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Abstract

We analyze the effect of structural change in the payment sector and of monetary policy on prices. Means of payment are obtained through portfolio choices and commodity sales and "liquified" through velocity choices. Interest rates, intermediation margins and costs of payment instrument use affect portfolios, velocities, liquidity, relative prices, and the aggregate price level. Money is neutral, interest rate policy is not. Scarcer liquidity need not drive up velocity. Payment instruments and velocities generate positive externalities. Commodity price aggregates mis-measure consumer price inflation, distinctly so over the business cycle.

JEL Classification: E31, E41, E44, E52, G11, G23

Keywords: Payments, inflation

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Payments and Prices*

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July 10, 2023

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We analyze the effect of structural change in the payment sector and of monetary policy on prices. Means of payment are obtained through portfolio choices and commodity sales and "liquified" through velocity choices. Interest rates, intermediation margins, and costs of payment instrument use affect portfolios, velocities, liquidity, relative prices, and the aggregate price level. Money is neutral, interest rate policy is not. Scarcer liquidity need not drive up velocity. Payment instruments and velocities generate positive externalities. Commodity price aggregates mis-measure consumer price inflation, distinctly so over the business cycle.

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1 Introduction

Households and businesses hold portfolios of payment instruments with transaction-specific costs and benefits. While checks or a standing order may be ideal to pay one's rent, cash offers added privacy at the risk of theft or loss, and card payments provide complementary services such as credit or discounts. The digital revolution has added new payment channels, from convenient mobile phone based real-time money transfers to slow but more censorship-resistant crypto-asset transfers.

Whatever the payment instruments, ongoing structural change as well as monetary policy alter their costs and benefits. Stiffer competition lowers the margins of payment service providers, monetary policy cyclically varies the opportunity cost of holding money, and the bundling of payments and information collection as well as regulatory pushback affect privacy costs. This raises the question of how payers respond by adjusting their

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portfolios, velocities, and spending habits and what effects this has on relative prices, inflation, and even production. This paper provides a first set of answers to these questions.

Our approach merges theories of money demand and price aggregation in general equilibrium. The money demand side of the model builds on centuries-old quantity-equation reasoning (Hume, 1752; Mill, 1848; Fisher, 1911), allowing for multiple means of payment. Similar to models in the tradition of Baumol (1952) and Tobin (1956) it relates nominal balances to transactions and velocity, which agents choose at a cost. Unlike that tradition, we allow for transactions costs in addition to the opportunity costs of holding money and our model features multiple payment instruments with potentially different costs. Like models in the tradition of Sidrauski (1967) or Clower (1967), it characterizes money demand as the outcome of a portfolio choice problem. However, unlike the former "money-in-the-utility-function" approach, it explicitly represents the payments role of money, and unlike the latter conventional "cash-in-advance" constraint (CIA) framework, it does not fix velocity but allows for money demand conditional on transactions to vary with opportunity costs. Unlike all these models, our framework allows for multiple commodities in addition to assets.

Price aggregation in the model reflects the standard cost minimization problem of households that value a consumption bundle. Unlike in Dixit and Stiglitz (1977), however, the costs of acquiring a commodity exceed its price, for two reasons. First, purchases require means of payment, and the financial return on the latter may be lower than on pure stores of value, as in a standard CIA setting. Second, using means of payment may generate additional costs. We refer to these costs as "leakage costs" that reflect foregone privacy, but they could also arise for many other reasons, e.g., due to the inconvenience of using specific means of payment for particular types of transactions.[1]

These elements imply several new results relative to the standard CIA setting. First, the shadow value of liquidity not only reflects the intermediation margins of payment service providers (which the standard model posits to equal the market interest rate) but also leakage costs. Even in the absence of intermediation margins, or under the Friedman (1969) rule, the need to use means of payment is therefore costly.

Second, the presence of multiple payment instruments and commodity-specific transactions cost introduces a portfolio choice problem for media of exchange. In addition to the standard C-CAPM result for pure stores of value, our framework yields a consumption-based "means-of-payment pricing model" according to which the covariance between return differences (post intermediation margin) and the shadow value of wealth is proportional to differences in leakage costs.

Third, endogenous and asset-specific velocities decouple the stocks of means of payment from their importance as payment instruments. Rather than allocating large portfolio shares to specific means of payment, households can hold them for shorter periods of time for added liquidity.

Fourth, the relation between interest rates, velocities, and shadow values of the good-specific liquidity constraints is nontrivial. In the conventional CIA setting, a higher interest rate tightens the liquidity constraint but does not affect velocity. In our framework,

[1]For an analysis of payments and privacy see Garratt and van Oordt (2021).

velocities typically increase in the spread between market interest rates and returns on payment instruments, but scarcer liquidity need not be associated with higher velocities. This is a consequence of the fact that households have two options to generate liquidity—exchanging pure stores of value for means of payment vs. increasing velocities, and the former option may be cheaper.

Fifth, velocities and payment instruments generate positive externalities. Since households take the inflow of means of payment as given, they perceive a private interest gain from lowering velocity, although gains and losses cancel out in the aggregate. Conversely, they perceive a private liquidity gain from increasing velocity, but the social gains are even larger because higher velocity also increases the inflow of means of payment for other households. Higher velocities thus generate positive externalities along two margins. Similarly, allocating a larger share of wealth to means of payment rather than pure stores of value also generates positive externalities.

Sixth, "money is neutral," but interest rate policy is not, although the model does not feature nominal or real rigidities. When all means of payment and prices are scaled by a common factor, the allocation remains unchanged. Changes in interest rates (reflecting changes in growth or inflation), in contrast, generally affect velocities, leakage costs, liquidity shadow values, and the allocation and price system.

Finally, our framework implies differences between the consumer price index defined as the minimum cost of acquiring a specific consumption bundle and the commonly-computed "naïve" index that solely aggregates goods prices. From a consumer perspective, the relative price between two commodities reflects not only goods prices but also payment costs associated with the purchases. Market clearing therefore forces goods price adjustments in response to changes in the payment sector or interest rates. The naïve index records such changes but does not take into account that consumers bear additional payment costs.

These theoretical results have multiple practical implications. Maybe most important, aggregate price measures (whether assessed comprehensively or naïvely) reflect not only factors such as quantities of goods and monies but also "payment fundamentals" such as intermediation margins, leakage costs, or inflation targets. Structural change in the payment sector thus affects the aggregate price level, as does interest rate policy. To illustrate magnitudes, we compute several examples. One baseline result suggests that over the business cycle, the naïve and comprehensive price indices fluctuate by roughly one percent relative to each other. Another result shows that equilibrium portfolio rebalancing among means of payment can imply nonmonotone comparative statics of the aggregate price level with respect to interest rates.

Most of our analysis considers endowment economies. In an extension, we endogenize production and show that "payment fundamentals" affect equilibrium production and consumption in addition to prices.

Related Literature The paper relates to an extensive literature in monetary economics on CIAs (Robertson, 1933; Tsiang, 1956; Clower, 1967; Grandmont and Younes, 1972) We adopt the timing assumption in Lucas (1982), according to which assets are traded at the beginning of the period before goods transactions take place, not vice versa as in

Lucas (1980) or Svensson (1985). This affords a sharp focus on velocity choice for liquidity reasons.[2]

Lucas (1982) considers an environment with multiple means of payment, one in each country, to study exchange rate determination. In contrast, we focus on multiple means of payment denominated in the same unit of account, which generates a portfolio choice "within currencies." We show that leakage costs can resolve issues of indeterminacy.

In Lucas and Stokey (1983, 1987), households choose between "cash goods" and "credit goods," i.e., between goods whose purchase requires different means of payment. Changes in the costs of holding money shift demands but not relative prices. Similarly, in Prescott (1987), households may purchase goods using cash or bank drafts.[3] In our framework, in contrast, relative goods prices adjust to clear markets, and changes in payment costs therefore affect goods prices. Moreover, in our setting, households choose velocities and do so for multiple means of payment.

In Alvarez and Lippi (2017), households switch between cash and credit payments depending on relative costs, which randomly vary.[4] We allow for multiple payment instruments, velocity choices, additional transaction costs and multiple goods to study the effect of payments on relative prices and the aggregate price level.

Structure of the Paper Section 2 presents a monetary economy with multiple goods, multiple means of payment, endogenous velocities, and leakage costs. We analyze the properties of equilibrium in Section 3 and study comparative statics in Section 4. Section 5 contains an extension to a production economy. Section 6 concludes the paper.

2 The Model

2.1 Agents, Goods, and Assets

We consider a discrete-time endowment economy with a unit measure of infinitely lived homogeneous households. (In Section 5 we consider a production economy.)

There is a set $\mathcal{J}$ of goods consumed in each period; goods are indexed by $j = 1, \ldots, J$. Consumption of the bundle $c \equiv \{c_j\}_{j \in \mathcal{J}}$ aggregates according to the CES function $C \equiv \text{CES}(c)$ and yields instantaneous utility $u(C)$ where u is smooth, strictly increasing and

[2] In Lucas (1980) or Svensson (1985) asset markets open at the end of the period. Risk introduces a precautionary motive to hold money. In Svensson (1985), households are subject to stochastic cash transfers. After a large transfer, the future purchasing power of money falls, the price level rises, and the CIA binds; after a smaller transfer, the constraint does not bind, and velocity drops below unity. In Lucas (1980), households are subject to idiosyncratic preference shocks. The CIA of a household with a low valuation of consumption does not bind, and its velocity falls below unity.

[3] In Lucas and Stokey (1987), producer costs are identical across goods. Moreover, currency receipts due to cash good sales are carried as overnight balances, while invoices due to credit good sales are settled in cash at the beginning of the next day; accordingly, both transaction types generate spendable cash at the same time on the following day. Prescott (1987) imposes symmetry assumptions to guarantee uniform goods prices.

[4] In Alvarez and Lippi (2017), households withdraw cash whenever withdrawal costs are low and spend it until balances are depleted before switching to credit.

concave, and satisfies $\lim_{C\downarrow 0} u'(C) = \infty$. The price of good j is denoted by p_j, and we let $p \equiv \{p_j\}_{j\in\mathcal{J}}$.

There is a set $\mathcal{S}$ of assets in each period; assets are indexed by $s = 1, \ldots, S$. Asset s carries the gross interest rate I_s, which may be history contingent.

Households cannot consume their own endowments (or production) but only the endowments (production) of other households. In equilibrium, households thus sell their endowments (production).

2.2 Household Program

At the beginning of a period, uncertainty about returns and endowments in the period is resolved, and each household is endowed with the consumption good bundle $e \equiv \{e_j\}_{j\in\mathcal{J}}$. The household also has financial wealth w (expressed in terms of the numeraire, dollars).

2.2.1 Portfolio Choice

In the subsequent portfolio choice stage, the household chooses the portfolio $a \equiv \{a_{s0}, \{a_{sj}\}_{j\in\mathcal{J}}\}_{s\in\mathcal{S}}$ (expressed in terms of the numeraire) subject to the budget constraint

$$w = \sum_{s\in\mathcal{S}} \left(a_{s0} + \sum_{j\in\mathcal{J}} a_{sj} \right) + \text{outlays} - \text{dividends}. \tag{1}$$

Here, a_{s0} denotes the quantity of asset s the household holds as a pure store of value, and $a_{sj} \geq 0$ denotes the quantity of asset s that the household will use as means of payment for purchases of good j.

The budget constraint contains two further terms, "outlays" and "dividends." The former represents outlays to service providers that the household incurs to increase velocity or address leakage, as described below. The service providers operate at zero cost, and their profits are immediately transferred back to households as dividends. That is, in equilibrium, outlays = dividends, but an individual household only internalizes the effect of its own actions on the former term.

The typical household in the economy chooses the portfolio $\bar{a} \equiv \{\bar{a}_{s0}, \{\bar{a}_{sj}\}_{j\in\mathcal{J}}\}_{s\in\mathcal{S}}$.

2.2.2 Trading

In the remainder of the period—the trading period, which lasts for one unit of time—the household sells its endowment and purchases goods; both types of transaction are intermediated by a payment service provider that holds, spends and receives means of payment on behalf of the household.

Spending by the typical household occurs uniformly throughout the trading period and symmetrically across all other households and is perfectly anticipated at the portfolio choice stage; the amount $\bar{a}_{sj} \geq 0$ of means of payment s is spent per interval $\bar{\delta}_{sj} \in (0, 1]$ on purchases of good j. As a consequence, the inflow of means of payment that any household receives for its endowment sales also occurs uniformly throughout the trading period. It totals $\sum_{j\in\mathcal{J}} p_j e_j = \sum_{j\in\mathcal{J}, s\in\mathcal{S}} \bar{a}_{sj}/\bar{\delta}_{sj}$. We let $\bar{\delta} \equiv \{\bar{\delta}_{sj}\}_{j\in\mathcal{J}, s\in\mathcal{S}}$.

The household is subject to good-specific liquidity constraints[5] with the Lucas (1982) timing convention: Purchases must be paid for with means of payment that have been acquired at the portfolio choice stage or sufficiently early during the trading period.[6] Specifically, the household may use means of payment of type s that it acquires when selling good j, to pay for its own purchases of good j but only after delay $\delta_{sj} \in (0, 1]$. Inflows that are not spent during the trading period augment financial wealth at the beginning of the following period; see below.

Formally, the constraints read

$$\sum_{s\in\mathcal{S}} a_{sj} + \frac{\bar{a}_{sj}}{\bar{\delta}_{sj}}(1-\delta_{sj}) \geq p_j c_j \quad \forall j \in \mathcal{J}. \tag{2}$$

The left-hand side of each constraint represents the available means of payment; they consist of the household's portfolio allocated to transactions at the portfolio choice stage and the sales receipts that materialize before fraction $1-\delta_{sj}$ of the trading period has passed. The right-hand side represents the outlays for c_j. Note that δ_{sj} and $\bar{\delta}_{sj}$ decouple the stock of means of payment from the transaction flow, as in the quantity theory. The inverses of $\bar{\delta}_{sj}$ and δ_{sj} represent velocity measures.

To reduce the quantity of assets earmarked for payments at the portfolio choice stage the household may increase the liquidity of its sales revenues by lowering δ_{sj}, but this is costly.[7] Letting $\delta \equiv \{\delta_{sj}\}_{j\in\mathcal{J},s\in\mathcal{S}}$, we represent the cost by the function $z(\delta; \bar{a}, \bar{\delta}) \equiv \sum_{j\in\mathcal{J},s\in\mathcal{S}} \tilde{z}_{sj}(\delta_{sj}; \bar{\delta})\bar{a}_{sj}/\bar{\delta}_{sj}$, which is a component of the outlays term in the budget constraint (1). We assume that $z(\delta; \bar{a}, \bar{\delta})$ is smooth, strictly convex in δ, and satisfies $\lim_{\delta_{sj}\downarrow 0} \partial\tilde{z}_{sj}(\delta_{sj}; \bar{\delta})/\partial\delta_{sj} = -\infty$ as well as $\partial\tilde{z}_{sj}(1; \bar{\delta})/\partial\delta_{sj} = \tilde{z}_{sj}(1; \bar{\delta}) = 0$. Intuitively, the cost of reducing δ_{sj} below 1 (increasing velocity) depends on the volume of payment inflows that the household may re-spend, $\bar{a}_{sj}/\bar{\delta}_{sj}$, and increases in velocity. By letting function $\tilde{z}$ depend on both δ and $\bar{\delta}$ we allow for general microfoundations. Moreover, since both δ and $\tilde{z}$ depend on s and j, our formulation allows for scenarios in which some means of payment are more or less useful for specific purchases than others.[8]

2.2.3 Continuation Wealth

Interest on pure stores of value is paid at the beginning of the subsequent period. The same holds true for interest on means of payment except that part of that interest is withheld by the payment service provider. The payment service provider operates at zero cost. Its profit is distributed lump sum to households at the beginning of the subsequent period.

Recall that the household allocates a_{sj} assets of type s at the portfolio choice stage for purchases of good j. These transaction media are depleted after the duration δ_{sj} in

[5] We refer to "liquidity constraints" rather than CIAs to avoid confusion with the standard CIA setting.

[6] Some assets may not be suitable as means of payment. We capture this by a cost of using means of payment, which we introduce below.

[7] For example, this could be because passing revenues quickly from the "seller" to the "shopper" in the household is costly or because there are effort costs of checking the accounts or accelerating settlement by a bank or validator.

[8] For added generality, we could assume that $\arg\min_d \tilde{z}_{sj}(d; \bar{\delta})$ differs depending on s, j.

the trading period, so the average holding period equals $\delta_{sj}/2$. Also during the trading period, the household accumulates new transaction media at rate $\bar{a}_{sj}/\bar{\delta}_{sj}$ in exchange for endowment sales. After duration δ_{sj}, it starts to spend these media, holding the stock $\bar{a}_{sj}/\bar{\delta}_{sj} \cdot \delta_{sj}$ constant. On average, the household thus holds an amount $\bar{a}_{sj}/\bar{\delta}_{sj} \cdot \delta_{sj}(\delta_{sj}/2 + 1 - \delta_{sj})$ of newly acquired assets; see Figure 1.

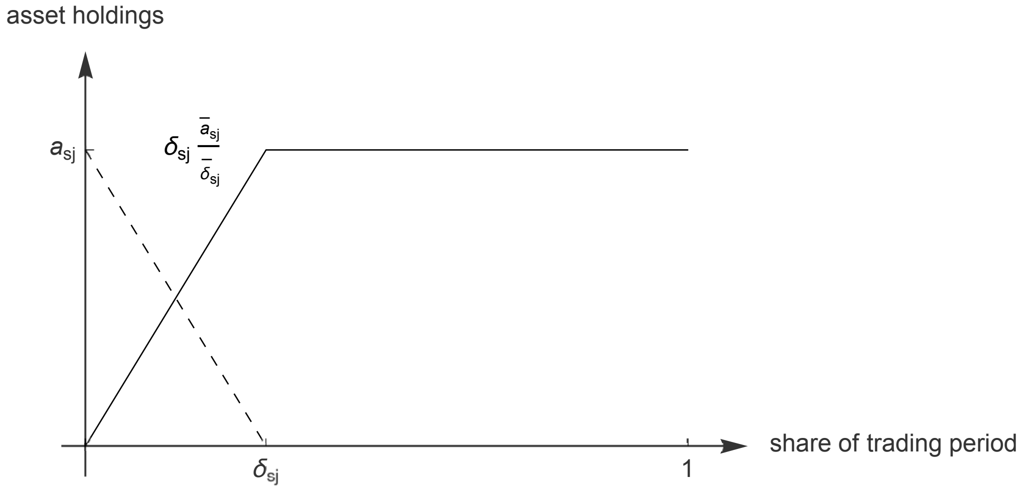

Figure 1: Stocks of means of payment during the trading period. The dashed line represents the remaining stock of means of payment acquired at the portfolio choice stage; this stock depletes after duration δ_{sj}. The solid line represents the stock of means of payment acquired and not yet re-spent during the trading period. In equilibrium, $a_{sj} = \bar{a}_{sj}$ and $\delta_{sj} = \bar{\delta}_{sj}$.

A fraction $\tau_{sj} \in [0, 1]$ of the interest income on transaction media of type s used for purchases of good j is withheld by the payment service provider; τ_{sj} may be history contingent and can be interpreted as the interest rate spread or intermediation margin. (A fixed user cost of transaction media can be subsumed under the cost term ϕ introduced below.) On media of exchange, the household therefore collects interest income

$$\sum_{j \in \mathcal{J}, s \in \mathcal{S}} (1 - \tau_{sj}) \left(a_{sj} \frac{\delta_{sj}}{2} + \frac{\bar{a}_{sj}}{\bar{\delta}_{sj}} \delta_{sj} \left(1 - \frac{\delta_{sj}}{2} \right) \right) i_s,$$

where $i_s \equiv I_s - 1$ denotes the net interest rate. Correspondingly, the payment service provider's profit equals

$$\sum_{j \in \mathcal{J}, s \in \mathcal{S}} \tau_{sj} \bar{a}_{sj} i_s.$$

Financial wealth at the beginning of the subsequent period, w', includes the profits distributed by the payment service provider.[9] It also includes new assets that are injected by transfer, T', if asset stocks grow more quickly than at the rate of interest. Financial

[9]Throughout the paper, a prime as in w' denotes the subsequent period.

wealth at the beginning of the subsequent period thus satisfies

$$\begin{aligned} w' \;=\; & \underbrace{\sum_{s\in\mathcal{S}} a_{s0} I_s}_{\text{return on pure stores of value}} + \underbrace{\sum_{j\in\mathcal{J},s\in\mathcal{S}} \left(a_{sj} + \frac{\bar{a}_{sj}}{\bar{\delta}_{sj}} \right) - \sum_{j\in\mathcal{J}} p_j c_j}_{\text{means of payment, revenues, expenditures}} + \underbrace{T'}_{\text{assets injected by transfer}} \\ & + \underbrace{\sum_{j\in\mathcal{J},s\in\mathcal{S}} (1-\tau_{sj}) \left(a_{sj}\frac{\delta_{sj}}{2} + \frac{\bar{a}_{sj}}{\bar{\delta}_{sj}} \delta_{sj} \left(1 - \frac{\delta_{sj}}{2}\right)\right) i_s}_{\text{interest on means of payment}} + \underbrace{\sum_{j\in\mathcal{J},s\in\mathcal{S}} \tau_{sj} \bar{a}_{sj} i_s}_{\text{payment service provider profit}} . \end{aligned} \tag{3}$$

Note that symmetry ($\bar{a}_{sj} = a_{sj}$ and $\bar{\delta}_{sj} = \delta_{sj}$) implies

$$w' = \sum_{s\in\mathcal{S}} \left(a_{s0} + \sum_{j\in\mathcal{J}} a_{sj} \right) I_s + T' + \sum_{j\in\mathcal{J},s\in\mathcal{S}} \frac{a_{sj}}{\delta_{sj}} - \sum_{j\in\mathcal{J}} p_j c_j,$$

and with binding liquidity constraints, the last two terms vanish. For future reference, we let $\omega_{sj} \equiv 1 + (1-\tau_{sj})\delta_{sj} i_s/2$ denote the holding-period weighted, post-intermediation margin gross interest rate on a_{sj}, i.e., the derivative of w' with respect to a_{sj}.

2.2.4 Leakage

Payments generate data, and addressing the consequences of data leakage is costly. (As mentioned earlier, leakage costs can be interpreted broadly as any payment-related costs, e.g., the inconvenience of using specific means of payment for purchases of certain goods.) Let

$$l_{sj} \equiv a_{sj} + \frac{\bar{a}_{sj}}{\bar{\delta}_{sj}}(1-\delta_{sj}) \quad \forall j \in \mathcal{J}, s \in \mathcal{S} \tag{4}$$

denote payments on market j using transaction medium s and, equivalently, data leakage caused by these payments. Let $l \equiv \{l_{sj}\}_{j\in\mathcal{J},s\in\mathcal{S}}$, and let $k \equiv \{k_{sj}\}_{j\in\mathcal{J},s\in\mathcal{S}}$ where k_{sj} denotes the stock of leakage due to past transactions. These stocks evolve according to the laws of motion

$$k'_{sj} = k_{sj}(1-\gamma_{sj}) + l_{sj} \quad \forall j \in \mathcal{J}, s \in \mathcal{S}, \tag{5}$$

where $\gamma_{sj} \in (0,1]$ denotes a "depreciation" rate.

The cost of addressing leakage is represented by the smooth function $\phi(l,k;x)$, which is a component of the outlays terms in the budget constraint (1). We assume that function ϕ is convex in contemporaneous leakage, l, and increasing in past leakage, k. The convexity assumption reflects the possibility of increasing returns in data collection; similarly, the assumption that ϕ weakly increases in k reflects the possibility of increasing returns over time or data leakage across means of payment or markets. The x argument allows for the possibility that exogenous (for the household) factors affect the cost. We assume that function ϕ exhibits constant returns to scale.[10]

[10]Note that ϕ represents the nominal costs of addressing leakage caused by nominal asset flows. Alternatively, we could have specified real costs of addressing leakage caused by real asset flows; the real costs would enter the budget constraint after multiplication with a price. The constant returns to scale assumption renders the two specifications interchangeable.

2.2.5 Bellman Equation

The household's endogenous state includes (w, k), and the exogenous state variables are $x \equiv (\bar{a}, \bar{\delta}, e, I, p, \phi, \tau, T)$ with $I \equiv \{I_s\}_{s\in\mathcal{S}}$ and $\tau \equiv \{\tau_{sj}\}_{j\in\mathcal{J},s\in\mathcal{S}}$. Elements of x might grow along deterministic trend paths, and/or their deviations from trend may follow a Markov process; we let X represent the implied law of motion.[11] Letting $V(w, k; x)$ denote the household's value function at the beginning of the period after observing the state, the household program reads

$$\begin{aligned} V(w, k; x) &= \max_{(c,a,\delta)\in\mathcal{A}} u(C) + \beta\mathbb{E}\left[V(w', k'; x')|w, k, x\right] \\ \text{s.t.} & \quad (1), (2), (3), (4), (5), X. \end{aligned}$$

Here, $\beta \in (0, 1)$ and $\mathbb{E}[\cdot|\cdot]$ denote the household's subjective discount factor and conditional expectation operator, respectively. $\mathcal{A}$ denotes the admissible set of household choices: Means of payment positions and consumption must be nonnegative, and each holding period must satisfy $\delta_{sj} \in (0, 1]$.

2.2.6 First-Order Conditions

Let λ denote the multiplier associated with the budget constraint at the portfolio choice stage, and let $\{\lambda\xi_j\}_{j\in\mathcal{J}}$ denote the multipliers attached to the liquidity constraints. Note that the household's envelope condition for w implies $\partial V(w, k; x)/\partial w = \lambda$, i.e., λ is the shadow value of (nominal) financial wealth. The envelope condition for k_{sj} implies $\lambda\kappa_{sj} = -\lambda\partial\phi(l, k; x)/\partial k_{sj} + \beta(1 - \gamma_{sj})\mathbb{E}[\lambda'\kappa'_{sj}|w, k, x]$ where $\kappa_{sj} \equiv \partial V(w, k; x)/\partial k_{sj}/\lambda$. For future reference, note that along a balanced growth path[12]

$$\kappa_{sj} = -\frac{\partial\phi(l, k; x)}{\partial k_{sj}} \frac{1}{1 - \beta(1 - \gamma_{sj})\frac{\lambda'}{\lambda}}.$$

The first-order conditions for asset s as a pure store of value or means of payment for good j, respectively, read

$$\lambda = \beta\mathbb{E}[\lambda' I_s|w, k, x] \quad \forall s \in \mathcal{S}, \tag{6}$$

$$\lambda \geq \lambda\xi_j + \beta\mathbb{E}[\lambda'\omega_{sj}|w, k, x] - \lambda\varphi_{sj}, \; a_{sj} \geq 0 \quad \forall j \in \mathcal{J}, s \in \mathcal{S}, \tag{7}$$

where ω_{sj} represents the holding-period weighted, post-intermediation margin gross interest rate introduced earlier and φ_{sj} denotes the present value of leakage costs,

$$\varphi_{sj} \equiv \frac{\partial\phi(l, k; x)}{\partial l_{sj}} - \beta\frac{\mathbb{E}[\lambda'\kappa'_{sj}|w, k, x]}{\lambda}.$$

Condition (6) is the standard stochastic Euler equation (in nominal terms). According to Condition (7) the household balances the shadow value of financial wealth and the

[11] The trend growth rates must be sufficiently small for the household's value function to be well defined.

[12] This follows from $\lambda\kappa_{sj} = -\lambda\partial\phi/\partial k_{sj} + \beta(1-\gamma_{sj})\{-\lambda'\partial\phi'/\partial k'_{sj} + \beta(1-\gamma_{sj})[\ldots]\} = -\lambda\partial\phi/\partial k_{sj}\{1 + \beta(1-\gamma_{sj})\frac{\lambda'}{\lambda} + \beta^2(1-\gamma_{sj})^2\frac{\lambda''}{\lambda} + \ldots\}$, where we use the constant-returns-to-scale property. The result requires $\beta(1-\gamma_{sj})\frac{\lambda'}{\lambda} < 1$, which we assume.

benefits from relaxing the liquidity constraint and carrying means of payment subject to reduced interest into the next period, net of leakage costs.[13]

The household's first-order condition with respect to c_j reads

$$\frac{\partial u(C)}{\partial C}\frac{\partial C}{\partial c_j} = p_j(\lambda \xi_j + \beta \mathbb{E}[\lambda'|w,k,x]) \ \ \forall j \in \mathcal{J}, \tag{8}$$

balancing the marginal utility of good j consumption and the marginal cost of tightening the liquidity constraint and reducing financial wealth in the subsequent period.[14] Finally, an optimal δ_{sj} choice satisfies

$$-\lambda \frac{\partial \tilde{z}_{sj}(\delta_{sj};\bar{\delta})}{\partial \delta_{sj}} \frac{\bar{a}_{sj}}{\bar{\delta}_{sj}} \ \geq \ \frac{\bar{a}_{sj}}{\bar{\delta}_{sj}}\lambda(\xi_j - \varphi_{sj}) - \left(\frac{a_{sj}}{2} + \frac{\bar{a}_{sj}}{\bar{\delta}_{sj}}(1-\delta_{sj})\right)\beta\mathbb{E}\left[\lambda'(1-\tau_{sj})i_s|w,k,x\right],$$
$$\delta_{sj} \leq 1 \ \ \forall j \in \mathcal{J}, s \in \mathcal{S}. \tag{9}$$

A higher δ_{sj} generates benefits because increasing velocity is costly (reflected by the term on the left-hand side), a longer holding period raises interest income (the right-most term), and a reduction in liquid revenues lowers leakage (the φ_{sj} term). It also generates a loss because lower velocity reduces liquid revenues and thus tightens the liquidity constraint (the ξ_j term).[15]

2.3 Asset Supply

The supply of assets is exogenous; see also below. We denote it by $A \equiv \{A_s\}_{s\in\mathcal{S}}$.

2.4 Equilibrium

The aggregate state is (A, e, I, ϕ, τ, T). Elements of the state might grow along deterministic trend paths, and/or their deviations from trend may follow a Markov process; X also represents the implied law of motion of the aggregate state. We consider symmetric equilibria, which satisfy

$$\left.\begin{aligned} a_{s0} &= \bar{a}_{s0} \\ a_{sj} &= \bar{a}_{sj} \\ \delta_{sj} &= \bar{\delta}_{sj} \end{aligned}\right\} \forall j \in \mathcal{J}, s \in \mathcal{S}. \tag{10}$$

Goods and asset market clearing implies

$$\left.\begin{array}{ll} e_j = c_j & \forall j \in \mathcal{J} \\ 0 = a_{s0} + \sum_{j\in\mathcal{J}} a_{sj} & \forall s \in \mathcal{S} \end{array}\right\}. \tag{11}$$

In the last set of equations in Condition (11), we have imposed that all assets are in zero net supply, $A_s = 0 \ \forall s \in \mathcal{S}$. To understand this condition, recall that households are homogeneous and rebated their expenditures for services; moreover, in equilibrium, their

[13] A complementary slackness condition is associated with Condition (7).

[14] We anticipate strictly positive equilibrium consumption, implying interior consumption choices.

[15] A complementary slackness condition is associated with Condition (9).

revenues from endowment sales equal their consumption expenditures. Since the economy is closed and there is no capital or other asset that could be accumulated, equilibrium requires net financial asset positions to equal zero. In other words, in equilibrium households hold long positions in means of payment and short positions in pure stores of value; the statement $a_{s0} \neq 0$ thus is equivalent to the statement $a_{sj} \neq 0$ for some $j \in \mathcal{J}$.

Finally, we introduce a nominal anchor. We consider different variants: first, a target for the quantity of the first means of payment, which we interpret as a target for central bank money or a "money peg";[16] second, a target for the shadow value of financial wealth, λ, which one may interpret as a conditional (on consumption) price level target (see below); and third, a target for the price level, which is formally specified later. That is,

$$\text{either} \quad a_{10} = \check{a}_{10} \quad \text{or} \quad \lambda = \check{\lambda} \quad \text{or} \quad \text{price index} = \text{price}\check{\ }\text{index} \tag{12}$$

for some strictly positive $-\check{a}_{10}$, $\check{\lambda}$, or price$\check{\ }$index.

We can now define equilibrium. Let $\xi \equiv \{\xi_j\}_{j\in\mathcal{J}}$; to simplify the notation, we omit reference to sequences over time:

Definition 1. An *equilibrium* conditional on $(A = 0, e, I, \phi, \tau, T = 0)$ is a value function V; allocation (c, δ); portfolio a; leakage (l, k); and (shadow) price system (λ, ξ, p) such that

- (V, c, a, δ, l, k) solve the Bellman equation, i.e., they satisfy Conditions (2), (4), (5), (6), (7), (8), (9) (and (1), (3)) and the admissibility and complementarity slackness conditions when expectations are consistent with X;
- symmetry prevails and markets clear, i.e., Conditions (10) and (11) hold; and
- the nominal anchor (12) is in effect.

3 Analysis

In this section, we analyze equilibrium properties.

3.1 Externalities

A household's choice of holding period δ_{sj} affects the household's liquidity and continuation wealth; see Conditions (2) and (3), respectively. In addition, it has welfare implications for other households. To see this, consider continuation wealth first. Conditional on aggregate $\bar{\delta}_{sj}$, the effect of a change in δ_{sj} on the interest income collected by a household,

$$\frac{d\,\text{interest on means of payment}}{d\,\delta_{sj}} = (1 - \tau_{sj})i_s \left(\frac{a_{sj}}{2} + \frac{\bar{a}_{sj}}{\bar{\delta}_{sj}}(1 - \delta_{sj}) \right),$$

[16]The government is a payment service provider in this case and refunds seignorage revenue to the public. For a discussion of the public good character of the unit of account function of central bank money see, e.g., Issing (1999).

is positive. This is also evident from Figure 1. In contrast, the effect subject to $\delta_{sj} = \bar{\delta}_{sj}$ collapses to zero,

$$\frac{d\,\text{interest on means of payment}}{d\,\delta_{sj}}|_{\delta_{sj}=\bar{\delta}_{sj}} = (1-\tau_{sj})i_s\left(\frac{a_{sj}}{2} - \frac{\bar{a}_{sj}}{2}\right),$$

because in equilibrium, changing the holding period is irrelevant for household interest income since aggregate holdings remain constant. While households take the inflow of means of payment as given and thus perceive a private interest gain from lengthening the holding period, gains and losses cancel out in the aggregate—lengthening the holding period generates a negative interest income externality.

The private incentive to lengthen the holding period to collect more interest contrasts with an opposing incentive to shorten it to increase liquidity: Conditional on aggregate $\bar{\delta}_{sj}$, a lower δ_{sj} marginally relaxes the liquidity constraint by $\bar{a}_{sj}/\bar{\delta}_{sj}$. Subject to the equilibrium constraint $\delta_{sj} = \bar{\delta}_{sj}$, this marginal effect is larger (as long as $\delta_{sj} < 1$) and equal to $\bar{a}_{sj}/\delta_{sj}^2$. Intuitively, an equilibrium reduction of δ_{sj} increases liquidity more than proportionally because it raises liquid inflows for other households. A reduction of δ_{sj}—higher velocity—thus generates positive liquidity externalities, in addition to the positive interest income externalities discussed above.

A household's choice of initial asset holdings for payment purposes, a_{sj}, also generates externalities. When $\bar{a}_{sj}$ is perceived as exogenous, the marginal effect on continuation wealth equals

$$\frac{d\,w'}{d\,a_{sj}} = \omega_{sj} \equiv 1 + (1-\tau_{sj})\frac{\delta_{sj}}{2}i_s.$$

Subject to the equilibrium constraint $a_{sj} = \bar{a}_{sj}$, in contrast, we find

$$\frac{d\,w'}{d\,a_{sj}}|_{a_{sj}=\bar{a}_{sj}} = \omega_{sj} + \frac{1}{\bar{\delta}_{sj}} + (1-\tau_{sj})\left(1 - \frac{\delta_{sj}}{2}\right)i_s + \tau_{sj}i_s = 1 + \frac{1}{\delta_{sj}} + i_s,$$

which is larger because initial stocks imply inflows for other households and because "lost" interest income due to outflows and intermediation margins generates income for others and profits.

A similar result holds for the liquidity constraint. Holding $\bar{a}_{sj}$ constant, a_{sj} marginally relaxes the constraint one-to-one, but a symmetric increase in a_{sj} and $\bar{a}_{sj}$ generates a marginal effect that is larger by $(1-\delta_{sj})/\bar{\delta}_{sj}$. Intuitively, liquidity is recycled as long as $\delta_{sj} < 1$. Higher initial asset holdings for transaction purposes therefore generate positive externalities in terms of both continuation wealth and liquidity.

We summarize these findings as follows:

Proposition 1. Shorter holding periods and higher initial stocks of means of payment generate positive externalities through greater continuation wealth and liquidity.

3.2 Portfolio Choice

The portfolio choice of a household is represented by Conditions (6) and (7). It reflects three types of tradeoffs. First, the tradeoff between two pure stores of value, say assets s

and $\hat{s}$: Equation (6) implies the standard C-CAPM condition

$$\mathbb{E}[\lambda'(I_s - I_{\hat{s}})|w, k, x] = 0.$$

Second, the tradeoff between two assets s and $\hat{s}$, which are both used as means of payment for the same good j. Equation (7) implies, for interior a_{sj} and $a_{\hat{s}j}$,

$$\beta\mathbb{E}[\lambda'(\omega_{sj} - \omega_{\hat{s}j})|w, k, x] = \lambda(\varphi_{sj} - \varphi_{\hat{s}j}), \tag{13}$$

i.e., leakage and holding-period weighted interest rate differentials after intermediation margins determine the portfolio composition across transaction media. This condition for means of payment parallels the C-CAPM condition for pure stores of value; one might refer to it as the "means-of-payment pricing model" or C-MPPM condition.

Finally, the tradeoff between an asset s held as pure store of value and an asset $\hat{s}$ used as means of payment for good j. Conditions (6) and (7) imply, for interior $a_{\hat{s}j}$,

$$\lambda\xi_j = \beta\mathbb{E}\left[\lambda'(I_s - \omega_{\hat{s}j})|w, k, x\right] + \lambda\varphi_{\hat{s}j} \tag{14}$$

such that (using Conditions (6), (7) and (8))

$$p_j\lambda\xi_j = \frac{\partial u(C)}{\partial C}\frac{\partial C}{\partial c_j}\frac{\beta\mathbb{E}\left[\lambda'(i_s - (1-\tau_{\hat{s}j})\delta_{\hat{s}j}i_{\hat{s}}/2)|w, k, x\right] + \lambda\varphi_{\hat{s}j}}{\beta\mathbb{E}\left[\lambda'(I_s - (1-\tau_{\hat{s}j})\delta_{\hat{s}j}i_{\hat{s}}/2)|w, k, x\right] + \lambda\varphi_{\hat{s}j}}.$$

In the deterministic case and when $\phi(l, k; x) = 0$ (such that $\varphi_{\hat{s}j} = 0$) and means of payment pay no interest (such that $\tau_{\hat{s}j} = 1$), the latter equation reduces to the standard condition according to which the shadow value of the liquidity constraint equals the marginal utility of consumption times i_s/I_s (e.g., Condition (9.6) in Niepelt (2019)). In the environment considered here, the liquidity constraint may bind even if holding means of payment does not cause interest losses ($i_s = i_{\hat{s}} = 0$) because leakage generates additional costs.

Condition (14) implies that a household only uses an asset s both as a pure store of value and means of payment if the benefits of the excess return $i_s(1-(1-\tau_{sj})\delta_{sj}/2)$ and of liquidity net of leakage exactly balance. When $i_s = 0$ this requires $\xi_j = \varphi_{sj}$, and from Condition (9), the household chooses $\delta_{sj} = 1$ in this case.

We summarize the main findings of this Subsection as follows:

Proposition 2. Consider an equilibrium in which s is used as store of value and $\hat{s}$ is used as means of payment for good j. In that equilibrium, $\xi_j - \varphi_{\hat{s}j} = \beta\mathbb{E}\left[\frac{\lambda'}{\lambda}\left(i_s - (1-\tau_{\hat{s}j})\frac{\delta_{\hat{s}j}}{2}i_{\hat{s}}\right)|w, k, x\right]$, and if $i_{\hat{s}} > 0$, the liquidity constraint for good j binds.

Proof. The first result restates Condition (14). The second result follows from the first result (with $s = \hat{s}$) and $\varphi_{\hat{s}j} \geq 0$, $\delta_{\hat{s}j}, \tau_{\hat{s}j} \in [0, 1]$. □

Note that $i_{\hat{s}} > 0$ is a sufficient condition for a binding liquidity constraint, not a necessary one. The liquidity constraint may also bind if $i_{\hat{s}}$ is zero or negative as long as $\varphi_{\hat{s}j}$ is sufficiently large.

3.3 Velocity and Liquidity

Recall that the inverse of δ_{sj} reflects the velocity of transaction medium s for purchases of good j. The average velocity of transaction medium s, v_s, therefore equals[17]

$$v_s \equiv \sum_{j \in \mathcal{J}} \frac{a_{sj}/\delta_{sj}}{\sum_{j \in \mathcal{J}} a_{sj}}.$$

According to Conditions (9) and (10), velocity reflects the shadow value of liquidity, leakage, the interest rate and the intermediation margin,

$$-\frac{\partial \tilde{z}_{sj}(\delta_{sj};\delta)}{\partial \delta_{sj}} \geq \xi_j - \varphi_{sj} - \left(1 - \frac{\delta_{sj}}{2}\right) \beta \mathbb{E}\left[\frac{\lambda'}{\lambda}(1-\tau_{sj}) i_s | w, k, x\right], \ \delta_{sj} \leq 1.$$

Furthermore, velocity affects the portfolio choice, and thus ξ_j and φ_{sj}, as is evident from the δ_{sj} term in Condition (7). In equilibrium, this circularity simplifies. Combining the condition stated in Proposition 2 (for $s = \hat{s}$ with $a_{sj} > 0$) and the inequality above and eliminating $\xi_j - \varphi_{sj}$ yields

$$-\frac{\partial \tilde{z}_{sj}(\delta_{sj};\delta)}{\partial \delta_{sj}} \geq \beta \mathbb{E}\left[\frac{\lambda'}{\lambda} i_s \tau_{sj} | w, k, x\right], \ \delta_{sj} \leq 1.$$

When $\delta_{sj} < 1$, the choice of velocity is interior such that the derivative of the $\partial \tilde{z}_{sj}$ function equals β times the expectation term. When $\delta_{sj} = 1$, in contrast, the derivative of the $\partial \tilde{z}_{sj}$ function equals zero, and the same result follows because $i_s, \tau_{sj} \geq 0$. We conclude that, independent of δ_{sj},

$$a_{sj} > 0 \quad \Rightarrow \quad -\frac{\partial \tilde{z}_{sj}(\delta_{sj};\delta)}{\partial \delta_{sj}} = \beta \mathbb{E}\left[\frac{\lambda'}{\lambda} i_s \tau_{sj} | w, k, x\right]. \tag{9'}$$

According to Condition (9'), households increase velocity (reduce δ_{sj} below one) only if interest rates and intermediation margins exceed zero. To understand this result, recall the effects on liquidity net of leakage for two strategies. The first is the strategy of increasing a_{sj} and decreasing a_{s0}. From Proposition 2, this yields an effect proportional to

$$i_s \left(1 - (1-\tau_{sj})\frac{\delta_{sj}}{2}\right).$$

The second is the strategy of reducing δ_{sj}. From Conditions (9) and (10), this yields an effect proportional to

$$i_s \left(1 - (1-\tau_{sj})\frac{\delta_{sj}}{2} - \tau_{sj}\right) - k\frac{\partial \tilde{z}_{sj}(\delta_{sj};\delta)}{\partial \delta_{sj}},$$

where $k > 0$. When both i_s and τ_{sj} are strictly positive, the second strategy yields a smaller effect than the first one through the interest channel; indifference between the

[17] The average velocity of *asset* s is not defined since $A_s = 0$.

two strategies then requires $\partial\tilde{z}_{sj}(\delta_{sj};\delta)/\partial\delta_{sj} < 0$, i.e., $\delta_{sj} < 1$. When either i_s or τ_{sj} equals zero, in contrast, both strategies yield the same effect through the interest channel and indifference requires $\delta_{sj} = 1$. This comparison also clarifies why the product $i_s\tau_{sj}$ enters Condition (9').

The same comparison clarifies why strictly positive interest rates render liquidity scarcer but may not drive up velocity. Note that with $i_s = 0$, $\delta_{sj} = 1$ and the liquidity constraint only is costly because of leakage ($\xi_j = \varphi_{sj}$); see Condition (9') and Proposition 2. When the interest rate rises but $\tau_{sj} = 0$, then δ_{sj} holds its value of one but ξ_j rises above φ_{sj}. This decoupling of the value of liquidity and optimal velocity again reflects the interest channels of the two strategies laid out above: As long as $\tau_{sj} = 0$, rebalancing the portfolio from stores of value to means of payment generates the same interest loss as higher velocity, but raising velocity generates additional costs. When $\tau_{sj} = 0$, it is therefore optimal to maintain $\delta_{sj} = 1$, although rising interest rates do increase the opportunity costs of holding means of payment. In summary we have the following:

Proposition 3. Consider an equilibrium in which asset s is used as means of payment for good j. Then, the holding period δ_{sj} satisfies Condition (9'); if $i_s\tau_{sj} > 0$ along some history, then δ_{sj} is interior. Scarcer liquidity (higher ξ_j) need not be associated with higher velocity (lower δ_{sj}).

Proof. When asset s is used as means of payment, then by market clearing, it is also held as pure store of value. The first result thus follows from the derivations given in the text. The second result restates the discussion in the text. □

We conjecture that a result in the spirit of Proposition 3—δ_{sj} reflects the stochastic discount factor, interest rates, and intermediation margins—holds under much more general conditions, as long as a key condition is met: The liquidity and leakage effects must be the same for means of payment that are allocated at the portfolio choice stage or acquired during the trading period.[18]

3.4 Price Index

The CES aggregator satisfies

$$\text{CES}(c) \equiv \left(J^{-\frac{1}{\eta}} \sum_{j\in\mathcal{J}} c_j^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}},$$

where $\eta > 0$ denotes the elasticity of substitution. Recall from Condition (8) that the marginal cost of c_j equals λq_j with

$$q_j \equiv p_j \left(\xi_j + \beta\mathbb{E}\left[\frac{\lambda'}{\lambda}|w,k,x\right]\right) = p_j \left(1 + \varphi_{sj} - \beta\mathbb{E}\left[\frac{\lambda'}{\lambda}\frac{1-\tau_{sj}}{2}\delta_{sj}i_s|w,k,x\right]\right),$$

[18]That is, variants of Conditions (7) and (9) must both contain the term $\lambda(\xi_j - \varphi_{sj})$.

where the equality follows from Conditions (6) and (7) as long as $a_{sj} > 0$. Standard derivations then imply that the price index associated with CES(c), Q, satisfies[19]

$$Q \equiv \left(J^{-1} \sum_{j \in \mathcal{J}} q_j^{1-\eta} \right)^{\frac{1}{1-\eta}} . \tag{15}$$

Price index Q differs from a naïve price index, P, that averages the commodity prices $\{p_j\}_{j\in\mathcal{J}}$ rather than the full marginal costs $\{q_j\}_{j\in\mathcal{J}}$. When the liquidity constraints are equally tight across goods such that ξ_j is constant across commodities, Q equals P scaled by the factor $\xi_j + \beta\mathbb{E}\left[\frac{\lambda'}{\lambda}|w,k,x\right]$, which can exceed unity or fall short of it, depending on $\varphi_{sj}, \tau_{sj}, \delta_{sj}, i_s$ and λ'/λ. In the standard CIA setting ($\varphi_{sj} = 0, \tau_{sj} = 1$, with or without symmetry), the scaling factor equals unity such that $q_j = p_j$, and the price index collapses to the naïve index.

As noted above, the multiplier λ associated with the budget constraint at the portfolio choice stage represents the shadow value of nominal financial wealth. Accordingly, $\lambda = u'(C)/Q$; this can be verified by substituting the expression for p_j in Equation (8) into Condition (15).

We summarize these findings as follows:

Proposition 4. The price index Q satisfies Condition (15) and differs from the naïve index P. With symmetrically tight liquidity constraints, $P/Q = p_j/q_j \ \forall j \in \mathcal{J}$. With asymmetrically tight liquidity constraints,

$$P = \left(J^{-1} \sum_{j \in \mathcal{J}} (q_j/f_j)^{1-\eta} \right)^{\frac{1}{1-\eta}} ,$$

where the factors f_j depend on $\varphi_{sj}, \tau_{sj}, \delta_{sj}, i_s$ and λ'/λ.

Proof. See the derivations and discussions in the text. □

3.5 Determinacy

Variation in interest rates across means of payment and variation in leakage across means-of-payment-good combinations renders portfolios determinate:

Proposition 5. Consider an equilibrium in which households use two means of payment, s and $\hat{s}$, to purchase good j. Suppose that $\xi_j > 0$. If the portfolio composition $(a_{sj}, a_{\hat{s}j})$ is indeterminate, then $I_s = I_{\hat{s}}$ in each history; l_{sj} and $l_{\hat{s}j}$ enter ϕ at most as a sum; k_{sj} and $k_{\hat{s}j}$ enter ϕ at most as a sum, and if they do, $\gamma_{sj} = \gamma_{\hat{s}j}$. Moreover, in that equilibrium, $\delta_{sj} = \delta_{\hat{s}j}$, $\varphi_{sj} = \varphi_{\hat{s}j}$ and $\mathbb{E}[\lambda' i_s(\tau_{sj} - \tau_{\hat{s}j})|w,k,x] = 0$.

[19] The first-order Condition (8) reads $u'(C)\frac{\partial C}{\partial c_j} = \lambda q_j$ with $\frac{\partial C}{\partial c_j} = (Jc_j/C)^{-\frac{1}{\eta}}$. Let z denote some given expenditure amount, and consider the program $\max_c$ CES(c) s.t. $\sum_{j\in\mathcal{J}} c_j q_j = z$. Let μ denote the multiplier associated with the constraint; by definition, $\mu = Q^{-1}$ and $QC = z$. The first-order conditions read $(Jc_j/C)^{-\frac{1}{\eta}} = \mu q_j \ \forall j \in \mathcal{J}$ such that $QC = \sum_{j\in\mathcal{J}} q_j(\mu q_j)^{-\eta} C/J$ or $Q^{1-\eta} = \sum_{j\in\mathcal{J}} q_j^{1-\eta}/J$.

Proof. If the equilibrium portfolio composition is indeterminate, then the equilibrium conditions continue to hold when the sum $a_{sj} + a_{\hat{s}j}$ is held constant but its composition changes. Since the liquidity constraint binds, this implies, from Conditions (2) and (10), that $\delta_{sj} = \delta_{\hat{s}j}$. Moreover, $I_s = I_{\hat{s}}$ in all histories. This is because if interest rates are deterministic, then, from Condition (6), they must be equal; and if they are stochastic, then they must have identical return characteristics because otherwise, from Conditions (3) and (10), w' could not remain unchanged in all histories. Consider next the implications for ϕ. Condition (7) requires that both φ_{sj} and $\varphi_{\hat{s}j}$ are unaffected by the change in composition. From Conditions (4) and (10), φ_{sj} and $\varphi_{\hat{s}j}$ are functions of $l_{sj} = a_{sj}/\delta_{sj}$, $l_{\hat{s}j} = a_{\hat{s}j}/\delta_{\hat{s}j} = a_{\hat{s}j}/\delta_{sj}$, k_{sj} and $k_{\hat{s}j}$. Independence of φ_{sj} and $\varphi_{\hat{s}j}$ from changes in the composition of $a_{sj} + a_{\hat{s}j}$ thus requires that l_{sj} and $l_{\hat{s}j}$ enter ϕ at most as a sum, and k_{sj} and $k_{\hat{s}j}$ also enter ϕ at most as a sum with $\gamma_{sj} = \gamma_{\hat{s}j}$. However, if only the sums enter ϕ, then $\varphi_{sj} = \varphi_{\hat{s}j}$. Equation (13) then implies $\mathbb{E}[\lambda' i_s(\tau_{sj} - \tau_{\hat{s}j})|w, k, x] = 0$. □

A similar reverse statement also holds true:

Proposition 6. Consider an equilibrium in which households use means of payment s to purchase good j. Consider another asset $\hat{s}$, and assume that neither s nor $\hat{s}$ is the nominal anchor and $x \perp a_{sj}, a_{\hat{s}j}$. If $I_s = I_{\hat{s}}$ in each history; $\tau_{sj} = \tau_{\hat{s}j}$ in each history; l_{sj} and $l_{\hat{s}j}$ enter ϕ at most as a sum; k_{sj} and $k_{\hat{s}j}$ enter ϕ at most as a sum, and if they do, $\gamma_{sj} = \gamma_{\hat{s}j}$, then the portfolio composition $(a_{sj}, a_{\hat{s}j})$ is indeterminate. Moreover, in that equilibrium, $\delta_{sj} = \delta_{\hat{s}j}$.

Proof. The assumptions about ϕ imply $\varphi_{sj} = \varphi_{\hat{s}j}$, and thus, from Condition (7) and the assumptions about interest rates and intermediation margins, $\delta_{sj} = \delta_{\hat{s}j}$. Equilibrium Conditions (1)–(11) thus continue to hold when the sum $a_{sj} + a_{\hat{s}j}$ is held constant but its composition changes. □

3.6 Money and Interest Rate Policy

Money is neutral in the sense that a change in the quantity of the nominal anchor induces equiproportionate changes in all portfolios without changing the equilibrium allocation (velocity and leakage costs), relative prices or relative costs:

Proposition 7. Consider an equilibrium, and suppose that the argument x of function ϕ only contains nominal variables. Then, there exists another equilibrium in which all portfolios, prices and nominal variables including the nominal anchor in (12) are scaled by $f > 0$, λ is scaled by f^{-1}, and the allocation remains unchanged.

Proof. Since ϕ exhibits constant returns to scale, all equilibrium conditions according to definition 1 continue to be satisfied. (The multipliers $\{\xi_j\}_{j \in \mathcal{J}}$ do not change.) □

In contrast, interest rate policy generally affects the allocation and price system. To see this, consider a uniform increase in interest rates in a deterministic setting. From Condition (6), this decreases λ'/λ, and from Condition (7), $\xi_j - \varphi_{sj}$ and/or δ_{sj} must change; velocity also changes for the reasons summarized in Proposition 3. Moreover, from Condition (8), this affects relative prices. We consider interest rate changes along balanced growth paths in greater detail below.

Proposition 8. Interest rate policy affects the allocation and relative prices. Under the assumptions of the standard CIA setting ($\varphi_{sj} = 0, \tau_{sj} = 1 \ \forall j \in \mathcal{J}, s \in \mathcal{S}$), this is not the case.

Proof. The first part follows from the discussion in the text and specific examples analyzed in the following section. For the second part, note that $\varphi_{sj} = 0$ and $\tau_{sj} = 1$ imply a constant ξ_j across all goods (from Condition (7)). From Condition (8), relative prices therefore solely reflect marginal utilities. □

We note that inflation, which is reflected in nominal rates, generally "has real effects" and affects relative prices—the model does not exhibit "superneutrality." However, in our endowment economy, inflation does of course not change the goods allocation. This changes in the production economy we analyze below.

3.7 Balanced Growth Path

Next, we turn to balanced growth paths (BGPs). Let $\varphi \equiv \{\varphi_{sj}\}_{j\in\mathcal{J},s\in\mathcal{S}}$.

Definition 2. A *BGP* is an equilibrium conditional on $(A = 0, e, I, \phi, \tau, T = 0)$ such that

- $e = c$ grow at gross rate Γ;
- p grows at gross rate Π;
- τ, δ, φ, ξ, and I are constant, and interest rates are strictly positive;
- a, w, l, and k grow at gross rate $\Gamma\Pi$;
- and the following parametric conditions are satisfied:

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma}, \ \sigma > 0; \ \beta < \Gamma^{\sigma-1}; \ \beta I_s = \Gamma^{\sigma}\Pi; \ 1 - \gamma_{sj} < \Gamma\Pi \ \ \forall j \in \mathcal{J}, s \in \mathcal{S}.$$

We impose the CIES preference assumption for convenience. The second parametric restriction guarantees that the objective function of households is bounded. The third restriction is implied by an equilibrium condition (see below) and the fourth guarantees the boundedness of φ_{sj} (see below).

With strictly positive interest rates, all liquidity constraints bind; see Proposition 2. Along a BGP, Conditions (1)–(5) and (10)–(11) then imply

$$0 = \sum_{s\in\mathcal{S}} \left(a_{s0} + \sum_{j\in\mathcal{J}} a_{sj} \right), \tag{16}$$

$$\sum_{s\in\mathcal{S}} \frac{a_{sj}}{\delta_{sj}} = p_j e_j \ \ \forall j \in \mathcal{J}, \tag{17}$$

$$T' = 0, \tag{18}$$

$$l_{sj} = \frac{a_{sj}}{\delta_{sj}} \ \ \forall j \in \mathcal{J}, s \in \mathcal{S}, \tag{19}$$

$$(\Gamma\Pi - 1 + \gamma_{sj})k_{sj} = l_{sj} \ \ \forall j \in \mathcal{J}, s \in \mathcal{S}. \tag{20}$$

Since $u(C)$ takes the CIES form, $u'(C)$ grows at gross rate $\Gamma^{-\sigma}$. Condition (8) therefore requires λ to grow at the gross rate $\Gamma^{-\sigma}\Pi^{-1}$, and Condition (6) implies

$$\Gamma^{\sigma}\Pi = \beta I_s \quad \forall s \in \mathcal{S}, \tag{21}$$

as we imposed.

Condition (7) implies[20]

$$\begin{aligned} 1 &\geq \xi_j + \beta\Gamma^{-\sigma}\Pi^{-1}\left(1 + \frac{1-\tau_{sj}}{2}\delta_{sj} i_s\right) - \varphi_{sj},\ a_{sj} \geq 0\ \forall j \in \mathcal{J}, s \in \mathcal{S}, \\ \Rightarrow\ \xi_j - \varphi_{sj} &\leq \frac{i_1}{I_1}\left(1 - \frac{1-\tau_{sj}}{2}\delta_{sj}\right),\ a_{sj} \geq 0\ \ \forall j \in \mathcal{J}, s \in \mathcal{S}. \end{aligned} \tag{22}$$

Conditions (8) and (11) imply[21]

$$\begin{aligned} E^{-\sigma}(Je_j/E)^{-\frac{1}{\eta}} &= p_j\lambda(\xi_j + \beta\Gamma^{-\sigma}\Pi^{-1})\ \ \forall j \in \mathcal{J} \\ &= p_j\lambda(\xi_j + 1/I_1)\ \ \forall j \in \mathcal{J} \\ &\leq p_j\lambda\left(1 + \varphi_{sj} - \frac{i_1}{I_1}\frac{1-\tau_{sj}}{2}\delta_{sj}\right),\ a_{sj} \geq 0\ \ \forall j \in \mathcal{J}, s \in \mathcal{S}, \end{aligned} \tag{23}$$

where the inequality uses Condition (22). Additionally, Condition (9) implies, from Proposition 3,

$$a_{sj} > 0 \quad \Rightarrow \quad -\frac{\partial \tilde{z}_{sj}(\delta_{sj};\delta)}{\partial \delta_{sj}} = \frac{i_1}{I_1}\tau_{sj} \ \ \forall j \in \mathcal{J}, s \in \mathcal{S}. \tag{24}$$

Finally, from the nominal anchor Condition (12),

$$\text{either} \ \ a_{10} = \check{a}_{10} \ \ \text{or} \ \ \lambda = \check{\lambda} \ \ \text{or} \ \ Q = \check{Q} \ \ \text{or} \ \ P = \check{P} \ \ \text{in initial period.} \tag{25}$$

Functional Form Assumptions for $\tilde{z}$ and ϕ Recall that $\lim_{\delta_{sj}\downarrow 0} \partial \tilde{z}_{sj}(\delta_{sj};\bar{\delta})/\partial \delta_{sj} = -\infty$ and $\tilde{z}_{sj}(1;\bar{\delta}) = \partial \tilde{z}_{sj}(1;\bar{\delta})/\partial \delta_{sj} = 0$. To represent these restrictions, we posit the following functional form:

Assumption 1. $\tilde{z}_{sj}(\delta_{sj};\bar{\delta}) = \zeta_{sj}(1-\delta_{sj})^2/\delta_{sj}$.

The first-order condition for δ_{sj}, Condition (24), then reduces to

$$a_{sj} > 0 \quad \Rightarrow \quad \delta_{sj} = \left(1 + \frac{i_1}{I_1}\frac{\tau_{sj}}{\zeta_{sj}}\right)^{-\frac{1}{2}},\zeta_{sj} > 0 \ \ \forall j \in \mathcal{J}, s \in \mathcal{S}. \tag{26}$$

Intuitively, higher interest rates lead the household to hold means of payment for a shorter duration. This effect is weaker if the means of payment also pays interest, $\tau_{sj} < 1$, or if reducing δ_{sj} is more costly (large ζ_{sj}).

[20] A complementary slackness condition is associated with Condition (22).
[21] A complementary slackness condition is associated with Condition (23).

Recall also that $\phi(l,k;x)$ is convex in l but exhibits constant returns to scale. To represent both these restrictions and also allow for settings in which some asset-good combinations generate zero leakage, we assume that $\phi(l,k;x)$ is the sum of a finite number of subfunctions, $\phi_n(l,k;x), n = 1,\dots,N$, where each subfunction exhibits constant returns to scale and takes arbitrary elements of l and k as arguments. The exponents of l_{sj} arguments weakly exceed unity, reflecting the convexity assumption, and the exponents of k_{sj} arguments weakly exceed zero. The x argument is chosen to guarantee constant returns to scale, i.e., ϕ_n is decreasing in x; we let $x = -\bar{a}_{10}$ for simplicity. In equilibrium, we thus have

$$\begin{aligned}\varphi_{sj} &= \sum_{n=1}^{N} \frac{\partial \phi_n(l,k;a_{10})}{\partial l_{sj}} - \beta \Gamma^{-\sigma}\Pi^{-1}\kappa'_{sj} \\ &= \sum_{n=1}^{N} \frac{\partial \phi_n(l,k;a_{10})}{\partial l_{sj}} + \frac{\partial \phi_n(l,k;a_{10})}{\partial k_{sj}} \frac{\beta\Gamma^{-\sigma}\Pi^{-1}}{1-\beta(1-\gamma_{sj})\Gamma^{-\sigma}\Pi^{-1}},\end{aligned}$$

where the second equality follows from the envelope condition for k_{sj} discussed previously. The infinite sum requires $\beta(1-\gamma_{sj})\Gamma^{-\sigma}\Pi^{-1} < 1$, which explains the parametric condition $1-\gamma_{sj} < \Gamma\Pi$ (in addition to the condition $\beta < \Gamma^{\sigma-1}$) that we imposed in the definition of a BGP.

Since each subfunction ϕ_n exhibits constant returns to scale, the partial derivatives in the expression for φ_{sj} are homogeneous of degree zero. In essence, φ_{sj} therefore reflects portfolio shares and parameters. With minimal loss of generality, we impose a Cobb-Douglas structure such that

$$\varphi_{sj} = \sum_{n=1}^{N} \frac{\phi_n(l,k;a_{10})}{l_{sj}} \left(\exp_{n,l,sj} + \exp_{n,k,sj} \frac{(\Gamma\Pi - 1 + \gamma_{sj})\beta\Gamma^{-\sigma}\Pi^{-1}}{1-\beta(1-\gamma_{sj})\Gamma^{-\sigma}\Pi^{-1}} \right),$$

where $\exp_{n,l,sj}$ and $\exp_{n,k,sj}$ denote the exponents (potentially zero) of the l_{sj} and k_{sj} arguments in $\phi_n(l,k;x)$, respectively, and where we also use Condition (20). Finally, for tractability, we assume quadratic costs of l_{sj} or k_{sj}. Simplifying then yields

$$\varphi_{sj} = \frac{\alpha_{sj}}{-a_{10}} \frac{a_{sj}}{\delta_{sj}}, \tag{27}$$

where the α_{sj} term is positive and reflects parameters.[22]

Assumption 2. ϕ satisfies the assumptions given in the text.

We summarize the restrictions imposed by a BGP as follows:

Proposition 9. A BGP subject to Assumptions (1) and (2) satisfies Conditions (16)–(17), (19)–(20), (23) and (25)–(27).

Proof. The result follows from the derivations in the text. □

[22]The restrictions imply cost functions of the form $\phi_n(l,k;a_{10}) = c_0 l_{sj}^{c_1} k_{sj}^{c_2} a_{10}^{-2}$, where c_0, c_1, c_2 are positive parameters with $c_1 + c_2 = 2$.

The BGP equilibrium conditions reduce to four key restrictions: The liquidity constraint (17) subject to the equilibrium expression for δ_{sj}, Condition (26); the consumption first-order Condition (23) subject to Conditions (26) and (27); as well as the budget and nominal anchor Conditions (16) and (25).

To solve for a BGP equilibrium conditional on $A = T' = 0$, $e = c$, I, τ, Γ and Π, we can conjecture the pairs $(s, j) \in \mathcal{S} \times \mathcal{J}$ for which $a_{sj} > 0$; use the equilibrium conditions to solve for the variables; and verify the conjecture by checking that no other (s, j) pairs are preferable. Under the assumptions of the standard CIA setting ($\phi(l, k; x) \equiv 0$), Condition (26) determines δ; (23) implies $p\lambda$; (17) implies $\{a_{sj}\lambda\}_{j\in\mathcal{J},s\in\mathcal{S}}$; (16) implies $\{a_{s0}\lambda\}_{s\in\mathcal{S}}$; and (25) pins down λ and the portfolios. Portfolios may be indeterminate; see Proposition 6. Leakage ($\phi(l, k; x) \neq 0$) may eliminate indeterminacy by introducing cross-asset restrictions.

4 Comparative Statics

To assess plausible magnitudes of the payments channel on prices and portfolios, we consider numerical examples. We focus on a BGP and impose Assumptions 1 and 2. Unless otherwise noted, we impose symmetry with respect to intermediation margins, the structure of leakage costs, and the costs of adjusting velocity. Our baseline calibration stipulates $e_1 = e_2 = 0.5$ such that $C = E = 1$; $\tau_{11} = \tau_{12} = 0.5$ and $\alpha_{11} = \alpha_{12} = 0.01$, i.e., intermediaries retain half of the interest on means of payment and leakage costs amount to one percent of the transaction volume; and $\zeta_{11} = \zeta_{12} = 0.05$ to match estimates of an interest semielasticity of money demand of roughly -5 (Ball, 2001).[23]

4.1 Price Effects

In a first step, we let $S = 1$ and focus on price effects before generalizing in the subsequent subsection. The liquidity constraints thus require $a_{1j} > 0$ for all goods, and there is no indeterminacy, even if $\phi(l, k; x) \equiv 0$. Condition (26) determines $\{\delta_{1j}\}_{j\in\mathcal{J}}$; $\{a_{1j}, p_j\}_{j\in\mathcal{J}}$ and (a_{10}, λ, P, Q) in the initial period are determined by the peg $a_{10} = -1$, the definitions of the price indices and the equilibrium conditions

$$0 = a_{10} + \sum_{j\in\mathcal{J}} a_{1j}, \tag{28}$$

$$a_{1j} = p_j e_j \delta_{1j} \text{ s.t. (26)} \quad \forall j \in \mathcal{J}, \tag{29}$$

$$E^{-\sigma}(Je_j/E)^{-\frac{1}{\eta}} = p_j\lambda\left(1 + \frac{\alpha_{1j}}{-a_{10}}\frac{a_{1j}}{\delta_{1j}} - \frac{i_1}{I_1}\frac{1-\tau_{1j}}{2}\delta_{1j}\right) \text{ s.t. (26)} \quad \forall j \in \mathcal{J}. \tag{30}$$

The remaining equilibrium conditions determine the other variables such as $\{\xi_j\}_{j\in\mathcal{J}}$.

[23] We also let $\eta = 0.5$ and $\sigma = 1$. Suppose that the money demand semielasticity is fully reflected in the negative semielasticity of velocity, $1/\delta_{sj}$. Condition (26) implies $\ln(1/\delta_{sj}) = 0.5 \ln(1 + i\tau_{sj}/I\zeta_{sj})$, implying a semielasticity of approximately $0.5\,\tau_{sj}/\zeta_{sj}$, and thus $\tau_{sj}/\zeta_{sj} = 10$.

Interest Rate Changes To study the effects of interest rate changes we consider a scenario with two goods that households are symmetrically endowed with, $e_1 = e_2$.[24] This comparative statics exercise represents standard monetary policy in an environment with endogenous velocity but without heterogeneity across means of payment or goods. (We consider asymmetric settings below.) Symmetry implies equal holding periods and commodity prices, $\delta_{11} = \delta_{12}$ and $p_1 = p_2$; from Conditions (28) and (29), it also implies $a_{11} = a_{12} = 0.5$.

Figure 2 verifies these predictions and illustrates other effects when the interest rate varies between zero and four percent. The top-left panel shows that the price level Q rises in response to higher interest rates but slightly less so than the naïve index P. Inversely with Q, the shadow value of nominal wealth, λ, falls. The positive effect of i_1 on the naïve index P reflects higher commodity prices (top-right panel). The slightly weaker effect on Q is because marginal leakage costs (bottom-left panel) rise more slowly than interest income on means of payment such that q_j falls below p_j; see Equation (30). Equivalently, the normalized shadow values of the liquidity constraints, ξ_{1j} (bottom-right panel), rise more slowly than the gross interest rate; see Condition (23). Both higher interest rates and leakage costs drive up the latter shadow values; see Condition (22). Velocities $1/\delta_{1j}$ rise with the interest rate (middle-right panel) because $\tau_{1j} > 0$; see Condition (9').

We conclude from this example that changes in interest rates have major effects on prices when means of payment serve as a nominal anchor. When the central bank targets P, this naturally changes. For the same parameter values, an increase in interest rates subject to a P peg raises λ, substantially lowers $-a_{10}$, and reduces Q by roughly one percent (all not shown). The latter result is consistent with the finding of slightly weaker Q inflation than P inflation under a a_{10} target (illustrated in the top-left panel of Figure 2). These findings suggest the following rule of thumb:

Result 1. During a typical interest rate cycle, a central bank that targets P reduces the consumer price index Q by one percent while hiking interest rates and similarly increases it while reducing rates.

One may argue about the quantitative significance of the cyclical variation in the wedge between P and Q; the major expansions that central banks engineered in the 2010s to slightly raise inflation certainly suggest that it is significant. Even if one came to the opposite conclusion, it would be wrong to interpret the similarity between P and Q as evidence of the irrelevance of the new model elements relative to a conventional CIA setting. This is because the link between interest rates and prices would be dramatically different if velocities were held constant. This is a direct consequence of the fact that the conventional CIA setting abstracts from changes in velocities while our model combines the transaction perspective of the conventional CIA setting with conditional interest elasticity due to endogenous velocity (and portfolio choice, analyzed below). If, realistically, velocities do change, then a central bank that targets P with a conventional CIA mindset

[24] Changes in interest rates are associated with changes in growth or inflation (see Condition (21)). We assume that these underlying changes in growth or inflation do not change φ_{sj} because ϕ does not depend on k_{sj}.

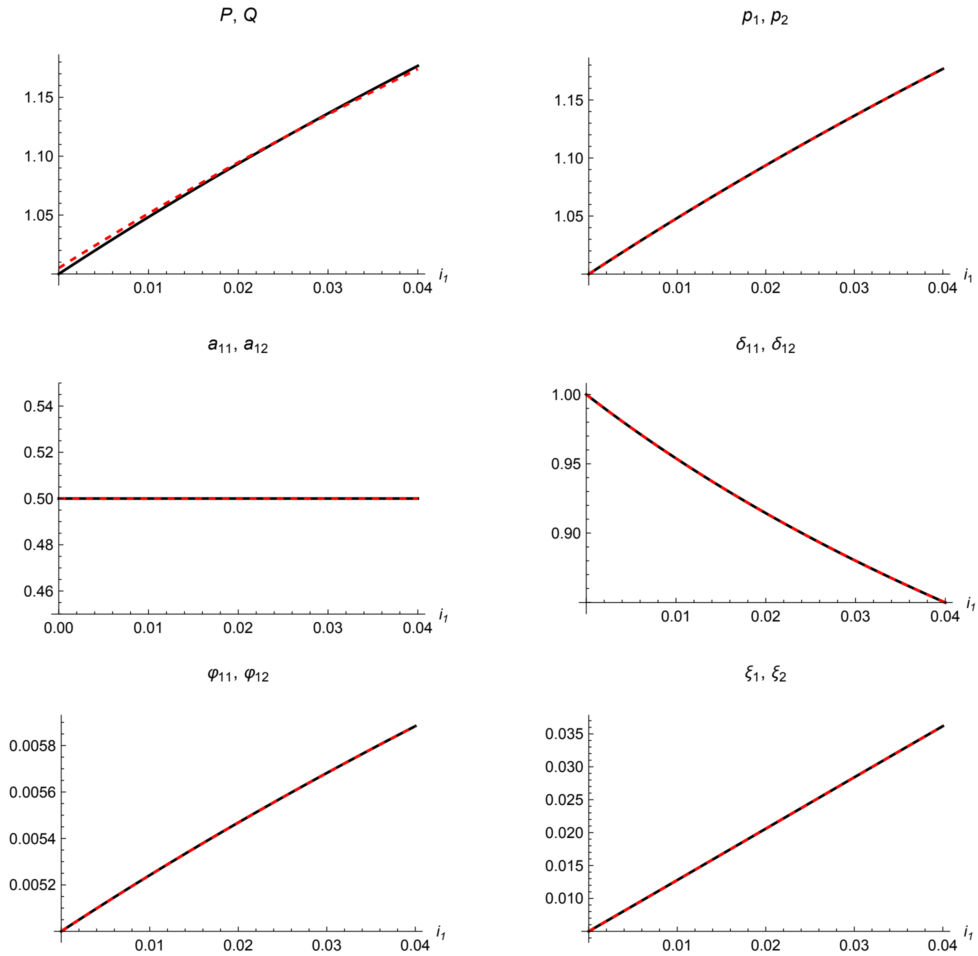

Figure 2: Comparative statics with respect to i_1: Single means of payment, symmetric $\alpha_{1j}, \tau_{1j}, \zeta_{1j}$. The first variable in a panel is indicated in black/solid, the second in red/dashed.

encounters price developments very much at odds with its model of the world.[25]

Endowment Changes Next, we focus on the consequences of endowment changes. We maintain the symmetry assumption for intermediation margins, the structure of leakage costs, and the costs of adjusting velocity. The symmetric intermediation margins continue to imply equal holding periods, $\delta_{11} = \delta_{12}$. However, a_{11} and a_{12} now generally differ from each other when the two endowments are unequal. Intuitively, maintaining symmetry ($a_{11} = a_{12}$) would require goods prices to move inversely with endowments ($p_1 e_1 = p_2 e_2$, from Equation (29)), but this would be inconsistent with Condition (30) unless $\eta = 1$.

When the elasticity of substitution falls short of unity ($\eta < 1$), a relative endowment change translates into a more than inverse proportional change in relative marginal utilities, and market clearing therefore requires a more than inverse proportional response of

[25]Some degree of conditional interest elasticity is present in the conventional CIA setting with Svensson (1985) timing.

relative costs. From Condition (30), changes in relative costs are driven by changes in relative goods prices in addition to changes in relative leakage, and relative goods prices therefore also respond more than inversely proportionally. This effect dominates the relative endowment effect and drives up the means of payment share earmarked for purchases of the good that has become scarcer. That is, when e_2/e_1 rises, then q_1/q_2 and p_1/p_2 rise more than proportionally, and from Condition (29), a_{11}/a_{12} also rises. Figure 3 illustrates this case for our baseline calibration with $\eta = 0.5$.[26]

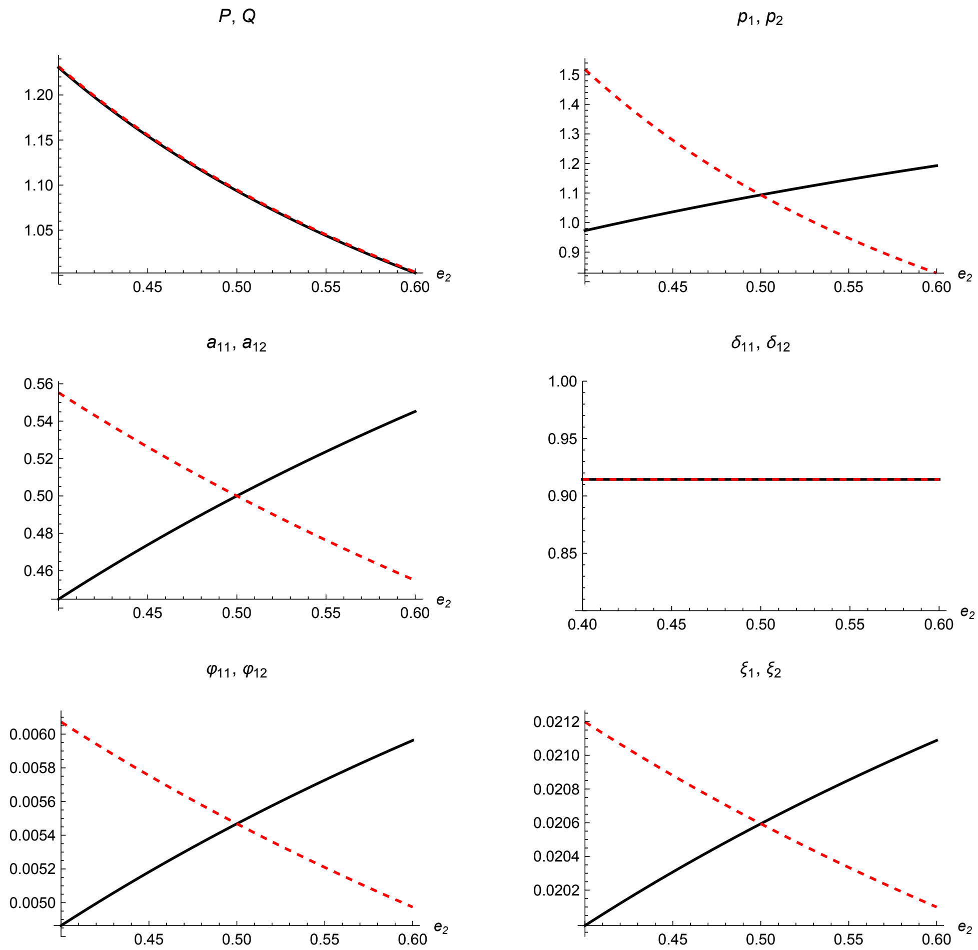

Figure 3: Comparative statics with respect to e_2: Single means of payment, symmetric $\alpha_{1j}, \tau_{1j}, \zeta_{1j}$, $\eta = 0.5$. The first variable in a panel is indicated in black/solid, the second in red/dashed.

When the elasticity of substitution exceeds unity ($\eta > 1$), in contrast, then relative endowment changes induce weaker inverse price adjustments. The endowment effect dominates the price effect on the demand for means of payment in this case, and relatively more means of payment are allocated to purchases of the good that has become more abundant good. That is, when e_2/e_1 rises, p_1/p_2 rises less than proportionally and a_{11}/a_{12} falls.

[26]We set $i = 0.02$.

In the limiting case of $\eta \downarrow 0$, endowment differences translate into very sharp price and portfolio effects: When $e_2 < e_1$, $p_1 \to 0$ and $a_{11} \to 0$ as $\eta \downarrow 0$ (and vice versa). In the limiting case of $\eta \to \infty$, in contrast, relative endowment changes translate into negligible changes in relative marginal utilities and relative consumer prices. This does not imply that goods prices remain unchanged because in that case a_{11}/a_{12} would have to change proportionally (from Condition (29)), and leakage costs then would imply changes in q_1/q_2 (from Condition (30)). In fact, equilibrium requires that both p_1 and p_2 vary with the endowment change at unchanged relative prices while a_{11}/a_{12} responds inversely proportionally to the relative endowment change (from Condition (29)). That is, when $e_2 < e_1$, $p_1 \to p_2$ and $a_{11}/a_{12} \to e_1/e_2$ as $\eta \to \infty$ (and vice versa).

These findings differ from the implications of a conventional CIA setting, in which relative marginal utilities equal relative goods prices, while in our model they equal relative consumer prices. The presence of leakage costs introduces a wedge, $p_1/p_2 \neq q_1/q_2$, even if the structure of those leakage costs is symmetric ($\alpha_{11} = \alpha_{12}$; see Condition (30)). This discrepancy also translates into aggregate prices: The index P differs from the index Q. The quantitative differences subject to the baseline calibration are very minor, however, as the top-left panel of Figure 3 shows.

Asymmetries When the structural parameters τ_{1j}, α_{1j} or ζ_{1j} differ across commodities, the comparative statics with respect to i_1 or e_2 also exhibit additional asymmetries. As an example, Figure 5 in the Appendix illustrates the comparative statics with respect to i_1 when the baseline calibration is modified to feature $\tau_{11} = 0, \tau_{12} = 1$.

From Proposition 3, $\tau_{11} = 0$ implies that $\delta_{11} = 1$, while $\delta_{12} < 1$ if $i > 0$; see the middle-right panel of the figure. Relative to the symmetric case in Figure 2, this implies differences between a_{11} and a_{12}, φ_{11} and φ_{12}, ξ_1 and ξ_2, as well as goods prices. Asymmetries with respect to α_{1j} rather than τ_{1j} yield $\delta_{11} = \delta_{12}$ but differences between a_{11} and a_{12}, φ_{11} and φ_{12}, ξ_1 and ξ_2, as well as goods prices. The discrepancies between P and Q are of a similar magnitude as in the symmetric case.

4.2 Portfolio Effects

Next, we turn to the portfolio choice among means of payment and the implications for prices. We continue to focus on a BGP subject to Assumptions 1 and 2. Unlike in the previous subsection, we do not impose restrictions on the number of assets. Condition (26) determines $\{\delta_{sj}\}_{j\in\mathcal{J},s\in\mathcal{S}}$; $\{a_{s0}, \{a_{sj}\}_{j\in\mathcal{J}}\}_{s\in\mathcal{S}}$, $\{p_j\}_{j\in\mathcal{J}}$, and (λ, P, Q) in the initial period are determined by the peg $a_{10} = -1$, the definitions of the price indices as well as the equilibrium conditions

$$0 = a_{s0} + \sum_{j\in\mathcal{J}} a_{sj} \quad \forall s \in \mathcal{S}, \tag{31}$$

$$\sum_{s\in\mathcal{S}} \frac{a_{sj}}{\delta_{sj}} = p_j e_j \text{ s.t. (26)} \quad \forall j \in \mathcal{J}, \tag{32}$$

$$E^{-\sigma}(Je_j/E)^{-\frac{1}{\eta}} \leq p_j \lambda \left(1 + \frac{\alpha_{sj}}{-a_{10}} \frac{a_{sj}}{\delta_{sj}} - \frac{i_1}{I_1} \frac{1-\tau_{sj}}{2} \delta_{sj}\right), \; a_{sj} \geq 0 \quad \text{s.t. (26)} \quad \forall j \in \mathcal{J}, s \in \mathcal{S}. \tag{33}$$

The remaining equilibrium conditions determine the other variables such as $\{\xi_j\}_{j\in\mathcal{J}}$.

The portfolio choice tradeoffs are apparent from Condition (33). Consider a generic j purchased with means of payment s and 1. Combining the condition for the two means of payment and using Equation (31) then yields an instance of Condition (13),

$$\frac{1}{-a_{10}} \left(\alpha_{sj} \frac{a_{sj}}{\delta_{sj}} - \alpha_{1j} \frac{a_{1j}}{\delta_{1j}}\right) = \frac{i_1}{I_1} \left(\frac{1-\tau_{sj}}{2} \delta_{sj} - \frac{1-\tau_{1j}}{2} \delta_{1j}\right) \text{ s.t. (26).} \tag{34}$$

Suppose first that $\zeta_{sj} = \zeta_{1j}$ and $\tau_{sj} = \tau_{1j}$ such that $\delta_{sj} = \delta_{1j}$ and the right-hand side of Condition (34) collapses to zero. The portfolio composition a_{sj}/a_{1j} then reflects leakage costs, with stronger leakage causing lower portfolio shares.[27]

Other parameter differences introduce further effects. One transmission mechanism operates through holding periods and their effect on leakage costs, represented on the left-hand side of Condition (34). Ceteris paribus, a higher δ_{sj} (due to a higher ζ_{sj} or lower τ_{sj}) lowers leakage costs, inducing a relatively higher a_{sj}. The other mechanism operates through interest income. Ceteris paribus, a higher $(1-\tau_{sj})\delta_{sj}$ (due to a higher ζ_{sj} or lower τ_{sj}) increases the financial return on a_{sj} and incentivizes its use.

Proposition 10. Consider a BGP subject to Assumptions (1) and (2) with $a_{1j}, a_{sj} > 0$. Ceteris paribus, the portfolio composition a_{sj}/a_{1j} is decreasing in α_{sj} and τ_{sj} and increasing in ζ_{sj}.

Proof. The result follows from the derivations in the text. □

Turning to the implications for goods prices, suppose for simplicity that there is a single good, $J = 1$, and a_{10} constitutes the nominal anchor such that a_{11} is fixed. From Proposition 10, parameter changes affect velocities and the portfolio composition. The induced change in $\sum_{s\in\mathcal{S}} a_{sj}/\delta_{sj}$ translates into an equiproportionate change in p_j; see Equation (32). For example, ceteris paribus, an increase in α_{sj} reduces a_{sj}/a_{1j} (from Proposition 10) but has no effect on velocities, implying that p_1 falls. When $J > 1$, portfolio substitution occurs not only within goods but also across them.

Interest rate changes have ambiguous effects on a_{sj}/a_{1j}. Holding δ_{1j} and δ_{sj} constant, the right-hand side of Condition (34) increases in i_1 if and only if that right-hand side is

[27] The exact inverse relationship between α_{1j}/α_{sj} and a_{sj}/a_{1j} is a consequence of our assumption that leakage enters quadratically into ϕ; see Condition (27).

positive. However, δ_{1j} and δ_{sj} may also respond, and this can reverse the comparative statics. Moreover, different elasticities of δ_{1j} and δ_{sj} with respect to i_1 may flip the sign of the left-hand side of the condition. It is therefore possible that a_{sj}/a_{1j}, p_j, P and Q are nonmonotone in i_1.

Figure 4 offers an example of such nonmonotone comparative statics.[28] As interest rates rise, velocities also rise, particularly for the second means of payment, whose holding period can be shortened at lower cost. Ceteris paribus, this drives up a_{21}/δ_{21} relative to a_{11}/δ_{11}, and thus φ_{21} to φ_{11}, but since a_{21} falls, the total effects are nonmonotone. Prices reflect this. The ratio of the price indices P and Q rises by roughly half a percent.

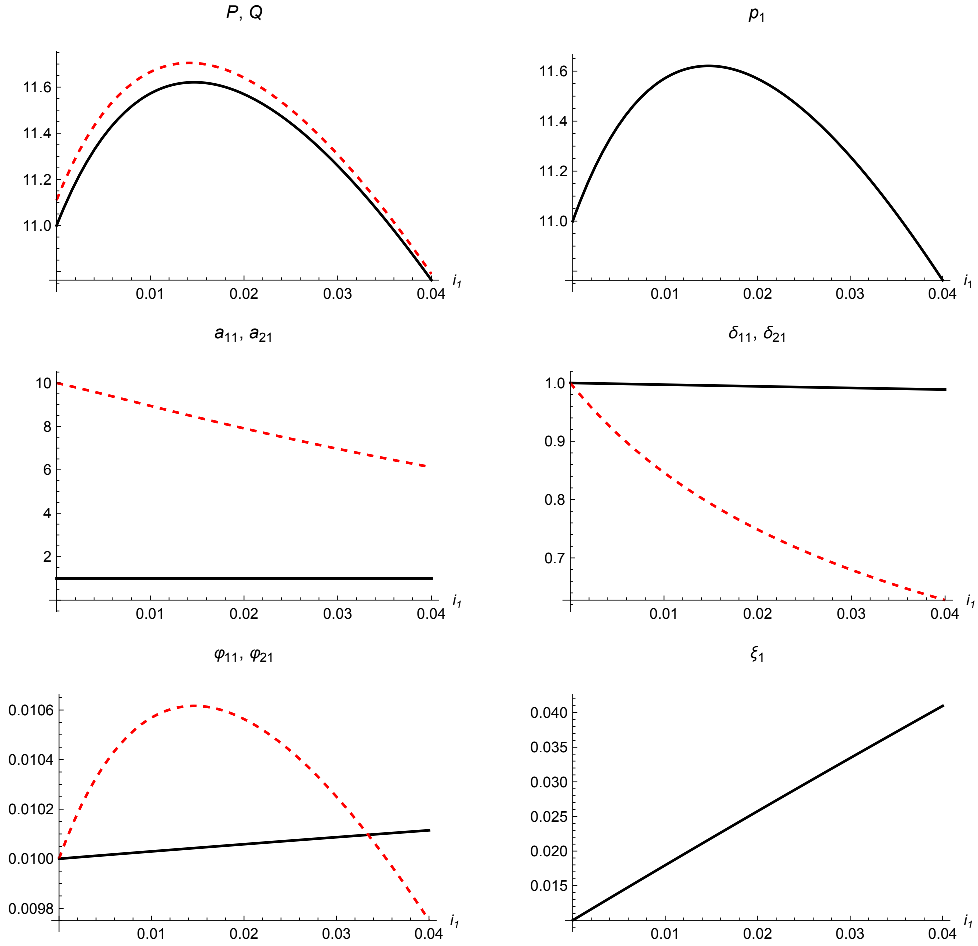

Figure 4: Nonmonotone comparative statics with respect to i_1: Multiple means of payment, single good, asymmetric $\alpha_{s1}, \tau_{s1}, \zeta_{s1}$. The first variable in a panel is indicated in black/solid, the second in red/dashed.

We summarize this finding as follows:

Result 2. Portfolio rebalancing of means of payment can give rise to nonmonotone comparative statics of prices with respect to interest rates.

[28] We set $\zeta_{11} = 1, \zeta_{21} = 0.01, \tau_{11} = 0.6, \tau_{21} = 0.4, \alpha_{11} = 0.01, \alpha_{21} = 0.001, e_1 = 1, \eta = 0.5, \sigma = 1, J = 1$.

Proof. The example establishes the result. □

5 Production

It is straightforward to introduce production into the model. Suppose that, rather than being endowed with e, households can produce the "endowment" at a utility cost $v(e)$ where v is strictly convex and satisfies $v(0) = 0$, $v_j \equiv \partial v(e)/\partial e_j \geq 0$, and $v_j|_{e_j=0} = 0$. The static objective of the household now reads $u(C) - v(e)$.

Except for the endogeneity of e, the constraints of the household's program are unchanged. Higher production e_j causes disutility but generates more liquidity, leakage and next-period financial wealth. Recall that $p_j e_j = \sum_{s\in\mathcal{S}} \bar{a}_{sj}/\bar{\delta}_{sj}$, and let $\bar{\sigma}_{sj}$ denote the share of the household's revenues from sales of good j that accrue in the form of means of payment s,

$$\bar{\sigma}_{\hat{s}j} \equiv \frac{\bar{a}_{\hat{s}j}/\bar{\delta}_{\hat{s}j}}{\sum_{s\in\mathcal{S}} \bar{a}_{sj}/\bar{\delta}_{sj}},$$

which the household takes as given. Using Conditions (2)–(5), the first-order condition with respect to e_j then reads

$$\begin{aligned} v_j &= \lambda p_j \sum_{s\in\mathcal{S}} \bar{\sigma}_{sj} \left\{ (\xi_j - \varphi_{sj})(1-\delta_{sj}) + \beta \mathbb{E}\left[\frac{\lambda'}{\lambda} (1 + \delta_{sj}\Omega_{sj}) \,|w,k,x \right] \right\} \quad \forall j \in \mathcal{J} \\ &= \lambda p_j \sum_{s\in\mathcal{S}} \bar{\sigma}_{sj} \left\{ -\frac{\partial \tilde{z}_{sj}(\delta_{sj};\bar{\delta})}{\partial \delta_{sj}} (1-\delta_{sj}) + \beta \mathbb{E}\left[\frac{\lambda'}{\lambda} (1 + \Omega_{sj}) \,|w,k,x \right] \right\} \quad \forall j \in \mathcal{J}, \end{aligned} \tag{35}$$

where we let $\Omega_{sj} \equiv (1-\tau_{sj}) i_s (1-\delta_{sj}/2)$ and the second equality holds if δ_{sj} is interior such that Condition (9) holds with equality. In equilibrium, the $\bar{\sigma}_{\hat{s}j}$ shares conform with optimal payment choices.

In the standard CIA setting, revenues are not available for re-spending in the same period and $\tau_{sj} = 1$. The first-order condition then collapses to

$$v_j = p_j \beta \mathbb{E}\left[\lambda' | w,k,x\right] \quad \forall j \in \mathcal{J}$$

and only goods prices and the value of financial wealth in the subsequent period shape production incentives. In our more general setting, features of the payment sector also affect production decisions, possibly in asymmetric ways. This is because revenues generate liquidity (but also leakage costs), which saves the household costs of reducing velocity, and since revenues generate post-intermediation margin interest income.

Heterogeneous intermediation margins or costs of adjusting velocity imply that the right-hand side of Condition (35) differs across goods whenever interest rates are strictly positive. Intuitively, different liquidity benefits net of leakage costs as well as different financial returns of means of payment invite the production of those goods that generate revenues in the most attractive media of exchange.

6 Conclusion

We introduced a medium-of-exchange portfolio choice and a choice of velocities into a multi-good model of payments that is inspired by the classical CIA framework. This yields a series of new theoretical insights: Transacting with means of payment is not only costly because of foregone interest; even under the Friedman (1969) rule, liquidity constraints may bind. Agents trade off two types of costs—foregone interest and, in the terminology of this paper, leakage costs; this trade off can be represented in the form of a consumption-based "means-of-payment pricing model." Endogenous and asset-specific velocities decouple means-of-payment stocks from their importance as payment instruments. This gives rise to a nontrivial relationship between interest rates, velocities, and shadow values of the good-specific liquidity constraints; scarcer liquidity need not be associated with higher velocities because it can be cheaper to generate liquidity by exchanging pure stores of value into means of payment rather than increasing velocities.

Our model also has important normative and policy implications. Regarding the former, velocities and payment instruments generate positive externalities because aggregate liquidity gains exceed private gains and perceived private interest gains from holding on to means of payment cancel out with corresponding losses of other agents. While "money is neutral," interest rate policy is not. Most important, there are important differences between the consumer price index as it would be computed in a standard CIA setting, that index in our more realistic environment, and the most comprehensive index in our environment.

A central bank that targets "the" consumer price index must not only take a stand on what exact concept it has in mind—an aggregate of goods prices vs. of the costs of acquiring the goods—but it must also recognize that the difference between the two index measures is endogenous; it varies over the business cycle and with monetary policy and responds to structural change in the payment sector. Independent of how aggregate prices are measured, relative prices—and with endogenous production, the allocation of goods as well—respond to growth, inflation, nominal interest rates and structural change in the payment sector, even in the absence of nominal or real rigidities.

Our model setup is purposefully general, capturing, e.g., multiple microfoundations for costly velocity choices. An important avenue for future research is to impose more structure and to calibrate the model in order to derive reliable quantitative implications. Our examples based on plausible parameter values have shown that the quantitative implications are likely to be significant for monetary policy makers, statistical agencies and other parties affected by the impact of payments on prices.

A Additional Figures

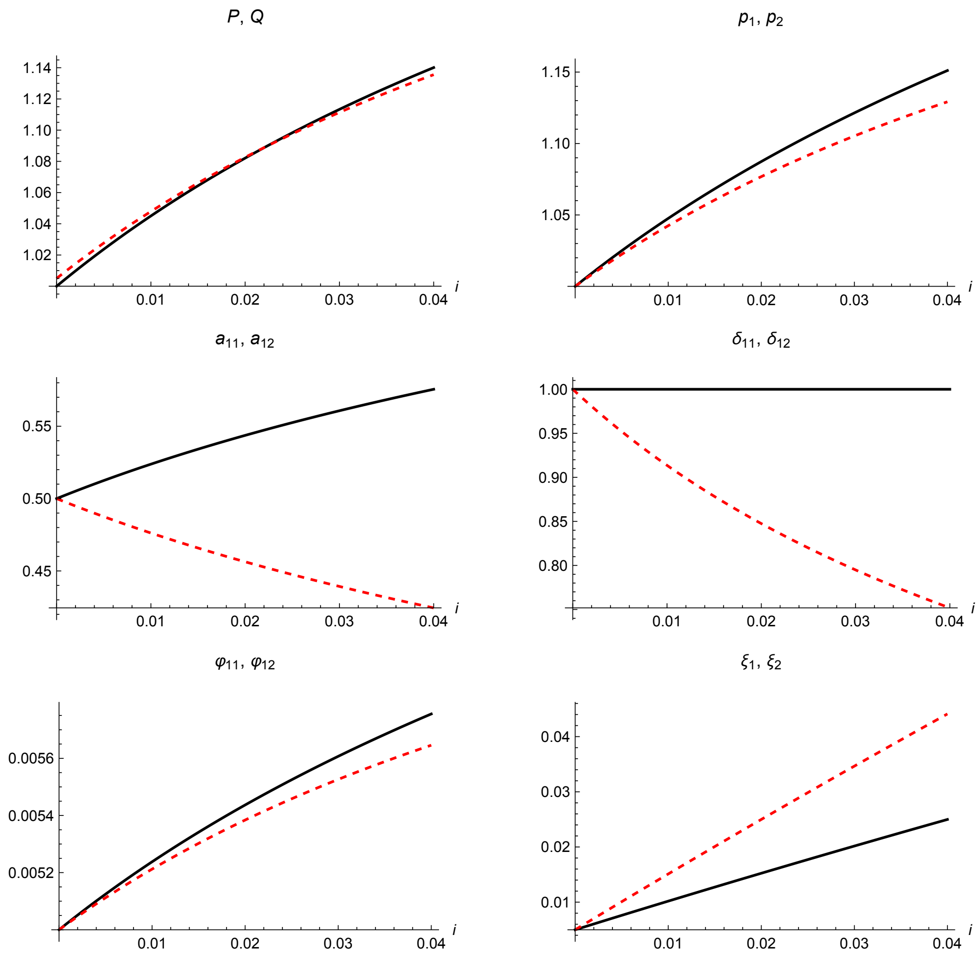

Figure 5: Comparative statics with respect to i_1: Single means of payment, symmetric α_{1j}, ζ_{1j}, asymmetric τ_{1j}. The first variable in a panel is indicated in black/solid, the second in red/dashed.

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