* **47**(4): 1343–1366. (Cited on p. 125.)

Sidrauski, M. (1967). Rational choice and patterns of growth in a monetary economy, *American Economic Review* **57**(2): 534–544. (Cited on p. 142.)

Simon, C. P. and Blume, L. (1994). *Mathematics for Economists*, Norton & Company, New York. (Cited on p. 240.)

Sims, C. A. (1980). Macroeconomics and reality, *Econometrica* **48**(1): 1–48. (Cited on p. 9.)

Sims, C. A. (1994). A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy, *Economic Theory* **4**(3): 381–399. (Cited on p. 185.)

Siu, H. E. (2004). Optimal fiscal and monetary policy with sticky prices, *Journal of Monetary Economics* **51**(3): 575–607. (Cited on p. 209.)

Stein, J. C. (2012). Monetary policy as financial stability regulation, *Quarterly Journal of Economics* **127**(1): 57–97. (Cited on p. 126.)

Stiglitz, J. E. (1982). The inefficiency of the stock market equilibrium, *Review of Economic Studies* **49**: 241–261. (Cited on p. 125.)

Stiglitz, J. E. (1987). Pareto efficient and optimal taxation and the new new welfare economics, *in* A. J. Auerbach and M. Feldstein (eds), *Handbook of Public Economics*, Vol. 2, North-Holland, Amsterdam, chapter 15, pp. 991–1042. (Cited on p. 258.)

Stiglitz, J. E. and Weiss, A. (1981). Credit rationing in markets with imperfect information, *American Economic Review* **71**(3): 393–410. (Cited on p. 125.)

Stigum, B. P. (1969). Competitive equilibria under uncertainty, *Quarterly Journal of Economics* **83**(4): 533–561. (Cited on p. 8.)

Stokey, N. L. (1989). Reputation and time consistency, *American Economic Review* **79**(2): 134–139. (Cited on p. 234.)

Stokey, N. L. (1991). Credible public policy, *Journal of Economic Dynamics and Control* **15**(4): 627–656. (Cited on p. 234.)

Stokey, N. L. and Lucas, R. E. (1989). *Recursive Methods in Economic Dynamics*, Harvard University Press, Cambridge, Massachusetts. (Cited on pp. 86, 240, and 258.)

Stolper, W. F. and Samuelson, P. A. (1941). Protection and real wages, *Review of Economic Studies* **9**(1): 58–73. (Cited on p. 96.)

Straub, L. and Werning, I. (2014). Positive long run capital taxation: Chamley-Judd revisited, *Working Paper 20441*, NBER, Cambridge, Massachusetts. (Cited on p. 208.)

Strotz, R. H. (1956). Myopia and inconsistency in dynamic utility maximization, *Review of Economic Studies* **23**(3): 165–180. (Cited on p. 24.)

Svensson, L. E. O. (1985). Money and asset prices in a cash-in-advance economy, *Journal of Political Economy* **93**(5): 919–944. (Cited on p. 142.)

Svensson, L. E. O. and Razin, A. (1983). The terms of trade and the current account: The Harberger-Laursen-Metzler effect, *Journal of Political Economy* **91**(1): 97–125. (Cited on p. 96.)

Taylor, J. B. (1979). Staggered wage setting in a macro model, *American Economic Review* **69**(2): 108–113. (Cited on p. 155.)

Taylor, J. B. (1993). Discretion versus policy rules in practice, *Carnegie-Rochester Conference Series on Public Policy* **39**: 195–214. (Cited on p. 185.)

Taylor, J. B. (1999). A historical analysis of monetary policy rules, *in* J. B. Taylor (ed.), *Monetary Policy Rules*, University of Chicago Press, Chicago, chapter 7, pp. 319–348. (Cited on p. 185.)

Thomas, J. and Worrall, T. (1988). Self-enforcing wage contracts, *Review of Economic Studies* **55**(4): 541–553. (Cited on p. 125.)

Tirole, J. (1982). On the possibility of speculation under rational expectations, *Econometrica* **50**(5): 1163–1181. (Cited on p. 63.)

Tirole, J. (1985). Asset bubbles and overlapping generations, *Econometrica* **53**(5): 1071–1100. (Cited on pp. 63 and 142.)

Tirole, J. (2006). *The Theory of Corporate Finance*, Princeton University Press, Princeton. (Cited on p. 125.)

Tobin, J. (1956). The interest elasticity of the transactions demand for cash, *Review of Economics and Statistics* **38**(3): 241–247. (Cited on p. 142.)

Tobin, J. (1969). A general equilibrium approach to monetary theory, *Journal of Money, Credit, and Banking* **1**(1): 15–29. (Cited on p. 124.)

Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification, *Journal of Economic Theory* **21**(2): 265–293. (Cited on p. 125.)

Townsend, R. M. (1980). Models of money with spatially separated agents, *in* J. H. Kareken and N. Wallace (eds), *Models of Monetary Economies*, Federal Reserve Bank of Minneapolis, Minneapolis, pp. 265–304. (Cited on p. 142.)

Uribe, M. and Schmitt-Grohé, S. (2017). *Open Economy Macroeconomics*, Princeton University Press, Princeton. (Cited on p. 96.)

Uzawa, H. (1961). Neutral inventions and the stability of growth equilibrium, *Review of Economic Studies* **28**(2): 117–124. (Cited on p. 85.)

Vaughan, D. R. (1970). A non recursive algorithm solution for the discrete Ricatti equation, *IEEE Transactions on Automatic Control* **AC-15**: 597–599. (Cited on p. 258.)

von Neumann, J. and Morgenstern, O. (1944). *Theory of Games and Economic Behavior*, Princeton University Press, Princeton. (Cited on p. 52.)

Wallace, N. (1980). The overlapping generations model of fiat money, *in* J. H. Kareken and N. Wallace (eds), *Models of Monetary Economies*, Federal Reserve Bank of Minneapolis, Minneapolis, pp. 49–82. (Cited on p. 142.)

Wallace, N. (1981). A Modigliani-Miller theorem for open-market operations, *American Economic Review* **71**(3): 267–274. (Cited on p. 185.)

Walras, L. (1874). *Éléments d'Économie Politique Pure, ou Théorie de la Richesse Sociale*, L. Corbaz, Lausanne. (Cited on p. 8.)

Walsh, C. E. (1995). Optimal contracts for central bankers, *American Economic Review* **85**(1): 150–167. (Cited on p. 235.)

Walsh, C. E. (2017). *Monetary Theory and Policy*, 3. edn, MIT Press, Cambridge, Massachusetts. (Cited on pp. 126, 142, 155, 185, 209, and 235.)

Weil, P. (1987). Confidence and the real value of money in an overlapping generations economy, *Quarterly Journal of Economics* **102**(1): 1–22. (Cited on p. 63.)

Weil, P. (1989). The equity premium puzzle and the risk-free rate puzzle, *Journal of Monetary Economics* **24**: 401–421. (Cited on p. 258.)

Werning, I. (2003). Standard dynamic programming for Aiyagari, Marcet, Sargent and Seppälä. Mimeo, MIT, Cambridge, Massachusetts. (Cited on p. 208.)

Werning, I. (2007). Optimal fiscal policy with redistribution, *Quarterly Journal of Economics* **122**(3): 925–967. (Cited on p. 208.)

Werning, I. (2015). Incomplete markets and aggregate demand, *Working Paper 21448*, NBER, Cambridge, Massachusetts. (Cited on p. 53.)

Williams, J. B. (1938). *The Theory of Investment Value*, Harvard University Press, Cambridge, Massachusetts. (Cited on p. 125.)

Woodford, M. (1990). Public debt as private liquidity, *American Economic Review* **80**(2): 382–388. (Cited on pp. 142 and 185.)

Woodford, M. (1995). Price level determinacy without control of a monetary aggregate, *Carnegie-Rochester Conference Series on Public Policy* **43**: 1–46. (Cited on p. 185.)

Woodford, M. (2001). The Taylor rule and optimal monetary policy, *American Economic Review, Papers and Proceedings* **91**(2): 232–237. (Cited on p. 185.)

Woodford, M. (2003). *Interest and Prices*, Princeton University Press, Princeton. (Cited on pp. 155, 185, 208, 209, and 235.)

Yaari, M. E. (1965). Uncertain lifetime, life insurance, and the theory of the consumer, *Review of Economic Studies* **32**(2): 137–150. (Cited on p. 38.)

Yun, T. (1996). Nominal price rigidity, money supply endogeneity, and business cycles, *Journal of Monetary Economics* **37**(2): 345–370. (Cited on p. 155.)

Zeldes, S. P. (1989a). Consumption and liquidity constraints: An empirical investigation, *Journal of Political Economy* **97**(2): 305–346. (Cited on p. 53.)

Zeldes, S. P. (1989b). Optimal consumption with stochastic income: Deviations from certainty equivalence, *Quarterly Journal of Economics* **104**(2): 275–298. (Cited on p. 53.)

Zhu, X. (1992). Optimal fiscal policy in a stochastic growth model, *Journal of Economic Theory* **58**(2): 250–289. (Cited on p. 208.)

Zilcha, I. (1990). Dynamic efficiency in overlapping generations models with stochastic production, *Journal of Economic Theory* **52**(2): 364–379. (Cited on p. 38.)