

DISCUSSION PAPER SERIES

No. 9203

ECONOMIC AND POLITICO- ECONOMIC EQUIVALENCE

Martin Gonzalez-Eiras and Dirk Niepelt

***INTERNATIONAL MACROECONOMICS
and PUBLIC POLICY***



Centre for Economic Policy Research

www.cepr.org

Available online at:

www.cepr.org/pubs/dps/DP9203.asp

ECONOMIC AND POLITICO- ECONOMIC EQUIVALENCE

Martin Gonzalez-Eiras, Universidad Adolfo Ibañez
Dirk Niepelt, Study Center Gerzensee and CEPR

Discussion Paper No. 9203
November 2012

Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

This Discussion Paper is issued under the auspices of the Centre's research programme in **INTERNATIONAL MACROECONOMICS and PUBLIC POLICY**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as an educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Martin Gonzalez-Eiras and Dirk Niepelt

CEPR Discussion Paper No. 9203

November 2012

ABSTRACT

Economic and Politico-Economic Equivalence*

We extend "economic equivalence" results, like the Ricardian equivalence proposition, to the political sphere where policy is chosen sequentially. We derive conditions under which a policy regime (summarizing admissible policy choices in every period) and a state are "politico-economically equivalent" to another such pair, in the sense that both pairs give rise to the same equilibrium allocation. The equivalence conditions help to identify factors that render institutional change non-neutral. We exemplify their use in the context of several applications, relating to social security reform, tax-smoothing policies and measures to correct externalities.

JEL Classification: E62, H55 and H63

Keywords: equivalence, government debt, politico-economic equilibrium, social security reform and tax policy

Martin Gonzalez-Eiras
Universidad Adolfo Ibañez
Diagonal Las Torres 2640
Peñalolén
Santiago
CHILE

Dirk Niepelt
Study Center Gerzensee
PO Box 21
3115 Gerzensee
SWITZERLAND

Email: mge@alum.mit.edu

Email: dirk.niepelt@szgerzensee.ch

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=166286

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=149298

* We thank John Hassler, Roland Hodler, Enrique Kawamura, Per Krusell, Andrés Neumeyer, Torsten Persson, Jaime Ventura as well as seminar and conference participants at EPRU (University of Copenhagen), IIES, Stockholm University, Universidad de San Andrés, Universidad Torcuato di Tella, ESSIM, LAMES and SED for helpful comments.

Submitted 23 October 2012

1 Introduction

Important neutrality results in public economics, macroeconomics and other fields establish equivalence classes of “economically equivalent” policies that support the same equilibrium allocation (conditional on an initial state). For example, in a simple model of household choice, policies relying on different combinations of consumption, capital-income and labor-income taxes form equivalence classes, and in the standard overlapping-generations model, pay-as-you-go social security policies are economically equivalent to certain policies relying on taxes and explicit government debt.

While proving very useful in a variety of contexts, these neutrality results are limited in their applicability to theories that treat policy as exogenous. In politico-economic models or theories of optimal (Ramsey) policy, this is not the case. Policy constitutes an equilibrium outcome in these models and the primitives of the analysis include institutions or policy regimes which define the admissible policy instruments available to political decision makers.

This raises the question whether equivalence classes over policy regimes can be defined and if so, how these equivalence classes relate to the conventional equivalence classes defined over policies. An answer to this question has important policy implications. Consider for example the proposal to “privatize” social security and debt finance the transition. From a narrow economic point of view, shifting from a pay-as-you-go financed social security regime to a regime with taxes and explicit government debt could be irrelevant because specific pay-as-you-go and debt policies belong to the same economic equivalence class. From a politico-economic point of view, however, one would expect that such a regime change could alter the equilibrium allocation. In fact, this is what the observed disagreement among policy makers concerned with institutional changes of social security systems suggests.

In this paper, we propose an answer to the question posed above. We define a policy regime and state to be “politico-economically equivalent” to another such pair if both pairs support politico-economic equilibria and the same equilibrium allocation. We derive conditions under which politico-economic equivalence follows. And we use these conditions in the context of several applications. While we focus on specific applications with taxes, public debt and corrective policy measures, the theoretical conditions we obtain are general in nature and apply in many other contexts featuring an endogenous choice of policies.

Our results are derived within a general dynamic framework comprising a household sector, firms and a government. We do not impose restrictions on the commitment power of political decision makers. As a consequence, the results apply to environments with both sequential policy choice and policy choice once and for all, as for example with Ramsey policies. Nor do we impose restrictions on political objective functions (except that these functions be defined over allocations) or political aggregation mechanisms. Our results therefore apply independently of specific assumptions about the political process or the structure of government, political parties and interest groups.

In a first step, we define economic equivalence of exogenous policies (conditional on states) and we extend well-known economic neutrality propositions (e.g., Barro, 1974; Sar-

gent, 1987; Rangel, 1997; Coleman, 2000; Ghiglino and Shell, 2000; Bassetto and Kocherlakota, 2004; Niepelt, 2005) to derive a general economic equivalence result. In the second step, we define politico-economic equivalence of policy regimes (conditional on states) and derive sufficient conditions for such equivalence. In parallel to the economic equivalence result which emphasizes the implications of exogenous policy for the choice sets of households and firms, the politico-economic equivalence result emphasizes the consequences of institutions for the choice sets of political decision makers. These choice sets are constrained fourfold: By the state; the admissibility restrictions on policy instruments under the control of political decision makers; the continuation policy functions of subsequent political decision makers; and by the requirement that policies support a competitive equilibrium. Accordingly, our conditions for politico-economic equivalence relate to state spaces and admissibility restrictions.

The first condition for politico-economic equivalence requires that the state spaces under an “initial” and a “new” policy regime are comparable in the sense that states can unambiguously be related. This guarantees that continuation policy functions can be compared across regimes as well. Verifying this first condition may not be immediate if policy instruments or commitment structures under the two institutions differ.

The other two conditions concern the admissible policy instruments. The conditions require the admissibility restrictions on instruments in the new regime to be both sufficiently loose and sufficiently tight: Sufficiently loose for political decision makers in the new regime to be able to choose policy instruments that, together with the continuation policy function of their successors, support the equilibrium allocation that political decision makers in the initial regime choose to implement; and sufficiently tight such that political decision makers in the new regime must not be able to implement competitive equilibria that cannot be implemented in the initial regime. If the three conditions are met in all periods and for all possible states revealed preference implies politico-economic equivalence.

The politico-economic equivalence result serves several purposes. On the one hand, it constitutes a useful tool for researchers interested in characterizing politico-economic equilibria. When high dimensional state and policy spaces render such a task difficult, the equivalence result can help by allowing to relate the equilibrium conditions of interest to their counterparts in a simpler setting that is easier to characterize. On the other hand, the politico-economic equivalence result helps to identify factors that render institutional change non-neutral. We exemplify this in the context of several applications, relating to social security reform, tax-smoothing policies and measures to correct externalities. Interestingly, we find that the politico-economic equivalence conditions can in some cases be applied even without prior knowledge of the politico-economic equilibrium in the initial regime.

Regarding social security reform, we start from the well-known fact that in overlapping generations economies, certain pay-as-you-go social security policies and debt policies are economically equivalent. Asking whether this equivalence extends to the political sphere, we contrast existing politico-economic models of social security (Cooley and Soares, 1999; Tabellini, 2000; Boldrin and Rustichini, 2000; Forni, 2005; Gonzalez-Eiras and Niepelt, 2008) with alternative models in which political decision makers may issue debt and choose

the repayment rate on maturing debt. We show that certain politico-economic theories of social security that have been proposed in the literature may be re-interpreted as politico-economic theories of government debt, and our analysis therefore contributes to a small but growing literature on debt in politico-economic equilibrium (e.g., Battaglini and Coate, 2008; Díaz-Giménez, Giovanetti, Marimon and Teles, 2008; Yared, 2010; Niepelt, 2011; Song, Storesletten and Zilibotti, 2012).¹ Other theories cannot be re-interpreted in that way. We identify (sufficiently rich) heterogeneity among households and differential tightness of admissibility restrictions across policy regimes as factors undermining politico-economic equivalence, and we argue that those factors can help rationalize why interest groups might favor or oppose the privatization of social security although from a narrow economic point of view, a regime change might appear irrelevant.

Regarding tax smoothing policies, we start from Bassetto and Kocherlakota's (2004) observation that the timing of distorting tax collections may be allocation neutral if taxes can be levied on contemporaneous and lagged incomes.² We find that this economic equivalence result extends to the political sphere if policy is chosen once and for all. With sequential policy choice, in contrast, a policy regime allowing for the taxation of current and lagged incomes generally is not politico-economically equivalent to a regime allowing for the taxation of contemporaneous income only. Intuitively, the ex-post elasticity of the tax base with respect to delayed taxes equals zero. When policy is chosen sequentially this renders political decision makers' choice set in the former regime larger than in the latter. Institutional change from one regime to the other therefore may be highly controversial.

Our final application concerning measures to correct externalities compares policy regimes with Pigovian taxes and cap-and-trade restrictions. We find that the two policy regimes are politico-economically equivalent, provided that permits in the cap-and-trade regime can be allocated in a way that replicates the distributive implications of the Pigovian tax and transfer scheme. This may be difficult to implement, however.

The remainder of the paper is structured as follows. Section 2 offers a simple example that introduces central concepts of the analysis and conveys the basic intuition. Section 3 lays out the model and discusses economic equivalence. Section 4 contains the paper's main result on politico-economic equivalence. Section 5 discusses applications and section 6 concludes.

2 A Simple Example

To motivate the analysis in the following sections, we start with a simple example of a two period economy with an OLG structure. Time is indexed by $t = 0, 1$. The economy is inhabited by workers and retirees. Workers in period $t = 0$ are retirees in period $t = 1$; workers born in the second period and retirees die at the end of the period. The ratio of workers to retirees equals $\nu > 0$. Workers supply labor inelastically, pay taxes, consume

¹Cukierman and Meltzer (1989) argue in a model with commitment that political decision makers are indifferent between social security and debt policies when the policy regime features both instruments and allows for lump sum taxes.

²Bassetto and Kocherlakota (2004) extend Barro's (1974) Ricardian (economic) equivalence result to environments with distorting taxes.

and save. Retirees consume the return on their savings as well as resources they might receive from government. Firms use labor supplied by workers and physical capital owned by retirees to produce the output good.

We compare a social security regime and a debt regime. In the social security regime, political decision makers may levy a non-negative social security tax τ_t on labor income and distribute the proceeds among retirees. In the debt regime, political decision makers may levy a tax τ'_t on labor income, issue debt b'_{t+1} and use the proceeds to repay outstanding debt b'_t (held by retirees) at the non-negative repayment rate z'_t . (Throughout the paper, we differentiate regimes by denoting variables in one of them by a “prime.”) Since the repayment rate on debt, and thus in equilibrium its price, is endogenous we can without loss of generality fix the stock of debt per retiree at exogenous levels, $\bar{b}'_t > 0$.

Economic equivalence relations link pairs of states and policy sequences that support the same competitive equilibrium allocation. In the example, the capital stock k is the single endogenous state variable. The state k_0 and a social security policy sequence (τ_0, τ_1) are economically equivalent to the state k'_0 and a debt policy sequence $(\tau'_0, z'_0, \tau'_1, z'_1)$ if $k'_0 = k_0$ (identical production possibilities), $\bar{b}'_0 z'_0 = \tau_0 w_0 \nu$ and $\bar{b}'_1 z'_1 = \tau_1 w_1 \nu$ (identical payments from the government to retirees at initial equilibrium prices), $\tau'_0 = \tau_0 - \frac{\tau_1 w_1 \nu}{R_1 w_0}$ (identical life time tax burdens for workers born in the first period at initial equilibrium prices), and $\tau'_1 = \tau_1$ (identical life time tax burdens for workers born in the second period). Here, R_t and w_t denote gross interest rates and wages in the social security equilibrium, respectively.

Political decision makers (who might represent voters) cannot commit and policy therefore is chosen sequentially, subject to admissibility restrictions that define the instruments political decision makers may use (labor income taxes in either regime and debt repayment in the debt regime) as well as numerical restrictions on those instruments ($\tau_t \geq 0$ and $z'_t \geq 0$ since lump sum taxes on retirees are ruled out), and subject to the constraint that their policy choices be feasible.³ Political decision makers in the first period also must take the continuation policy function of their successors into account. A politico-economic equilibrium in the social security regime conditional on initial capital stock k_0 is given by policy functions $\tau_0(\cdot), \tau_1(\cdot)$ and a competitive equilibrium with capital stock k_1 such that $k_0, \tau_0^* = \tau_0(k_0), \tau_1^* = \tau_1(k_1)$ support this competitive equilibrium and the policy choices are optimal with respect to the political objective functions $\Omega_t(\cdot)$ which are defined over allocations. A politico-economic equilibrium in the debt regime is defined similarly, with the policy functions given by $\tau'_0(\cdot), z'_0(\cdot), \tau'_1(\cdot), z'_1(\cdot)$.

Suppose that conditional on k_0 the social security regime supports a politico-economic equilibrium with sequentially chosen equilibrium policy sequence (τ_0^*, τ_1^*) . We want to assess *politico-economic equivalence* of the social security and debt regimes. More specifically, we want to assess whether (conditional on an initial capital stock k_0) the debt regime supports a politico-economic equilibrium with equilibrium policy sequence $(\tau_0'^*, z_0'^*, \tau_1'^*, z_1'^*)$ such that the pairs $(k_0, (\tau_0^*, \tau_1^*))$ and $(k_0, (\tau_0'^*, z_0'^*, \tau_1'^*, z_1'^*))$ are economically equivalent.

A brute force approach to checking this relies on solving for the politico-economic equilibrium in the debt regime. An alternative approach directly focuses on the choice sets

³The general analysis also applies to settings where political decision makers can commit, for example because a Ramsey planner chooses policy.

of political decision makers. Consider the situation in period $t = 1$. Since any equilibrium social security policy choice τ_1^* is non-negative the economically equivalent debt policy choice as implied by the economic equivalence relations, namely $(\tau'_1, z'_1) = (\tau_1^*, \frac{\tau_1^* w_1(k_1)\nu}{b'_1})$, necessarily is admissible.⁴ Moreover, for any admissible and feasible debt policy choice (τ'_1, z'_1) there exists an admissible social security policy choice, $\tau_1 = \tau'_1$, such that (k_1, τ_1) and $(k_1, (\tau'_1, z'_1))$ are economically equivalent. Conditional on a given capital stock in the second period the admissibility restrictions in the debt regime thus are sufficiently loose for the equilibrium allocation in the social security regime to be implementable in the debt regime. At the same time, these admissibility restrictions are sufficiently tight that any allocation that can be implemented in the debt regime can also be implemented in the social security regime. With the choice set of political decision makers in the debt regime thus being sufficiently large but not too large, and since political objectives only depend on allocations, the debt policy replicating the equilibrium social security policy must be an equilibrium outcome in the debt regime. Politico-economic equivalence in the second period follows.

Proceeding to the first period, the brute force characterization of the debt equilibrium would have to deal with the fact that the equilibrium price of debt reflects expectations about the equilibrium repayment choice z'_1 , workers are indifferent between accumulating capital or buying debt, and government spending is financed out of taxes and funds raised from newly issued debt. Focusing on the choice sets of political decision makers again simplifies the task although continuation policy functions now have to be taken into account as well. From the perspective of political decision makers in the debt regime, these continuation policy functions are given by *equivalent continuation policy functions* $\tilde{\tau}'_1(\cdot)$ and $\tilde{z}'_1(\cdot)$ that implement the same competitive equilibrium allocation as the equilibrium policy choice in the social security regime.⁵ This follows directly from the finding above that the equilibrium debt policy choice in the second period is economically equivalent to the equilibrium policy choice in the social security regime.

Since the continuation policy functions in the debt regime, $\tilde{\tau}'_1(\cdot), \tilde{z}'_1(\cdot)$, are equivalent to the continuation policy function in the social security regime, $\tau_1(\cdot)$, the allocation supported by $k_0, \tau_1(\cdot)$ and the social security tax rate τ_0^* can also be supported by $k_0, \tilde{\tau}'_1(\cdot), \tilde{z}'_1(\cdot)$ and a debt policy choice (τ'_0, z'_0) . This follows from the economic equivalence relations when the continuation policy functions are evaluated at k_1^* . In fact, this debt policy choice is admissible.⁶ The economic equivalence relations also imply that any allocation that can be supported by $k_0, \tilde{\tau}'_1(\cdot), \tilde{z}'_1(\cdot)$ and some admissible debt policy choice (τ'_0, z'_0) can equally be supported by k_0 , the continuation policy function $\tau_1(\cdot)$ and an admissible social security policy choice τ_0 .⁷ Conditional on the same initial capital stock, the equilibrium allocation in the social security regime therefore can also be implemented

⁴We denote by $w_t(k_t)$ and $R_t(k_t)$ the equilibrium wage and gross interest rate, respectively, as a function of the capital stock.

⁵The equivalent continuation policy functions are given by $\tilde{\tau}'_1(\cdot) = \tau_1(\cdot)$ and $\tilde{z}'_1(\cdot) = \tau_1(\cdot)w_1(\cdot)\nu/\bar{b}'_1$. Continuation policy functions determine policy choices over the complete continuation history. In the example here, this history only comprises the second period.

⁶It is given by $(\tau'_0, z'_0) = (\tau_0^* - \frac{\tau_1(k_1^*)w_1(k_1^*)\nu}{R_1(k_1^*)w_0(k_0)}, \tau_0^*w_0(k_0)\nu/\bar{b}'_0)$. Note that $\tau_0^* \geq 0$ implies $z'_0 \geq 0$.

⁷The policy choice is given by $\tau_0 = \frac{z'_0(k_0)\bar{b}'_0}{w_0(k_0)\nu}$. Note that $z'_0 \geq 0$ implies $\tau_0 \geq 0$.

in the debt regime, and any allocation that can be implemented in the debt regime can also be implemented in the social security regime. With the choice set of political decision makers in the debt regime thus being sufficiently large but not too large, politico-economic equivalence follows.

This simple example illustrates the general logic behind the politico-economic equivalence conditions derived in section 4. In line with the above reasoning, these conditions require that the choice sets of political decision makers in the new regime (the debt regime in the example above) are sufficiently large, but not too large. To compare these choice sets, we heavily rely on economic equivalence relations. A revealed preference argument then implies politico-economic equivalence. A complication we have to address in section 4 relates to state spaces. In the example considered here, both regimes feature the same endogenous state—the capital stock—but this need not be the case in general. As a prerequisite for the definition of equivalent continuation policy functions and thus, the choice set conditions we therefore require that elements of the state spaces under different regimes can be related to each other. As shown in section 5, both the condition on state spaces and the choice set conditions hold in some of the applications we consider but not in others.

3 Economics

We consider a deterministic, discrete-time economy with time indexed by $t = 0, 1, \dots$ ⁸ The economy is inhabited by a government, households, and firms which are owned by households. Let \mathcal{I} denote the set of household types or households, for short, and let $\mathcal{I}_t \subseteq \mathcal{I}$ denote the set of households that are economically active (“alive”) in period t ; households are indexed by $i \in \mathcal{I}$. Let \mathcal{J} denote the set of firms; firms are indexed by $j \in \mathcal{J}$.

Preferences of household $i \in \mathcal{I}$ in period t are described by the function Ω_t^i . We do not impose any restrictions on preferences except that they are defined over allocations. This rules out the possibility that households care about the choice of policy instruments itself as opposed to the consequences of this choice for the equilibrium allocation. Firms maximize profits, and their production functions do not depend on policy instruments either.

Central objects in our analysis are states, policy sequences and the competitive equilibrium allocations that the states and policy sequences support. Let μ_t denote the *state* in period t . This state may encompass both economic and political restrictions. Examples of the former type include physical or human capital stocks; household choices in previous periods if current or future taxes are functions of those; or financial asset holdings. Examples of political state variables include policy instruments chosen in the past by political decision makers that could commit. For instance, under (partial) commitment, the contemporaneous income tax schedule or the repayment rate on maturing government

⁸The extension to the stochastic case is immediate if the number of states in each period is finite. Our analysis applies both to finite and infinite horizons. We distinguish between the two cases where necessary.

debt may be part of the state. We denote state variables that determine production possibilities (at the aggregate level and/or the level of individual firms) by $\mu_t^J \subseteq \mu_t$, and we denote by $\mu_t^i \subseteq \mu_t$ the state of household $i \in \mathcal{I}$.

Let p^{t-1} denote the *policy sequence* from period t onward (that is, after period $t - 1$). The policy sequence includes all policy instruments that are chosen in period t or later. Absent commitment on the part of political decision makers, p^{t-1} could for example include contemporaneous and future income tax schedules or the repayment rate on government debt maturing in period t . A policy sequence p^{t-1} is *feasible* conditional on μ_t if the pair (μ_t, p^{t-1}) supports a competitive equilibrium. We denote a competitive equilibrium allocation supported by a pair of state and feasible policy sequence by $CE(\mu_t, p^{t-1})$ and equilibrium prices by q_t . The competitive equilibrium implies a law of motion of the state that we denote by

$$\mu_{s+1} = \hat{g}_s(\mu_s, p^{s-1}), \quad s \geq t. \quad (1)$$

Equivalence classes in economic models typically relate policy sequences (and, at least implicitly, states) that support the same equilibrium allocation. For example, when taxes are not distorting and other conditions are satisfied, the Ricardian equivalence proposition defines such a class of tax policies with constant present discounted tax revenues. We refer to the equivalence of policy sequences (and states) as *economic equivalence*, as defined below. The definition allows for the possibility that a state and policy sequence support multiple equilibrium allocations.

Definition 1. A state and policy sequence, (μ_t, p^{t-1}) , is *economically equivalent* to another state and policy sequence, (μ'_t, p'^{t-1}) , if

- i. p^{t-1} is feasible conditional on μ_t ;
- ii. p'^{t-1} is feasible conditional on μ'_t ;
- iii. the set of competitive equilibrium allocations supported by (μ_t, p^{t-1}) and the set of competitive equilibrium allocations supported by (μ'_t, p'^{t-1}) are identical.

The direct approach to establishing economic equivalence of (μ_t, p^{t-1}) and (μ'_t, p'^{t-1}) consists of characterizing the competitive equilibrium allocations for each pair (if they exist) and comparing them. An indirect approach relies on a comparison of choice sets, as formalized in proposition 1 below. Let $\mathcal{B}_t^i(\mu_t^i, p^{t-1}, q_t, e_t^i)$ denote the choice set of household i as of period t as a function of the household's state variables, the policy sequence, the equilibrium prices supported by (μ_t, p^{t-1}) and the household's endowments, e_t^i .⁹ The choice set encompasses all restrictions imposed by the dynamic and intertemporal budget constraints as well as other constraints, for instance the consumption set or quotas instituted by policy. The choice set of firms is defined by their production functions and, potentially, restrictions imposed by policy on the level of inputs and/or outputs.

⁹For simplicity, we assume here that (μ_t, p^{t-1}) supports a unique equilibrium allocation and set of equilibrium prices. For a household not yet active in period t , the state μ_t^i is empty. The endowment vector e_t^i captures, for example, wealth at birth or time endowments.

Proposition 1. Consider a state and policy sequence, (μ_t, p^{t-1}) , that support a competitive equilibrium with allocation $\text{CE}(\mu_t, p^{t-1})$ and prices q_t . Consider a new state and policy sequence, (μ'_t, p'^{t-1}) , that satisfies the following conditions:

- i. state variables that determine production possibilities are identical across states:

$$\mu_t^{\mathcal{J}} = \mu'_t{}^{\mathcal{J}'};$$

- ii. policy restrictions on inputs and/or outputs of firms are identical across policies;
 iii. at the equilibrium prices, households' choice sets are identical:

$$\mathcal{B}_t^i(\mu_t^i, p^{t-1}, q_t, e_t^i) = \mathcal{B}_t^i(\mu'_t{}^i, p'^{t-1}, q_t, e_t^i) \text{ for all } i \in \bigcup_{s \geq t} \mathcal{I}_s;$$

- iv. at the equilibrium allocation and prices, (μ'_t, p'^{t-1}) satisfies the government budget constraints.

Then, (μ'_t, p'^{t-1}) is economically equivalent to (μ_t, p^{t-1}) .

Proof. With household choice sets unchanged and preferences defined over allocations, household choices are unaltered. With firm production functions unaffected by policy, policy restrictions on inputs and/or outputs unchanged, and both prices and state variables determining production possibilities at the firm level unchanged, firm choices are unaltered. With state variables determining production possibilities at the aggregate level unchanged, (new and old) household and firm choices satisfy the resource constraints in period t . Since household and firm choices are unchanged, future state variables $\mu_s^{\mathcal{J}'}$, $s > t$, are unchanged as well, implying that future resource constraints are also satisfied. Moreover, household choices and the government's new policy satisfy the relevant budget constraints. The pair (μ'_t, p'^{t-1}) therefore supports the same competitive equilibrium allocation $\text{CE}(\mu_t, p^{t-1})$.¹⁰ \square

Proposition 1 provides sufficient conditions for economic equivalence. We emphasize the result for two reasons. On the one hand, because it summarizes and extends well known (economic) equivalence results in the literature (e.g., Barro (1974), Sargent (1987, ch. 8), Bassetto and Kocherlakota (2004), Rangel (1997), Niepelt (2005)). On the other hand, because the strategy of comparing choice sets rather than directly characterizing equilibria mirrors the strategy we adopt below when establishing equivalence relations across policy regimes.

¹⁰While supporting the same competitive equilibrium allocation, the two pairs (μ_t, p^{t-1}) and (μ'_t, p'^{t-1}) might support price systems that differ from each other because certain goods or assets may be present in one equilibrium but not in the other. For example, one policy sequence but not the other may involve government debt implying that the price of government debt is an equilibrium object in one but not the other competitive equilibrium.

4 Politics

As discussed in the introduction, policy regimes rather than policy sequences constitute primitives of the analysis when policy is endogenous. In models with an endogenous policy choice, it therefore seems reasonable to consider equivalence classes over policy regimes (and states). In this section, we define equivalence of policy regimes and derive sufficient conditions for it. In line with the maintained assumption that household preferences are functions of allocations and thus, do not directly depend on policy, we assume that the same holds true for the objective functions maximized by political decision makers, $\Omega_t(\cdot)$.

Our analysis applies to settings where policy is chosen once and for all (as, for example, when a benevolent government chooses a Ramsey policy) and to environments with sequential policy choice. To keep the notation simple, we present the case where policy choices are made in every period. As we discuss below, the case with full commitment or intermediate cases require slight adjustments in the definition of politico-economic equilibrium and the politico-economic equivalence conditions. Specifically, the equivalence conditions then only need to be satisfied in periods when policy choices actually are made.

In period t , political decision makers are confronted with state μ_t and opt for a *policy choice* p_t . The private sector then learns about this policy choice and anticipates the *continuation policy choice* p^t . Jointly, the policy choice and continuation policy choice form the policy sequence $p^{t-1} \equiv (p_t, p^t)$. If this policy sequence is feasible conditional on μ_t , then a competitive equilibrium with allocation $\text{CE}(\mu_t, p^{t-1})$ results.

Let \mathcal{P}_t denote the set of *admissible* policy choices p_t . The restrictions embedded in \mathcal{P}_t specify the policy instruments under the control of political decision makers in period t as well as restrictions on the numerical values of those instruments. A *policy regime* is defined by $\mathcal{P} \equiv \prod_{t \geq 0} \mathcal{P}_t$ and the set of admissible continuation policy choices in period t is denoted by $\mathcal{P}^t \equiv \prod_{s \geq t+1} \mathcal{P}_s$. The *policy space* in period t , \mathcal{Q}_t , is defined as the superset of \mathcal{P}_t that results if restrictions on the numerical values of the policy instruments in \mathcal{P}_t are dropped. Let $\mathcal{Q} \equiv \prod_{t \geq 0} \mathcal{Q}_t$ and $\mathcal{Q}^t \equiv \prod_{s \geq t+1} \mathcal{Q}_s$.

Recall that an admissible continuation policy choice $p^t \in \mathcal{P}^t$ is feasible conditional on μ_{t+1} if the pair supports a competitive equilibrium. Let $\mathcal{P}^t(\mu_{t+1}) \subseteq \mathcal{P}^t$ denote the set of admissible and feasible continuation policy choices conditional on μ_{t+1} . An admissible policy choice $p_t \in \mathcal{P}_t$ is feasible conditional on μ_t if there exists an admissible continuation policy $p^t \in \mathcal{P}^t$ such that $p^{t-1} = (p_t, p^t)$ is feasible conditional on μ_t . Let $\mathcal{P}_t(\mu_t) \subseteq \mathcal{P}_t$ denote the set of admissible and feasible policy choices conditional on μ_t . Every admissible and feasible continuation policy choice at time 0, $p^{-1} = (p_0, p^0) \in \mathcal{P}^{-1}(\mu_0)$, and the allocation it supports correspond with a sequence of the state, $\{\mu_t\}_{t \geq 0}$. Let \mathcal{M}_t denote the set of values that the state may take in period t across all such admissible and feasible continuation policy choices.¹¹

¹¹Leaving physical state variables aside, one might expect the state to summarize the cumulative restrictions on households' budget sets as implied by policy choices in previous periods. These restrictions would be given by the present value of those tax functions that are predetermined. This view is not correct, for two reasons. First, the economic equilibrium conditions do not only include the net present value of the predetermined tax functions (and the physical state variables) but also the contemporaneous

Sequential decision making implies that policy choices in period t are functions of the state with the *policy function* $p_t(\cdot)$ mapping \mathcal{M}_t into $\bigcup_{\mu_t \in \mathcal{M}_t} \mathcal{P}_t(\mu_t) \subseteq \mathcal{P}_t$. Similarly, a *continuation policy function* $p^t(\cdot)$ is a mapping from \mathcal{M}_{t+1} into $\bigcup_{\mu_{t+1} \in \mathcal{M}_{t+1}} \mathcal{P}^t(\mu_{t+1}) \subseteq \mathcal{P}^t$.¹² Finally, from equation (1), we define a conditional law of motion that describes the evolution of the state as a function of the current policy choice and parametrized by the continuation policy function:

$$\mu_{s+1} = g_s(\mu_s, p_s; p^s(\cdot)), s \geq t.$$

For every continuation policy function $p^s(\cdot)$, this conditional law of motion derives from the fixed point condition $\mu_{s+1} = \hat{g}_s(\mu_s, (p_s, p^s(\mu_{s+1})))$.

We are now ready to define politico-economic equilibrium.

Definition 2. A *politico-economic equilibrium* as of period t conditional on $\mu_t \in \mathcal{M}_t$ as well as policy regime \mathcal{P} , denoted as $\text{PEE}(\mu_t, \mathcal{P})$ for short, consists of a sequence of policy functions $\{p_s(\cdot)\}_{s \geq t}$, a sequence of continuation policy functions $\{p^s(\cdot)\}_{s \geq t-1}$, policy choices p^{*t-1} , and a competitive equilibrium allocation $\text{CE}(\mu_t, p^{*t-1})$ such that

- i. policy functions are optimal subject to continuation policy functions:

$$p_s(\mu_s) \in \arg \max_{p_s \in \mathcal{P}_s} \Omega_s(\text{CE}(\mu_s, (p_s, p^s))) \text{ s.t. } p^s = p^s(g_s(\mu_s, p_s; p^s(\cdot))) \text{ for all } \mu_s \in \mathcal{M}_s, s \geq t;$$

- ii. continuation policy functions are consistent with policy functions:

$$p^{s-1}(\mu_s) = (p_s(\mu_s), p^s(g_s(\mu_s, p_s(\mu_s); p^s(\cdot)))) \text{ for all } \mu_s \in \mathcal{M}_s, s \geq t;$$

- iii. equilibrium policy choices are generated by the continuation policy function,

$$p^{*t-1} = p^{t-1}(\mu_t),$$

and (μ_t, p^{*t-1}) support the competitive equilibrium allocation $\text{CE}(\mu_t, p^{*t-1})$.

The definition of politico-economic equilibrium allows policy functions to depend on time. In environments with an infinite horizon and a recursive, time-autonomous structure the policy and continuation policy functions may be time-autonomous functions of the state as well, $\psi(\cdot)$ and $\Psi(\cdot)$ say. The consistency requirement in part ii. of the above definition then reads

$$\Psi(\mu_s) = (\psi(\mu_s), \psi[g(\mu_s, \psi(\mu_s); \Psi(\cdot))], \psi\{g(g(\mu_s, \psi(\mu_s); \Psi(\cdot)), \psi[g(\mu_s, \psi(\mu_s); \Psi(\cdot))]; \Psi(\cdot)\}, \dots)$$

with the conditional law of motion $\mu_{s+1} = g(\mu_s, p_s; \Psi(\cdot))$. Clearly, the function $\psi(\cdot)$ is sufficient for $\Psi(\cdot)$ in that case. The conditional law of motion therefore can be re-expressed

predetermined tax functions since these enter into the government's dynamic budget constraint. Second, absent commitment to debt repayment, the ownership structure of debt also enters the state (although it is in general not under the control of previous governments).

¹²To streamline notation, we define continuation policy functions not only for $t \geq 0$ but also for $t = -1$.

as $\mu_{s+1} = g(\mu_s, p_s; \psi(\cdot))$ and conditions i. and ii. of politico-economic equilibrium can be combined to the fixed point requirement¹³

$$\begin{aligned} \psi(\mu_s) &\in \arg \max_{p_s \in \mathcal{P}_s} \Omega(\text{CE}(\mu_s, (p_s, p^s))) \\ \text{s.t. } p^s &= (\psi[g(\mu_s, p_s; \psi(\cdot))], \psi\{g(g(\mu_s, p_s; \psi(\cdot)), \psi[g(\mu_s, p_s; \psi(\cdot))]; \psi(\cdot))\}, \dots) \end{aligned}$$

for all $\mu_s \in \mathcal{M}_s$ and $s \geq t$.

Returning to the motivating question, consider an “initial” policy regime \mathcal{P} with associated politico-economic equilibrium $\text{PEE}(\mu_0, \mathcal{P})$, and a “new” policy regime \mathcal{P}' . We are interested in conditions that, if satisfied, guarantee politico-economic equivalence as specified in the following definition:

Definition 3. A state and policy regime, (μ_t, \mathcal{P}) , is *politico-economically equivalent* to another state and policy regime, (μ'_t, \mathcal{P}') , if

- i. (μ_t, \mathcal{P}) supports a politico-economic equilibrium $\text{PEE}(\mu_t, \mathcal{P})$ with policy choices p^{*t-1} ;
- ii. (μ'_t, \mathcal{P}') supports a politico-economic equilibrium $\text{PEE}(\mu'_t, \mathcal{P}')$ with policy choices p'^{*t-1} ;
- iii. for each sequence of policy choices p^{*t-1} in i. there exists a sequence of policy choices p'^{*t-1} in ii. such that (μ_t, p^{*t-1}) is economically equivalent to (μ'_t, p'^{*t-1}) , and vice versa.

Note that politico-economic equivalence is defined with respect to pairs of a state and policy regime whereas economic equivalence is defined with respect to pairs of a state and policy sequence. This reflects the fact that the primitives of competitive equilibrium on the one hand and politico-economic equilibrium on the other differ. Note also that definition 3 allows for multiplicity among politico-economic equilibria. Such multiplicity may arise for a unique equilibrium policy choice and equilibrium continuation policy function if the pair supports multiple sequences of the state with different associated equilibrium allocations; or it may arise if the equilibrium policy function and equilibrium continuation policy function themselves are not unique. Importantly, multiplicity of equilibrium allocations conditional on an exogenous policy sequence (as allowed for in definition 1) need not imply multiplicity of politico-economic equilibrium. Intuitively, different allocations typically are associated with different sequences of the state; a given policy function would map those different sequences into different policy choices, undermining the possibility of multiple politico-economic equilibria.

A sufficient condition for politico-economic equivalence is that the choice set of political decision makers in the new regime satisfies two requirements. On the one hand, this choice set must be sufficiently large in the sense that political decision makers in the new regime can implement those competitive equilibria that political decision makers in the initial

¹³To avoid confusion, we keep the subscripts of the objects \mathcal{P}_s and \mathcal{M}_s although the environment is time-autonomous.

regime find optimal to implement, on or off the equilibrium path. On the other hand, the choice set in the new regime must not be too large. In particular, political decision makers in the new regime must not be able to implement competitive equilibria that cannot be implemented in the initial regime. If both requirements are satisfied, then revealed preference (and regime independent preferences) implies that political decision makers in the new regime implement policies that support the same competitive equilibrium as in the initial regime.

As mentioned in section 2 and is clear from the definition of politico-economic equilibrium, political decision makers choose policy subject to four types of constraints. The state; admissibility restrictions; continuation policy functions of their successors; and the requirement that the state and equilibrium policy sequence support a competitive equilibrium. Accordingly, our first politico-economic equivalence condition, condition 1, imposes cross-regime restrictions on the state spaces. If these restrictions are met, equivalent continuation policy functions can be defined. Conditions 2 and 3 then stipulate that the choice sets of political decision makers in the new regime, as implied by the state, admissibility and feasibility restrictions as well as the equivalent continuation policy function, are sufficiently large but not too large.

We start by defining a relation between states under the different regimes.

Definition 4. For a state $\mu_t \in \mathcal{M}_t$ under the policy regime \mathcal{P} , a state μ'_t under the policy regime \mathcal{P}' is an *associated state* if there exists a $p^{t-1} \in \mathcal{Q}^{t-1}$ such that $(\mu_t, p^{t-1}(\mu_t))$ is economically equivalent to (μ'_t, p^{t-1}) .

The set of states under the policy regime \mathcal{P}' that are associated with $\mu_t \in \mathcal{M}_t$ is denoted $\tilde{\mathcal{M}}'_t(\mu_t)$. Note that the continuation policy choice p^{t-1} in definition 4 is constrained by the policy space \mathcal{Q} and not by the policy regime \mathcal{P}' . That is, while the continuation policy choice must contain policy instruments available in the new regime, the numerical values of these instruments do not need to satisfy the admissibility restrictions in the new regime.¹⁴ If $\tilde{\mathcal{M}}'_t(\mu_t)$ is empty for some $\mu_t \in \mathcal{M}_t$ then the policy instruments in the new policy regime are not flexible enough to support the equilibrium allocation given μ_t in the initial policy regime, even disregarding numerical restrictions on the instruments.

Condition 1. The following holds true for all t :

- i. $\mathcal{M}'_t \subseteq \cup_{\mu_t \in \mathcal{M}_t} \tilde{\mathcal{M}}'_t(\mu_t)$;
- ii. if $\text{CE}(\mu_t, p^{t-1}(\mu_t)) \neq \text{CE}(\hat{\mu}_t, p^{t-1}(\hat{\mu}_t))$, $\mu_t, \hat{\mu}_t \in \mathcal{M}_t$, then $\tilde{\mathcal{M}}'_t(\mu_t) \cap \tilde{\mathcal{M}}'_t(\hat{\mu}_t) \cap \mathcal{M}'_t = \emptyset$;
- iii. $\tilde{\mathcal{M}}'_t(\mu_t) \cap \mathcal{M}'_t \neq \emptyset$ for all $\mu_t \in \mathcal{M}_t$.

The first part of condition 1 requires that every state under the new policy regime can be associated with a state under the initial regime, and the second part requires that a state under the new policy regime can be associated with more than one state under

¹⁴For example, if admissibility restrictions in the new regime prescribe non-negative proportional tax rates, μ'_t could be associated with μ_t even if the corresponding p^{t-1} involves proportional but negative tax rates.

the initial regime only if the latter induce identical competitive equilibrium allocations. If these requirements are met, we can define an *equivalent continuation policy function* $\tilde{p}^{t-1}(\cdot)$ that maps the state μ'_t which is associated with μ_t into a continuation policy choice $\tilde{p}^{t-1}(\mu'_t) \in \mathcal{Q}^{t-1}$ such that $(\mu_t, p^{t-1}(\mu_t))$ is economically equivalent to $(\mu'_t, \tilde{p}^{t-1}(\mu'_t))$. Similarly, we can define an *equivalent policy function* $\tilde{p}'_t(\cdot)$ that maps the state μ'_t into a policy choice $\tilde{p}'_t(\mu'_t) \in \mathcal{Q}'_t$ that corresponds to the time- t component of $\tilde{p}^{t-1}(\mu'_t)$. Both functions have domain $\cup_{\mu_t \in \mathcal{M}_t} \tilde{\mathcal{M}}'_t(\mu_t)$.¹⁵

The third part of condition 1 guarantees that for every state under the initial regime, there exists a state under the new regime that is associated with the former. Initial conditions that may be encountered on or off the equilibrium path in the initial regime therefore can be related to initial conditions in the new regime.

We now turn to the conditions regarding the choice sets of political decision makers. Condition 2 formalizes the requirement that the choice set of political decision makers in the new regime be sufficiently large. It postulates that the policy choice in the new regime as prescribed by the equivalent policy function is admissible for any state, on or off the equilibrium path.

Condition 2. The following holds true for all $\mu'_t \in \mathcal{M}'_t$ and all t : $\tilde{p}'_t(\mu'_t) \in \mathcal{P}'_t$.

Condition 3 formalizes the requirement that the choice set not be too large. It stipulates that every competitive equilibrium supported by μ'_t , an admissible policy choice in the new policy regime and the equivalent continuation policy function, can also be supported in the initial regime. To simplify notation, we write $\mu_{t+1}(p_t)$ for the state implied by the conditional law of motion $g_t(\mu_t, p_t; p^t(\cdot))$, leaving the current state and the continuation policy function implicit.

Condition 3. The following holds true for all $\mu'_t \in \mathcal{M}'_t$ and all t , where $\mu'_t \in \tilde{\mathcal{M}}'_t(\mu_t)$, $\mu_t \in \mathcal{M}_t$: If there exists a $p'_t \in \mathcal{P}'_t$ such that μ'_t , p'_t and $\tilde{p}'^t(\cdot)$ support the competitive equilibrium allocation $\text{CE}(\mu'_t, (p'_t, \tilde{p}'^t(\mu'_{t+1}(p'_t))))$, then there exists a $p_t \in \mathcal{P}_t$ such that $(\mu_t, (p_t, p^t(\mu_{t+1}(p_t))))$ is economically equivalent to $(\mu'_t, (p'_t, \tilde{p}'^t(\mu'_{t+1}(p'_t))))$.

Note that the states $\mu_{t+1} = g_t(\mu_t, p_t; p^t(\cdot))$ and $\mu'_{t+1} = g'_t(\mu'_t, p'_t; \tilde{p}'^t(\cdot))$ satisfy $\mu'_{t+1} \in \tilde{\mathcal{M}}'_{t+1}(\mu_{t+1})$ because economic equivalence of $(\mu_t, (p_t, p^t(\mu_{t+1})))$ and $(\mu'_t, (p'_t, \tilde{p}'^t(\mu'_{t+1})))$ implies economic equivalence of $(\mu_{t+1}, p^t(\mu_{t+1}))$ and $(\mu'_{t+1}, \tilde{p}'^t(\mu'_{t+1}))$.

We can now state the politico-economic equivalence result:

Proposition 2. Consider a state and policy regime, (μ_0, \mathcal{P}) with $\mu_0 \in \mathcal{M}_0$, that support a politico-economic equilibrium $\text{PEE}(\mu_0, \mathcal{P})$. Consider a new state and policy regime, (μ'_0, \mathcal{P}') with $\mu'_0 \in \tilde{\mathcal{M}}'_0(\mu_0)$, and suppose that conditions 1–3 are satisfied. Then, (μ_0, \mathcal{P}) is politico-economically equivalent to (μ'_0, \mathcal{P}') .

Proof. We show that there exists a politico-economic equilibrium in the new regime with policy and continuation policy functions $\{\tilde{p}'_t(\cdot), \tilde{p}^{t-1}(\cdot)\}_{t \geq 0}$, policy choices $p^{*-1} \equiv \tilde{p}^{-1}(\mu'_0)$, and the same competitive equilibrium allocation as in $\text{PEE}(\mu_0, \mathcal{P})$.

¹⁵If policy instruments in the new policy regime are redundant then the equivalent continuation policy function and the equivalent policy function generally are correspondences rather than functions. To keep notation simple, we disregard this possibility in what follows.

Conjecture that in the new regime in period t , future policy choices are expected to be determined according to the equivalent continuation policy function $\tilde{p}'^t(\cdot)$. (From condition 1, this function is well defined over the domain \mathcal{M}'_{t+1} .) We claim that the policy function in the new regime then is given by $\tilde{p}'_t(\cdot)$. To verify the claim by contradiction, suppose instead that the policy function is given by another function, $\pi'_t(\cdot)$ say, such that for some $\mu'_t \in \mathcal{M}'_t$ with $\mu'_t \in \tilde{\mathcal{M}}'_t(\mu_t)$, $\mu_t \in \mathcal{M}_t$, the allocation $\text{CE}(\mu'_t, (\pi'_t(\mu'_t), \tilde{p}'^t(\mu'_{t+1}(\pi'_t(\mu'_t))))))$ is strictly preferred over $\text{CE}(\mu'_t, (\tilde{p}'_t(\mu'_t), \tilde{p}'^t(\mu'_{t+1}(\tilde{p}'_t(\mu'_t))))))$ and $\pi'_t(\mu'_t) \in \mathcal{P}'_t$. From condition 3, there exists an admissible policy choice $\pi_t \in \mathcal{P}_t$ in the initial regime such that $(\mu'_t, (\pi'_t(\mu'_t), \tilde{p}'^t(\mu'_{t+1}(\pi'_t(\mu'_t))))))$ is economically equivalent to $(\mu_t, (\pi_t, p^t(\mu_{t+1}(\pi_t))))$. By definition of the policy function, $\text{CE}(\mu_t, p^{t-1}(\mu_t))$ is preferred (at least weakly) over $\text{CE}(\mu_t, (\pi_t, p^t(\mu_{t+1}(\pi_t))))$. Political decision makers in the new regime share this preference, as they only care about allocations. From condition 2, political decision makers in the new regime can support the former equilibrium by choosing $\tilde{p}'_t(\mu'_t)$ rather than $\pi'_t(\mu'_t)$. This establishes the desired contradiction and thus, verifies the claim.

We conclude that for all $\mu'_t \in \mathcal{M}'_t$ and all t , political decision makers in the new regime implement policy choices according to the policy function $\tilde{p}'_t(\cdot)$ if the continuation policy function $\tilde{p}'^t(\cdot)$ is expected. We show next that such expectations are consistent with equilibrium. As noted earlier, $\mu'_t \in \tilde{\mathcal{M}}'_t(\mu_t)$ as well as economic equivalence of $(\mu_t, p^{t-1}(\mu_t))$ and $(\mu'_t, (\tilde{p}'_t(\mu'_t), \tilde{p}'^t(\mu'_{t+1}(\tilde{p}'_t(\mu'_t))))))$ implies that $\mu'_{t+1} \in \tilde{\mathcal{M}}'_{t+1}(\mu_{t+1})$. By induction, the above argument for period t therefore extends to subsequent periods and the conjectured expected continuation policy functions are consistent with the policy functions governing actual policy choices. Accordingly, the functions $\tilde{p}'_t(\cdot)$ and $\tilde{p}'^t(\cdot)$ satisfy the conditions of politico-economic equilibrium.

Economic equivalence of $(\mu'_0, (\tilde{p}'_0(\mu'_0), \tilde{p}'^0(\mu'_1(\tilde{p}'_0(\mu'_0))))))$ and $(\mu_0, (p_0(\mu_0), p^0(\mu_1(p_0(\mu_0))))))$ implies that the equilibrium policy choices in the new policy regime support the same competitive equilibrium allocation as in the initial policy regime. The result then follows. \square

Three remarks are in order. First, conditions 1–3 are sufficient for politico-economic equivalence but not all three conditions are necessary. Failure of condition 2 necessarily undermines politico-economic equivalence since it implies that equivalent policy choices in the new regime are not admissible. Failure of condition 1 implies that equivalent continuation policy functions cannot be defined or that it might be impossible to relate initial conditions across regimes. While our strategy to prove equivalence cannot be pursued in this case, equivalence nevertheless may hold. Failure of condition 3 implies that some allocations may only be implementable in the new regime such that the choice set of political decision makers in the new regime is not a subset of the choice set in the initial regime. Equivalence still may hold since the *equilibrium* allocation in the new regime may be implementable in the initial regime as well.

Second, conditions 1–3 can sometimes easily be verified even without prior knowledge of the politico-economic equilibrium in one regime. We show this in the context of several examples in the applications section. In these examples, the endogenous state coincides across regimes and it can directly be seen that the same holds true for the state spaces, implying that condition 1 is satisfied. Moreover, in these examples *any* admissible policy choice in either regime maps into an admissible, economically equivalent policy choice in

the other regime. The requirements of condition 2, which relates to *equilibrium* policy choices, and of condition 3 therefore must be satisfied as well, and this conclusion holds for arbitrary political aggregation mechanisms.

Finally, as mentioned earlier, proposition 2 applies in the special case where policy choices are made once and for all, as for example when a benevolent government chooses a Ramsey policy. In this special case, $\mathcal{P}_0 = \mathcal{P}$ and the definition of politico-economic equilibrium requires that political decision makers choose a policy sequence p^{*-1} that together with the initial state, supports a competitive equilibrium maximizing their objective function. Moreover, condition 1 reduces to the requirement that $\mu'_0 \in \tilde{\mathcal{M}}'_0(\mu_0)$ (which is assumed in the statement of proposition 2). Finally, since choices are made once and for all, conditions 2 and 3 only require that $\tilde{p}^{*-1} \in \mathcal{P}'$ and that for every feasible p'^{-1} , there exists an admissible p^{-1} such that (μ_0, p^{-1}) is economically equivalent to (μ'_0, p'^{-1}) .

5 Applications

We now show how the theoretical framework developed above can be put to work. We consider three applications, relating to social security reform, tax-smoothing policies and measures to correct externalities. Unless otherwise noted, we let w_t , l_t and l_t^i denote the wage, labor supply of the representative worker, and labor supply of worker i in period t , respectively; $r_{t,s}$ the inverse of the gross interest rate between periods t and s , $s \geq t$; k_t the capital stock per worker; and ν_t the ratio of workers to retirees.

5.1 Social Security Reform

As noted in the introduction and the introductory example in section 2, pay-as-you-go financed social security policies are economically equivalent to certain debt-and-tax policies. At the same time, observed disagreement among political decision makers about the merits of social security reform suggests that “privatizing” and “pre-funding” social security may result in a change of allocation in politico-economic equilibrium. Our first application examines this apparent contradiction in more detail.

The analysis identifies a basic economic environment—the workhorse overlapping generations model with minimal household heterogeneity and non distorting taxes—that robustly generates politico-economic equivalence, for arbitrary political aggregation mechanisms. In this basic environment, social security reform always is allocation neutral and therefore politically uncontentious. This is no longer the case if realistic extensions of the basic environment are considered. In the first extension, the admissibility restrictions on policy instruments are asymmetrically tight across policy regimes, for example because of natural restrictions on tax rates when taxes are distorting. As a consequence, conditions 2 or 3 may be violated, depending on the political aggregation mechanism in place, and politico-economic equivalence may or may not hold. Since condition 1 is satisfied in this environment, however, economic equivalence relations may nevertheless be employed to construct novel politico-economic theories of government debt based on hypothetical

equilibria in social security regimes with relaxed admissibility restrictions. In the second extension we consider, richer household heterogeneity in combination with lack of commitment implies that the debt ownership structure constitutes a non-trivial state variable. As a consequence, condition 1 does not hold and the politico-economic equivalence result is undermined.

5.1.1 Robust Politico-Economic Equivalence

The economy is inhabited by two-period lived overlapping generations that are homogeneous within cohorts. Workers inelastically supply labor, $l_t = 1$, and accumulate capital; production is neoclassical.

In the social security regime, a proportional labor income tax τ_t funds transfers to retirees, $\nu_t w_t \tau_t$. The admissibility restrictions $\mathcal{P}_t = \{\tau_t | \tau_t \in \mathbb{R}_+\}$ rule out transfers from retirees to workers. In the debt regime, the government repays maturing debt at the rate z'_t , issues an exogenous stock of short-term debt $\bar{b}'_{t+1} > 0$ per retiree and levies a proportional labor income tax τ'_t .¹⁶ The admissibility restrictions $\mathcal{P}'_t = \{(\tau'_t, z'_t) | (\tau'_t, z'_t) \in \mathbb{R} \times \mathbb{R}_+\}$ again rule out transfers from retirees to workers.¹⁷

In the social security regime the state is given by $\mu_t = k_t$, and in the debt regime by $\mu'_t = k'_t$ since \bar{b}'_t is exogenous. Economic equivalence requires identical initial capital stocks, identical government cash flows in each period, and identical present values of net tax payments for each cohort. These cross-regime restrictions reduce to

$$\begin{aligned} k'_t &= k_t, \\ z'_s &= \tau_s \nu_s w_s / \bar{b}'_s \text{ for all } s \geq t, \\ \tau'_s &= \tau_s - \frac{r_{t,s+1} \tau_{s+1} \nu_{s+1} w_{s+1}}{r_{t,s} w_s} \text{ for all } s \geq t. \end{aligned}$$

To assess politico-economic equivalence, note first that condition 1 is satisfied regardless of whether the initial regime is the social security regime or the debt regime. This can be seen as follows: For any state $k_t \in \mathcal{M}_t$ and the corresponding equilibrium continuation policy sequence $\tau^{*t-1} = \tau^{t-1}(k_t)$ in the social security regime, there exists a state under the debt regime, $k'_t = k_t$, and a continuation policy sequence $(\tau'^{t-1}, z'^{t-1}) \in \mathcal{Q}'^{t-1}$ in the debt regime such that the two pairs are economically equivalent. Associated states therefore satisfy $k'_t = k_t$ for all $k_t \in \mathcal{M}_t$. In fact, the relevant continuation policy sequence in the debt regime is admissible, $(\tau'^{t-1}, z'^{t-1}) \in \mathcal{P}'^{t-1}$. As long as $k'_0 = k_0$, any state that may result under some feasible policy sequence in the social security regime therefore

¹⁶The quantity of debt can be normalized without loss of generality. To see this, note that a competitive equilibrium pins down the market value of newly issued debt as well as total debt repayment. Multiplying debt prices and repayment rates by a constant and dividing the total stock of debt by the same constant therefore does not affect the competitive equilibrium conditions. Moreover, adopting the proposed normalization does not constrain the effective choice set of political decision makers—with $\bar{b}_t > 0$, the amount of resources transferred to bond holders can fully be controlled by the choice of repayment rate—nor does it constrain the ownership structure of government debt and thus, the relative exposure of different groups of households to public debt.

¹⁷To streamline notation, we do not distinguish between debt repayment in periods $t \geq 1$ and “debt repayment” to retirees in the initial period who simply receive a transfer.

may also result under a feasible policy sequence in the debt regime, $\mathcal{M}_t \subseteq \mathcal{M}'_t$. Since the reverse argument holds as well, $\mathcal{M}'_t \subseteq \mathcal{M}_t$. We conclude that $k'_0 = k_0$ implies $\mathcal{M}_t = \mathcal{M}'_t$.

Condition 2 is satisfied as well regardless of the initial regime. This follows immediately from the fact that equivalent continuation policy sequences are admissible, both in the debt regime and the social security regime, as argued above. In fact, a stronger condition than condition 2 is satisfied because the equivalent continuation policy sequences of arbitrary admissible policy sequences (not only the equilibrium continuation policy sequence) are admissible, and this holds true regardless of the initial regime. But this stricter version of condition 2 for the debt regime as the initial regime is equivalent to condition 3 for the social security regime as the initial regime, and vice versa. As long as $k'_0 = k_0$, conditions 1–3 therefore are all satisfied and politico-economic equivalence is guaranteed. Since this conclusion does not rely on assumptions about the political aggregator function, politico-economic equivalence is guaranteed for *any* political aggregator function. Moreover, politico-economic equivalence also robustly holds with symmetric commitment.¹⁸

Forni (2005) analyzes the baseline setup under the assumption that a median voter is politically decisive. He shows that, for some parameter constellations, multiple equilibria with self-fulfilling expectations may exist in which strictly positive social security tax rates are sustained. Contemporaneous political decision makers support strictly positive taxes if they expect future social security benefits to be a decreasing function of the capital stock.¹⁹ From the above discussion, we can immediately conclude that the social security regime in Forni’s (2005) model is politico-economically equivalent (conditional on some initial capital stock) to a debt regime.

Boldrin and Rustichini (2000) analyze the baseline setup augmented by a trigger strategy under the assumption that a young median voter is politically decisive. They assume that political decision makers are confronted with a “suggested” social security tax rate as determined by their predecessors, and that political decision makers choose an updated suggestion for their successors in addition to the actual social security tax rate. Equilibrium policy choices depend on whether the policy choice in the preceding period conformed with the respective suggestion or not. Boldrin and Rustichini (2000) show that this trigger strategy provides sufficiently strong incentives for political decision makers to support equilibria with strictly positive social security transfers.

In the working paper (Gonzalez-Eiras and Niepelt, 2012) we demonstrate that proposition 2 extends to environments with trigger strategies if a corresponding trigger strategy in the new regime can be appropriately specified.²⁰ In Boldrin and Rustichini’s (2000) environment, such a corresponding trigger strategy could be based on a comparison of

¹⁸With one-period, symmetric commitment the state in the social security regime is given by $\mu_t = (k_t, \tau_t)$ and in the debt regime by $\mu'_t = (k'_t, z'_t)$. The economic equivalence relations continue to hold, with the exception that τ_t and z'_t are part of the respective states rather than the continuation policy sequences from period $t - 1$ onwards. With this qualification, and as long as $k'_0 = k_0$ and $z'_0 = \tau_0 \nu_0 w_0 / \bar{b}'_0$, all arguments establishing the validity of conditions 1–3 in the case without commitment extend.

¹⁹Forni (2005) considers the case where the initial capital stock evolves within a certain range of parameter-dependent values. See Gonzalez-Eiras (2011) for a general characterization of equilibrium.

²⁰Loosely speaking, suggested policies in the initial and new regimes need to be economically equivalent, conditional on appropriately defined states.

actually chosen and suggested debt repayment rates. Conditional on this corresponding trigger strategy and appropriate initial states, the social security regime in Boldrin and Rustichini's (2000) model therefore is politico-economically equivalent to a debt regime.

5.1.2 Asymmetrically Tight Admissibility Restrictions

In the setup with robust politico-economic equivalence, the equivalent continuation policy sequences of arbitrary admissible policy sequences are themselves admissible. This is no longer the case if institutional constraints generate asymmetrically tight admissibility restrictions across regimes. We illustrate this fact in a simple extension of the baseline setup with elastic labor supply.

Maintaining the assumption of proportional labor income taxes, economic equivalence now also requires that marginal tax rates be identical across regimes. This necessitates a second tax instrument. Let θ_t and θ'_t denote a second proportional tax in the social security and debt regime, respectively, whose proceeds are redistributed lump sum among workers. Ruling out lump-sum taxes on workers implies non-negative values for these tax rates, and ruling out lump-sum taxes on retirees implies non-negative social security benefits or debt repayment rates. The admissibility restrictions in the two regimes therefore are given by $\mathcal{P}_t = \{(\tau_t, \theta_t) | (\tau_t, \theta_t) \in \mathbb{R}_+^2\}$ and $\mathcal{P}'_t = \{(\tau'_t, \theta'_t, z'_t) | (\tau'_t, \theta'_t, z'_t) \in \mathbb{R} \times \mathbb{R}_+^2\}$. The state in the social security regime is $\mu_t = k_t$, and in the debt regime $\mu'_t = k'_t$ since \bar{b}'_t is exogenous. Economic equivalence requires

$$\begin{aligned} k'_t &= k_t, \\ z'_s &= \tau_s \nu_s w_s l_s / \bar{b}'_s \text{ for all } s \geq t, \\ \tau'_s &= \tau_s - \frac{r_{t,s+1}}{r_{t,s}} \frac{\tau_{s+1} \nu_{s+1} w_{s+1} l_{s+1}}{w_s l_s} \text{ for all } s \geq t, \\ \theta'_s &= \theta_s + \frac{r_{t,s+1}}{r_{t,s}} \frac{\tau_{s+1} \nu_{s+1} w_{s+1} l_{s+1}}{w_s l_s} \text{ for all } s \geq t, \end{aligned}$$

such that marginal tax rates are identical across regimes.

Every possible state in either of the two regimes is associated with a unique state in the other regime, $\tilde{\mathcal{M}}'_t(k_t) = k_t$ and $\tilde{\mathcal{M}}_t(k'_t) = k'_t$. Moreover, in either of the two regimes, the set of states in period t that can be attained by feasible policies ranges from zero (the capital stock subject to confiscatory taxation) to a maximum value, $\bar{k}_t(k_0)$ or $\bar{k}'_t(k'_0)$. Since the latter results in the absence of any taxation (assuming that consumption is a normal good), we have $\bar{k}_t(k_0) = \bar{k}'_t(k'_0)$ as long as $k_0 = k'_0$. As a consequence, $\mathcal{M}_t = \mathcal{M}'_t$ and condition 1 is satisfied, regardless of whether the initial regime is the social security regime or the debt regime.

For every admissible policy sequence (and thus, for the equilibrium policy sequence under any political aggregator function) in the social security regime the equivalent policy sequence in the debt regime is admissible as well because $\tau_s, \theta_s \geq 0$ for all $s \geq t$ implies that $z'_s, \theta'_s \geq 0$ and $\tau'_s \in \mathbb{R}$ for all $s \geq t$. Condition 2 therefore is satisfied for arbitrary political aggregator functions if the initial regime is the social security regime. In contrast, there exist admissible and feasible policy sequences in the debt regime such that the equivalent

policy sequences in the social security regime are not admissible. In particular, if the political aggregator function implies $z'_{t+1} > 0$ under the continuation policy function $p'^t(\cdot)$, then one feasible policy choice p'_t involves zero contemporaneous taxes, $\tau'_t + \theta'_t = 0$, but strictly positive debt repayment, $z'_t > 0$, which can be financed out of new debt issues. The economically equivalent policy in the social security regime, which satisfies $\theta_t = \tau'_t + \theta'_t - z'_t \bar{b}'_t / (\nu_t w'_t l'_t) < 0$, is not admissible in this case. Condition 2 therefore fails to hold for arbitrary political aggregator functions if the initial regime is the debt regime. As a consequence, condition 3 also fails to hold for arbitrary political aggregator functions if the initial regime is the social security regime.

Gonzalez-Eiras and Niepelt (2008) analyze a social security regime in the model with endogenous labor supply under the assumption that preferences are aggregated through probabilistic voting. They show that strictly positive social security transfers are sustained in politico-economic equilibrium. In a debt regime, these transfers would correspond with a positive debt repayment rate, and as discussed above, politico-economic equivalence therefore cannot be guaranteed.²¹ The opposite conclusion would follow if one assumed that a young median voter is politically decisive. Restricting attention to (the limit of) a finite horizon economy, equilibrium social security transfers then would equal zero in all periods and as a consequence, politico-economic equivalence would be guaranteed.²²

Even if admissibility restrictions are asymmetrically tight, the economic equivalence relations may be used to easily construct politico-economic theories of government debt. To that end, one may first characterize the politico-economic equilibrium in a social security regime with relaxed admissibility restrictions and then interpret the equilibrium allocation in the “relaxed” social security regime as the equilibrium allocation in the debt regime. For example, it is straightforward to characterize the equilibrium in Gonzalez-Eiras and Niepelt’s (2008) model subject to the admissibility restrictions $(\tau_t, \theta_t) \in \mathbb{R}_+ \times \mathbb{R}$ rather than $(\tau_t, \theta_t) \in \mathbb{R}_+^2$. If the debt policy sequences that are economically equivalent to the equilibrium policy sequences in the relaxed social security regime are admissible then conditions 2 and 3 are satisfied, politico-economic equivalence holds, and the politico-economic equilibrium with government debt is fully characterized.

5.1.3 Richer Household Heterogeneity

If household heterogeneity may be reflected in a non-trivial debt ownership structure, and absent commitment to the repayment rate, politico-economic equivalence generally cannot be guaranteed. Consider an environment with debt where households within a cohort are

²¹In fact, politico-economic equivalence fails since the tax rate θ_t sometimes is in a corner. Relaxing the non-negativity constraint $\theta_t \geq 0$ therefore would result in a different equilibrium allocation, and this different allocation would also be supported in the debt regime where the admissibility restrictions are less tight.

²²A continuation policy in the social security regime with tax rates equal to zero is economically equivalent to a continuation policy in the debt regime with repayment rates of zero. But if future debt repayment rates equal zero, any feasible policy in the debt regime must finance contemporaneous debt repayment out of current taxes, $z'_t = \tau'_t \nu_t w'_t l'_t / \bar{b}'_t$. The admissibility restriction $z'_t \geq 0$ then implies $\tau'_t \geq 0$. As a consequence, the economically equivalent policy choice in the social security regime satisfies $\tau_t = \tau'_t \geq 0$ and $\theta_t = \theta'_t \geq 0$ which satisfies all admissibility restrictions.

non-representative or households live for more than two periods. The debt ownership structure then is endogenous (in contrast to a setup with homogeneous, two-period lived households) and without commitment, it constitutes a state variable because it determines the extent to which a change in the repayment rate affects the wealth distribution.²³ The set of implementable policies in the debt regime then varies with an endogenous state variable that is not present in the social security regime. Evidently, this discrepancy would tend to undermine condition 3. More fundamentally, it undermines condition 1.

To see how an endogenous, non-trivial debt ownership structure undermines condition 1 in the absence of commitment, consider a state $\mu_t = \{a_t^i\}_{i \in \mathcal{I}_t}$ in a social security regime.²⁴ This state is associated with the state $\mu'_t = \{a_t^{i'}, b_t^{i'}\}_{i \in \mathcal{I}_t}$ in a debt regime if the following conditions are satisfied: $\int_{i \in \mathcal{I}_t} a_t^i di = \int_{i \in \mathcal{I}_t} a_t^{i'} di$ (identical capital stocks); $\int_{i \in \mathcal{I}_t} (b_t^{i'} - \bar{b}_t^i) di = 0$ (debt market clearing); and there exists a debt continuation policy $p'^{t-1} \in \mathcal{Q}^{t-1}$ such that households' budget sets are identical under the two regimes,

$$a_t^i - \text{NTF}_t^i(\cdot; \mu_t, p^{t-1}(\mu_t)) = a_t^{i'} + b_t^{i'} z_t^i - \text{NTF}_t^i(\cdot; \mu'_t, p'^{t-1}) \text{ for all } i \in \mathcal{I}_s, s \geq t.$$

Here, $\text{NTF}_t^i(\cdot; \mu_t, p^{t-1})$ denotes the “net tax function” of household i as of period t . This net tax function gives the present value of taxes net of transfers of household i as a function of i 's choices in period t and later; it is parameterized by the state (which includes asset holdings), the continuation policy as well as prices and interest rates which in turn depend on the state and the continuation policy through the competitive equilibrium. Note that any associated $\mu'_t \in \mathcal{M}'_t$ if $\mu'_0 \in \tilde{\mathcal{M}}'_0(\mu_0)$.

Suppose that the state μ_t in a social security regime is associated with a state μ_t^1 in a debt regime. (If no such state μ_t^1 exists, then condition 1 iii. is violated and we do not need to proceed.) Suppose further that a different state $\hat{\mu}_t$ in the social security regime—supporting a different competitive equilibrium allocation, but with the same capital stock as μ_t (that is, $\int_{i \in \mathcal{I}_t} a_t^i di = \int_{i \in \mathcal{I}_t} \hat{a}_t^i di$ and $\text{CE}(\mu_t, p^{t-1}(\mu_t)) \neq \text{CE}(\hat{\mu}_t, p^{t-1}(\hat{\mu}_t))$)—is associated with some other state μ_t^2 in the debt regime. (Ditto.) Letting $e_t^{i'1} \equiv a_t^{i'1} + b_t^{i'1} z_t^1$ and $e_t^{i'2} \equiv a_t^{i'2} + b_t^{i'2} z_t^2$, consider the $\mu'_t \equiv \{a_t^{i'}, b_t^{i'}\}_{i \in \mathcal{I}_t}$ satisfying $a_t^{i'} = (e_t^{i'2} z_t^1 - e_t^{i'1} z_t^2) / (z_t^1 - z_t^2)$ and $b_t^{i'} = (e_t^{i'1} - e_t^{i'2}) / (z_t^1 - z_t^2)$. Generically, such a $\mu'_t \in \mathcal{M}'_t$ exists.²⁵ Moreover, it satisfies

$$\left. \begin{aligned} a_t^{i'} + b_t^{i'} z_t^1 &= a_t^{i'1} + b_t^{i'1} z_t^1 \\ a_t^{i'} + b_t^{i'} z_t^2 &= a_t^{i'2} + b_t^{i'2} z_t^2 \end{aligned} \right\} \text{ for all } i \in \mathcal{I}_s, s \geq t,$$

as well as $\int_{i \in \mathcal{I}_t} a_t^{i'} di = \int_{i \in \mathcal{I}_t} a_t^{i'1} di$ and $\int_{i \in \mathcal{I}_t} (b_t^{i'} - \bar{b}_t^i) di = 0$. The capital stock in state μ'_t therefore corresponds with the capital stock in state μ_t^1 (or in state μ_t^2); debt

²³With commitment to the repayment rate, debt holdings are not an element of the state (in addition to households' financial assets) since political decision makers cannot affect the relative wealth positions of households by choosing the repayment rate. Cukierman and Meltzer (1989) analyze a model with social security and debt where political decision makers can commit to policy instruments one period in advance. They argue that voters are indifferent between social security and debt policies.

²⁴The state does not separately include the capital stock since the latter equals aggregate private asset holdings.

²⁵The μ'_t exists if $z_t^1 \neq z_t^2$. If the Jacobian of the system of equations relating $(\mu_t, p^{t-1}(\mu_t))$ to $(\mu_t^1, p'^{1,t-1})$ is of full rank then variations in μ_t (e.g., to $\hat{\mu}_t$) generically result in a change of z_t^1 (e.g., to z_t^2). See, for example, Mas-Colell, Whinston and Green (1995, p. 593).

markets clear in state μ'_t ; each household's financial wealth under $(\mu'_t, p'^{1,t-1})$ corresponds to its financial wealth under $(\mu_t^1, p'^{1,t-1})$; and each household's financial wealth under $(\mu'_t, p'^{2,t-1})$ corresponds to its financial wealth under $(\mu_t^2, p'^{2,t-1})$. This implies that state μ'_t is associated with both μ_t and $\hat{\mu}_t$. We conclude that condition 1 ii. necessarily is violated as soon as condition 1 iii. is satisfied (such that a μ_t^1 and μ_t^2 exist) and $z_t^1 \neq z_t^2$ (which holds generically). Clearly, this negative result may be overturned if exogenous restrictions on the ownership structure of debt are imposed.²⁶

The possibility of an endogenous, non-trivial debt ownership structure arises in the environment considered by Tabellini (2000) who analyzes a two-period lived overlapping generations economy with heterogeneous time endowments among young households.²⁷ It also arises in the environment considered by Cooley and Soares (1999) who analyze a four-period lived overlapping generations economy.²⁸ To satisfy condition 1 in these environments and possibly guarantee equivalence, debt holdings could be restricted to be symmetric across retirees (in the former model) or to be targeted to workers in their last period before retirement (in the latter). But even if debt could be issued in accordance with these restrictions, secondary markets could easily compromise those efforts.²⁹

5.2 Tax Smoothing Policies

When taxes distort economic decisions, government debt can be instrumental to reduce the negative welfare implications (Barro, 1979; Lucas and Stokey, 1983). An important economic equivalence result of Bassetto and Kocherlakota (2004) states that, nevertheless, policies that differ with respect to the timing but not the present value of tax collections can be economically equivalent. It is natural to ask whether this economic equivalence result extends to the political sphere. We find that this is only the case for regimes with commitment, as for example in the case of Ramsey policies.

Bassetto and Kocherlakota (2004) show that variations in the timing of distorting tax collections need not alter the equilibrium allocation if taxes on lagged labor income are admissible. Consider for example the case where labor income in period t might either be taxed at rate $\tau_{t,t}$ in period t or both at rate $\tau'_{t,t}$ in period t and at rate $\tau'_{t,t+1}$

²⁶For example, one may restrict debt issuance to be symmetric across certain types of households, or targeted to some but not others, and impose that secondary markets be closed.

²⁷Tabellini (2000) shows that, in a median voter framework with weak intergenerational altruism, a coalition of poor young and old households may sustain a social security system whose size increases with the degree of inequality, but decreases with the rate of population growth. Tabellini (2000) assumes proportional taxes and lump-sum benefits such that lifetime taxes of a household are an affine function of income during young age. Replicating households' budget sets in a debt regime without old-age benefits thus requires an affine tax function.

²⁸Cooley and Soares (1999) assume that the median voter in the initial period chooses a tax rate that serves as time-invariant suggested social security tax rate in all subsequent periods whereas successive median voters only choose between implementing the proposed tax rate or dismantling the social security system forever. Numerically solving a calibrated version of their model, Cooley and Soares (1999) find that the median voter is of age two and sustains positive intergenerational transfers.

²⁹In a different setting where the repayment rate on debt can vary across investors, Broner, Martin and Ventura (2010) show that debt may be reallocated on secondary markets to the politically most influential investors.

in period $t + 1$. If $\tau_{t,t} = \tau'_{t,t} + r_{t,t+1}\tau'_{t,t+1}$, switching from the former to the latter tax policy changes the timing of tax collections and the level of debt but does not alter effective marginal or average tax rates on period t labor income. A policy change of this kind therefore preserves households' budget sets and the equilibrium allocation. More generally, economic equivalence requires

$$\left. \begin{aligned} \tau_{s,s} &= \tau'_{s,s} + \frac{r_{t,s+1}}{r_{t,s}}\tau'_{s,s+1} \\ z_t b_t^i &= z'_t b_t^{i'} - \tau'_{t-1,t} l_{t-1}^{i'} \\ z_s &\text{ satisfies government DBC} \end{aligned} \right\} \text{ for all } i \in \mathcal{I}_s, s \geq t.$$

Consider now the implications for politico-economic equivalence. With full commitment (as for example in the case of Ramsey policies), the admissibility restriction in a regime with taxes on contemporaneous income only is given by $\mathcal{P}_0 = \mathcal{P} = \{(\tau, z) | (\tau_{t,t}, z_t) \in \mathbb{R} \times \mathbb{R}_+\}$, whereas in a regime with taxes on contemporaneous and lagged income, it is given by $\mathcal{P}'_0 = \mathcal{P}' = \{(\tau', \tau'_-, z') | (\tau'_{t,t}, \tau'_{t-1,t}, z'_t) \in \mathbb{R}^2 \times \mathbb{R}_+\}$. Note that any allocation that can be supported by a policy sequence in the former regime can also be supported by a policy sequence in the latter regime, and vice versa, provided that $l'_{-1} = 0$ for all $i \in \mathcal{I}$. In this case, where no inelastic tax base can be taxed, conditions 2 and 3 hold in both directions and economic equivalence of policy sequences translates into politico-economic equivalence of policy regimes as long as $\mu'_0 \in \tilde{\mathcal{M}}'_0(\mu_0)$ (i.e. $b_0^i = b_0^{i'}$ for all i).

When political decisions are taken sequentially, politico-economic equivalence may still hold as long as there is one period commitment to $\tau'_{t-1,t}$. The situation is different without commitment. In this case, the admissibility restrictions in the policy regime without and with taxes on lagged income, respectively, are given by $\mathcal{P}_t = \{(\tau_{t,t}, z_t) | (\tau_{t,t}, z_t) \in \mathbb{R} \times \mathbb{R}_+\}$ and $\mathcal{P}'_t = \{(\tau'_{t,t}, \tau'_{t-1,t}, z'_t) | (\tau'_{t,t}, \tau'_{t-1,t}, z'_t) \in \mathbb{R}^2 \times \mathbb{R}_+\}$. In the former regime, the state is $\mu_t = \{b_t^i\}_{i \in \mathcal{I}_t}$, and in the latter it is $\mu'_t = \{b_t^{i'}, l_{t-1}^{i'}\}_{i \in \mathcal{I}_t}$. Condition 1 ii. fails in this environment because two different states in the former regime can be associated with one and the same state in the latter regime. This can be shown by following a parallel strategy as in section 5.1.3: If μ_t is associated with μ_t^1 and $\hat{\mu}_t$ with μ_t^2 , then the state $\mu'_t = \{b_t^{i'}, l_{t-1}^{i'}\}_{i \in \mathcal{I}_t}$ satisfying

$$\left. \begin{aligned} -z_t^1 b_t^{i'1} + \tau_{t-1,t}^1 l_{t-1}^{i'1} &= -z_t^1 b_t^{i'} + \tau_{t-1,t}^1 l_{t-1}^{i'} \\ -z_t^2 b_t^{i'2} + \tau_{t-1,t}^2 l_{t-1}^{i'2} &= -z_t^2 b_t^{i'} + \tau_{t-1,t}^2 l_{t-1}^{i'} \end{aligned} \right\} \text{ for all } i \in \mathcal{I}_s, s \geq t,$$

is associated with both μ_t and $\hat{\mu}_t$ since each household's financial wealth net of lump sum taxes under $(\mu_t^1, p^{1,t-1})$ and $(\mu_t^1, p^{1,t-1})$ coincides and the same holds true for financial wealth net of lump sum taxes under $(\mu_t^2, p^{2,t-1})$ and $(\mu_t^2, p^{2,t-1})$. Politico-economic equivalence therefore is not guaranteed.

Intuitively, condition 1 ii. is violated in this environment because the tax on lagged income in the latter regime is both non distorting at the time it is levied and a function of a tax base that varies across households. This generates substitutability between debt holdings and the tax base which lies at the source of the condition's violation. Battaglini and Coate (2008) present a politico-economic model with sequential choice of fiscal policy in an environment with tax distortions. Our results indicate that the equilibrium in this

model cannot be re-interpreted as equilibrium in a model where taxes are additionally raised on lagged income.³⁰

5.3 Corrective Taxes versus Cap-and-Trade

In economies with external effects, alternative mechanisms may induce agents to internalize the social consequences of their actions. In the current debate about global warming and the appropriate policy responses to it, corrective taxes and cap-and-trade systems feature prominently among these mechanisms. In this final application, we consider the equivalence properties of those instruments.

Consider an economy with a dynamic production externality. Output y_t is produced with a neoclassical production function, using capital k_t (and potentially other factors, in fixed supply) as input. For simplicity, capital is assumed to fully depreciate after one period. Total factor productivity decreases in lagged output, for example because of a climate externality.

A Pigovian tax regime features a corrective tax τ_t whose revenues are distributed across households according to a sharing rule, $\{\sigma_t^i\}_{i \in \mathcal{I}_t}$, with $\sum_{i \in \mathcal{I}} \sigma_t^i = 1$. In contrast, a cap-and-trade regime features caps on each agent's output level and the permission to trade output permits in competitive markets. To simplify the analysis, we assume that taxes and caps apply with respect to capital inputs rather than output; we denote capital input permits by $\{k_t^i\}_{i \in \mathcal{I}_t}$. For now, we also assume that policy is chosen under commitment. The admissibility restrictions in the Pigouvian tax regime therefore are given by $\mathcal{P}_0 = \mathcal{P} = \{(\tau, \{\sigma_t^i\}_{i \in \mathcal{I}}) | (\tau_t, \{\sigma_t^i\}_{i \in \mathcal{I}}) \in \mathbb{R}_+^{\mathcal{I}+1}, \sum_{i \in \mathcal{I}} \sigma_t^i = 1 \text{ for all } t \geq 0\}$, and the restrictions in the cap-and-trade regime are given by $\mathcal{P}'_0 = \mathcal{P}' = \{\{\bar{k}_t^{i'}\}_{i \in \mathcal{I}} | \{\bar{k}_t^{i'}\}_{i \in \mathcal{I}} \in R_+^{\mathcal{I}} \text{ for all } t \geq 0\}$.

With a Pigovian tax, a household saving k_t^i for production in the subsequent period incurs investment outlays k_t^i and tax payments $k_t^i \tau_t$ while receiving transfers $\sigma_t^i \tau_t \sum_{i \in \mathcal{I}} k_t^i$. With a cap-and-trade system, in contrast, this household incurs outlays for the investment, $k_t^{i'}$, and for the net purchases of permits, $(k_t^{i'} - \bar{k}_t^{i'}) \rho_t$, where ρ_t denotes the market price of a permit. The impact of the two policies on household i 's budget set is identical if $\tau_t (\sigma_t^i \sum_{i \in \mathcal{I}} k_t^i - k_t^i) = \rho_t (\bar{k}_t^{i'} - k_t^i)$ for all $k_t^i \in \mathbb{R}_+$. For a given Pigovian tax policy, an economically equivalent cap-and-trade policy therefore satisfies $\bar{k}_t^{i'} = \sigma_t^i \sum_{i \in \mathcal{I}} k_t^i$. Conversely, for a given cap-and-trade policy, an economically equivalent Pigovian tax policy satisfies $\tau_t = \rho_t$ and $\sigma_t^i = \bar{k}_t^{i'} / \sum_{i \in \mathcal{I}} \bar{k}_t^{i'}$.

These economic equivalence relations imply that conditions 2 and 3 necessarily are satisfied. As a consequence, politico-economic equivalence of the two regulatory regimes follows for arbitrary social welfare functions if $\{k_0^i\}_{i \in \mathcal{I}} = \{\bar{k}_0^{i'}\}_{i \in \mathcal{I}}, y_{-1} = y'_{-1}$. If policy were chosen sequentially, then this conclusion would continue to hold since $\{k_0^i\}_{i \in \mathcal{I}} = \{\bar{k}_0^{i'}\}_{i \in \mathcal{I}}, y_{-1} = y'_{-1}$ implies $\mathcal{M}_t = \mathcal{M}'_t$ for all $t \geq 0$ and thus, that condition 1 is satisfied.

³⁰Yared (2010) presents another politico-economic model with sequential choice of fiscal policy in an environment with tax distortions. In his model, there is only one type of household holding debt, rendering it impossible for a state in the new regime to be associated with more than one state in the initial regime. Condition 1 ii. therefore is satisfied, as is condition 2. In contrast, condition 3 is violated since in the new regime, negative net transfers (beyond non payment on outstanding debt) can be implemented while this is not admissible in the initial regime. Politico-economic equivalence therefore cannot be guaranteed.

Golosov, Hassler, Krusell and Tsyvinski (2011) analyze optimal corrective taxation of carbon emissions under commitment in a growth model when these emissions affect world climate. Their model features a representative infinitely lived agent, two sectors of production and a carbon cycle. While capturing the same type of dynamic externality as the simple example considered above, Golosov et al.’s (2011) model is considerably more complex because the authors aim at estimating the optimal corrective tax—equal to the discounted value of marginal external damages—in a plausibly calibrated quantitative framework.

Our results suggest that a cap-and-trade regime would be politico-economically equivalent in the environment studied by Golosov et al. (2011), as noticed by the authors. Our results also suggest that equivalence would continue to hold with household heterogeneity, provided that emission permits could be allocated among households in a way that replicates the distributive implications of the Pigovian tax and transfer scheme. This might require, for example, that permits are allocated to consumers that do not contribute towards the externality generating production. If constraints were to prevent such an allocation, politico-economic equivalence may fail. We leave an analysis of this topic for future research.

6 Conclusions

We have derived sufficient conditions for politico-economic equivalence of institutions or policy regimes (conditional on initial states). These conditions apply in general dynamic environments with endogenous policy choice, independently of whether this choice occurs sequentially or once and for all, as for example with Ramsey policies. Exploiting relations between economically equivalent policy sequences, the conditions rely on an intuitive comparison of choice sets faced by political decision makers. These choice sets are constrained by the state, admissibility restrictions on policy instruments under the control of political decision makers, continuation policy functions of successive political decision makers, and the requirement that equilibrium policy be feasible. Accordingly, the politico-economic equivalence conditions impose constraints on state spaces and admissibility restrictions.

The conditions do not only provide a useful tool to characterize politico-economic equilibria in “new” policy regimes but they also allow to assess the consequences of institutional change. We have exemplified this in the context of several applications, relating to social security reform, tax-smoothing policies and measures to correct externalities, and we have shown that the politico-economic equivalence conditions can in some cases be applied even without prior knowledge of the politico-economic equilibrium in the “initial” regime. As far as social security reform is concerned, the analysis identifies a class of environments—characterized by minimal household heterogeneity and non distorting taxes—where politico-economic equivalence holds independently of the political aggregation mechanism. But it also makes clear that sufficient heterogeneity among households or differentially tight admissibility restrictions across regimes may undermine equivalence.

It is frequently argued that pre-funding of social security (a shift from a social security regime to a debt regime) may improve outcomes by reducing labor supply distortions. This

argument relies on the assumption, which often remains implicit, that certain competitive equilibria may be supported by admissible debt policies but not by admissible social security policies, that is, the argument presupposes violations of economic equivalence.³¹ Our conclusion regarding the failure of politico-economic equivalence differs from that standard argument but is related. According to our conclusion, political decision makers in a debt regime have larger choice sets. As a consequence, political decision makers may implement more distorting policies in a social security regime than in a debt regime if the restriction to smaller choice sets is binding. In turn, this may generate political support for institutional change towards pre-funding.

When applied to environments with tax distortions, our results make clear that economic equivalence generally does not extend to the political sphere. From a narrow economic point of view, the timing of distorting tax collections may be irrelevant if only the net present value of taxes (or tax functions) enters households' budget sets and determines the equilibrium allocation. From a politico-economic point of view, in contrast, timing is crucial because the ex-post elasticity of the tax base typically is non zero with respect to contemporaneous taxes while it equals zero with respect to taxes levied later in time. When policy is chosen sequentially this difference matters and renders institutional change that temporally decouples income generation and tax collections politically contentious. If policy is chosen once and for all, however, as for example with Ramsey policies, then the difference is irrelevant and politico-economic equivalence holds.

Finally, regarding measures to correct externalities, we have found that regimes with Pigovian taxes and cap-and-trade schemes are politico-economically equivalent, provided that permits can be allocated among households in a way that replicates the distributive implications of the Pigovian tax and transfer scheme.

The applicability of our equivalence conditions extends beyond the particular environments we have considered and is not confined to the realm of fiscal and regulatory policy. Before the background of an appropriately defined equivalence class of policies—be they fiscal, monetary or other—the conditions may be applied to any model featuring an endogenous choice of such policies.

References

- Barro, R. J. (1974), 'Are government bonds net wealth?', *Journal of Political Economy* **82**(6), 1095–1117.
- Barro, R. J. (1979), 'On the determination of the public debt', *Journal of Political Economy* **87**(5), 940–971.
- Bassetto, M. and Kocherlakota, N. (2004), 'On the irrelevance of government debt when taxes are distortionary', *Journal of Monetary Economics* **51**(2), 299–304.

³¹See Feldstein and Liebman (2002) for an overview over the literature and Rangel (1997) for a critical analysis of this argument.

- Battaglini, M. and Coate, S. (2008), ‘A dynamic theory of public spending, taxation, and debt’, *American Economic Review* **98**(1), 201–236.
- Boldrin, M. and Rustichini, A. (2000), ‘Political equilibria with social security’, *Review of Economic Dynamics* **3**, 41–78.
- Broner, F., Martin, A. and Ventura, J. (2010), ‘Sovereign risk and secondary markets’, *American Economic Review* **100**(4), 1523–1555.
- Coleman, W. J. (2000), ‘Welfare and optimum dynamic taxation of consumption and income’, *Journal of Public Economics* **76**(1), 1–39.
- Cooley, T. F. and Soares, J. (1999), ‘A positive theory of social security based on reputation’, *Journal of Political Economy* **107**(1), 135–160.
- Cukierman, A. and Meltzer, A. H. (1989), ‘A political theory of government debt and deficits in a Neo-Ricardian framework’, *American Economic Review* **79**(4), 713–732.
- Díaz-Giménez, J., Giovanetti, G., Marimon, R. and Teles, P. (2008), ‘Nominal debt as a burden on monetary policy’, *Review of Economic Dynamics* **11**, 493–514.
- Feldstein, M. and Liebman, J. B. (2002), Social security, *in* A. J. Auerbach and M. Feldstein, eds, ‘Handbook of Public Economics’, Vol. 4, Elsevier Science, Amsterdam, chapter 32.
- Forni, L. (2005), ‘Social security as Markov equilibrium in OLG models’, *Review of Economic Dynamics* **8**, 178–194.
- Ghiglino, C. and Shell, K. (2000), ‘The economic effects of restrictions on government budget deficits’, *Journal of Economic Theory* **94**, 106–137.
- Golosov, M., Hassler, J., Krusell, P. and Tsyvinski, A. (2011), Optimal taxes on fossil fuel in general equilibrium, Working Paper 17348, NBER, Cambridge, Massachusetts.
- Gonzalez-Eiras, M. (2011), ‘Social security as Markov equilibrium in OLG models: A note’, *Review of Economic Dynamics* **14**(3), 549–552.
- Gonzalez-Eiras, M. and Niepelt, D. (2008), ‘The future of social security’, *Journal of Monetary Economics* **55**(2), 197–218.
- Gonzalez-Eiras, M. and Niepelt, D. (2012), Economic and politico-economic equivalence, Working Paper 12.02, Study Center Gerzensee.
- Lucas, R. E. and Stokey, N. L. (1983), ‘Optimal fiscal and monetary policy in an economy without capital’, *Journal of Monetary Economics* **12**(1), 55–93.
- Mas-Colell, A., Whinston, M. D. and Green, J. R. (1995), *Microeconomic Theory*, Oxford University Press, New York.

- Niepelt, D. (2005), Social security reform: Economics and politics, *in* G. Greulich, M. Lössch, C. Müller and W. Stier, eds, ‘Empirische Konjunktur- und Wachstumsforschung’, Rüegger, Zürich, pp. 181–195.
- Niepelt, D. (2011), Debt maturity without commitment. Mimeo, Study Center Gerzensee.
- Rangel, A. (1997), Social security reform: Efficiency gains or intergenerational redistribution. Mimeo, Harvard University.
- Sargent, T. J. (1987), *Dynamic Macroeconomic Theory*, Harvard University Press, Cambridge, Massachusetts.
- Song, Z., Storesletten, K. and Zilibotti, F. (2012), ‘Rotten parents and disciplined children: A politico-economic theory of public expenditure and debt’, *Econometrica* **forthcoming**.
- Tabellini, G. (2000), ‘A positive theory of social security’, *Scandinavian Journal of Economics* **102**(3), 523–545.
- Yared, P. (2010), ‘Politicians, taxes and debt’, *Review of Economic Studies* **77**(2), 806–840.